RELIABILITY-BASED DESIGN OPTIMIZATION OF V-BELT DRIVES USING INVERSE RELIABILITY ANALYSIS AND MONTE CARLO SIMULATION

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Abstract - V-belt drives are widely used in industrial and agricultural machinery, yet conventional deterministic design often ignores uncertainties in materials, operating conditions, and tolerances. This study proposes a Reliability-Based Design Optimization (RBDO) framework for V-belt drives, integrating Inverse Reliability Analysis with a Genetic Algorithm and validated by Monte Carlo Simulation. Design variables include driver and driven pulley diameters, belt length, and number of belts, with objectives of minimizing mass and maximizing transmitted power under a reliability target of R* = 99.9%. Results indicate that RBDO reduces system mass by 18.2% and increases power from 4.75 kW to 5.65 kW compared to deterministic optimization, while ensuring reliability. The proposed framework provides a practical tool for lightweight, reliable, and high-performance V-belt drive design.

Key words - V-belt drive; Reliability-based design optimization; Inverse reliability analysis; Monte Carlo simulation

1. Introduction

V-belt drives are among the most common mechanical transmission systems, widely employed in industrial equipment, agricultural machinery, and domestic devices. Traditional studies have primarily focused on the design and performance evaluation of V-belt drives based on fundamental parameters such as transmitted power, speed ratio, belt tension, and service life [1, 2]. Classical references and technical standards generally provide deterministic design formulas to ensure reasonable sizing and safe operation. However, these methods assume that design parameters are fixed values, without accounting for variations caused by operating conditions, material properties, or manufacturing tolerances. As a result, deterministic designs may lead to over-conservative or, conversely, unsafe solutions [3 - 5].

In recent years, several studies have investigated the optimization of V-belt drive design to improve efficiency and reduce weight. Various optimization algorithms, such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and gradient-based methods, have been applied to determine optimal design parameters [6 - 9]. These approaches have demonstrated significant potential in enhancing performance and saving material. Nevertheless, most of these works remain within the scope of deterministic optimization, where only the mean values of design variables are considered. Consequently, such designs may lack reliability under uncertain operating conditions.

With the advancement of computational methods, Reliability-Based Design Optimization (RBDO) has been extensively studied and applied in diverse engineering fields, including mechanical, structural, and aerospace design [10 - 13]. Widely used reliability assessment methods include the First-Order Reliability Method (FORM), the Second-Order Reliability Method (SORM), and sampling-based techniques such as Monte Carlo Simulation (MCS). While FORM and SORM are computationally efficient, their accuracy is limited when the limit-state function exhibits strong nonlinearity. Conversely, MCS provides highly accurate results but requires significant computational effort, particularly when coupled with iterative optimization processes [4, 6, 14, 15].

To overcome these limitations, recent studies have introduced Inverse Reliability Analysis (IRA) as an effective alternative. IRA transforms probabilistic constraints into equivalent deterministic constraints in the standard normal space, thereby significantly reducing computational cost while maintaining high accuracy [16 - 18]. IRA has been successfully applied in steel structure design, mechanical components subjected to random loads, and nonlinear dynamic systems. However, a comprehensive review of the literature reveals that the application of IRA to V-belt drive design and optimization remains limited. In particular, no systematic study has combined IRA with MCS to simultaneously optimize, validate, and conduct sensitivity analysis for belt drive systems.

From the above discussion, it can be concluded that although RBDO methods have been widely applied in various engineering domains, there is still a research gap in the context of V-belt drive design. Specifically, current studies have not: (1) Developed a dedicated RBDO framework for V-belt drives that incorporates key uncertainties such as friction coefficient, belt tension; (2) Applied IRA to simplify the optimization process while preserving accuracy; (3) Performed a systematic comparison between deterministic optimization and RBDO approaches and (4) Integrated MCS verification to evaluate the robustness of optimized designs and conduct sensitivity analysis.

Therefore, this research is conducted to address these limitations and to extend the application of RBDO in the field of mechanical power transmission systems.

2. Theoretical Background

2.1. V-belt drives

2.1.1. Belt drive parameters

Belt drives are commonly used as the first transmission stage in machinery due to their flexibility and vibration damping capability. Power and torque are transmitted through frictional contact between the belt and the driving and driven pulleys. The key design inputs include transmitted power, motor speed, and output speed (or desired transmission ratio). Based on these parameters, the geometric design of the belt drive, such as pulley diameters, center distance, wrap angle, and belt length, is determined, as illustrated in Figure 1 [1].

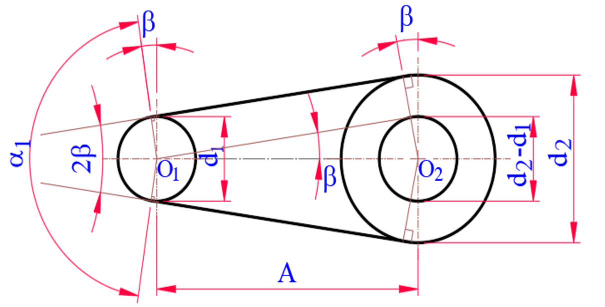

Figure 1. *Geometrical parameters of the V-belt drive*

Center distance between the two pulleys:

$$A=\frac{1}{4}\left(L-\frac{\pi(d_1+d_2)}{2}\right)+\frac{1}{4}\sqrt{\left(L-\frac{\pi(d_1+d_2)}{2}\right)^2-2(d_2-d_1)^2} \quad (1)$$

Where, L is the belt length (mm), d_1 is the driver pulley diameter, d_2 is the driven pulley diameter.

Belt wrap angle on the driver pulley:

$$\alpha_1=180-57\frac{(d_2-d_1)}{A} \quad (2)$$

Belt pulley width:

$$B=(z-1)e+2f \quad (3)$$

Where, z is the number of V-belts, e is the distance between two belt grooves, and f is the distance from the belt groove to the pulley edge.

Maximum calculated stress (including tensile and bending stress) occurring in the belt at the point where the belt first contacts the driver pulley:

$$\sigma_{max}=\frac{F}{S}\frac{e^{f\alpha_1}}{e^{f\alpha_1}-1}+\sigma_v+\sigma_u \quad (4)$$

Where, F is the circumferential force, S is the belt cross-sectional area; σ_u is the bending stress in the belt on the smaller pulley, σ_v is the stress caused by the centrifugal force, α is the slip angle, f is the coefficient of friction between the belt and pulley.

Limit stress according to strength:

$$\sigma_{lim}=\sigma_{lim0}\sqrt[m]{\frac{N_0}{N}} \quad (5)$$

Where, σ_{lim} is the fatigue limit corresponding to the reference number of cycles N_0; N is the total number of operating cycles of the belt; m is the fatigue strength exponent.

2.1.2. Output power of the belt drive

Based on the belt tension equilibrium relationship, considering the effects of slip, centrifugal force, and adhesion conditions, the output power can be expressed as a nonlinear function of the design variables (driver and driven pulley radii) along with parameters such as the friction coefficient, groove angle, initial tension, and wrap angle. According to [5], the general expression for the output power is written as follows:

$$P_{out}=P(d_1,d_2)=2v_b\left(F_0-\rho v_b^2\right)(1-\xi)\left(\frac{1-e^{\frac{f}{\sin\gamma}\alpha_1}}{1+e^{\frac{f}{\sin\gamma}\alpha_1}}\right) \quad (6)$$

Where: f is the friction coefficient, γ is the half-groove angle of the V-belt, α_1 is the wrap angle, F_0 is the initial tension, v_b is the belt speed, and $\rho v^2{}_2$ represents the effect of the centrifugal force; ξ is the slip coefficient.

Assuming that f, γ, F_0, and vb are constants, then $\alpha_1=\alpha_1(d_1,d_2)$ plays a decisive role in power transmission and is used as the basis for formulating the objective function in the optimization problem.

2.1.3. Limit state function of the belt drive

The limit state function serves as the basis for evaluating the reliability of a structure, defining the boundary between safe operation and failure. For a belt drive, the limit state function is typically constructed based on the strength criterion, that is, by comparing the load-carrying capacity with the applied load. A general form of the limit state function for the belt drive under strength conditions can be expressed as follows:

$$g(X)=\sigma_{lim}-\sigma_{max}=\sigma_{lim0}\sqrt[m]{\frac{N_0}{N}}-\left(\frac{F}{S}\frac{e^{\frac{f}{\sin\gamma}\alpha_1}}{e^{\frac{f}{\sin\gamma}\alpha_1}-1}+\sigma_v+\sigma_u\right) \quad (7)$$

Where, g(X) > 0 indicates that the belt drive operates safely, while g(X) ≤ 0 indicates that the belt drive is in a failure state.

2.2. Reliability-based design optimization (RBDO) framework

2.2.1. RBDO problem formulation

The general form of the reliability-based design optimization (RBDO) problem is expressed as:

Objective function: $\min f(X)$

Subject to:

$$\begin{cases} h_k(X,m_p)\le 0 & k=1,\ 2,\ \dots\ n_h \\ X_i^l\le X_i\le X_i^n & i=1,\ 2,\ \dots\ n \\ P\left(g_j(X,p)\ge 0\right)\ge R_j^* & j=1,\ 2,\ \dots\ n_g \end{cases} \quad (8)$$

Where, $X=\{X_1, X_2, \dots X_n\}$ is the design variable (deterministic or random), **p** represents the random parameter vector with mean value m_p, $f(\mathbf{X})$ is the objective function, $h_k(X,m_p)$ denotes deterministic constraints, $g_j(\mathbf{X},\mathbf{p})$ is the limit state function, n are the numbers of design variables, n_g is the numbers of probability constraints, R_j^* is the desired reliability, X_i^l, X_i^n are the lower and upper limits of the design variable.

The inverse reliability analysis method combined with the

Genetic Algorithm (GA) is applied to solve the reliability-based design optimization (RBDO) problem for Belt drives.

2.2.2. Inverse reliability analysis (IRA) in the standard normal space

a. Transformation into the standard normal space

To perform the IRA, all random variables must first be transformed into the standard normal space. In this study, the random variables are assumed to be statistically independent. Therefore, the mapping from the physical space (X-space), as illustrated in Figure 2a, to the standard normal space (U-space), shown in Figure 2b, is performed using the marginal cumulative distribution functions (CDFs) of each variable, according to:

$$U_i = \Phi^{-1}\left(F_{X_1}(x_i)\right) \quad where \quad X_i = m_{X_i} + U_i S_{X_i} \tag{9}$$

Where, $F_{Xi}(\cdot)$ is the cumulative distribution function of the random variable X_i, $\Phi^{-1}(\cdot)$ is the inverse of the standard normal CDF, m_{xi} is the mean value of the variable X_i, and S_{xi} is the standard deviation of the variable X_i.

This transformation enables the reliability analysis and the most probable point (MPP) search to be conducted in a normalized and uncorrelated space, which simplifies the computation and enhances the numerical stability of the algorithm. After the transformation, the MPP search is performed in the U-space, where the limit state surface g(U) = 0 is identified, and the shortest distance from the origin to this surface represents the reliability index β of the system.

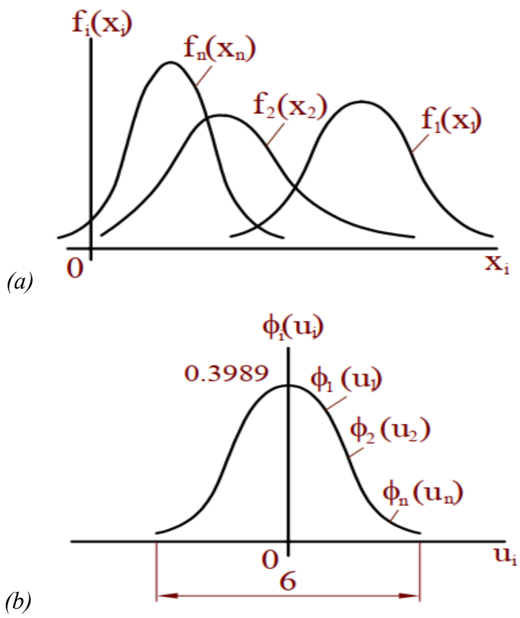

Figure 2. *Distribution density function:a) Space X and b) Space U*

b. Inverse reliability analysis (IRA)

Direct estimation of the failure probability using Monte Carlo Simulation (MCS) often requires prohibitive computational costs due to the need for a large number of samples, especially when integrated into iterative optimization processes. To overcome this limitation, the inverse reliability analysis (IRA) method is employed as an efficient alternative.

In IRA, instead of directly evaluating the failure probability P = P(g(X) ≥ 0), a target reliability index β (equivalent to a required reliability level R*=Φ(β)) is prescribed. The task is then to determine the Most Probable Point (MPP) in the standard normal space, as illustrated in Figure 3. The procedure to find MPP according to the inverse reliability analysis method is illustrated in Figure 4.

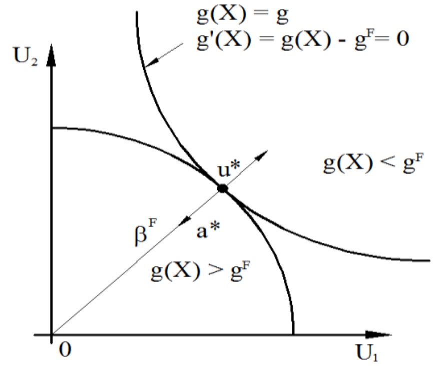

Figure 3. *Most Probable Point (MPP)* ***u**** *in the standard normal space*

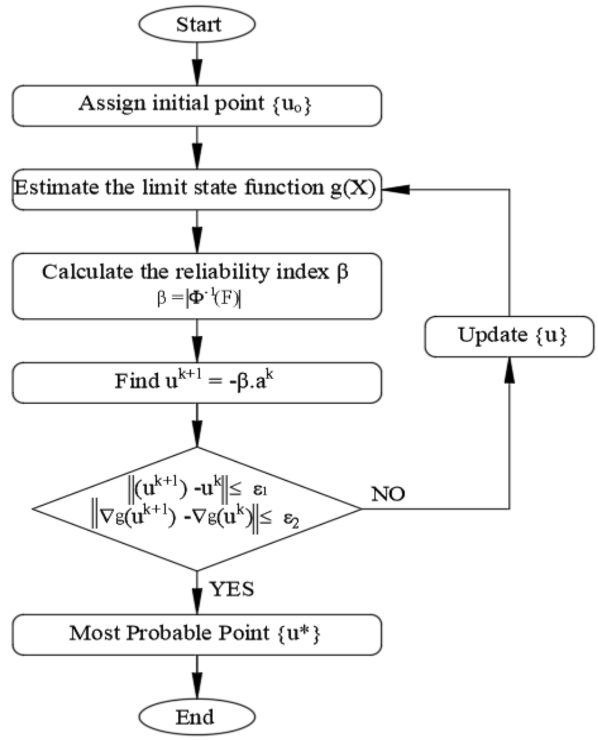

Figure 4. *Procedure of the inverse reliability analysis method to determine the Most Probable Point (MPP) in the standard normal space*

Specifically, the MPP is determined using the iterative Hasofer–Lind–Rackwitz–Fiessler (HL–RF) algorithm, with the following convergence conditions applied to terminate the iteration process:

$$\left\|u^{k+1} - u^k\right\| \le \varepsilon_1 \text{ and } \left\|\nabla g\left(u^{k+1}\right) - \nabla g\left(u^k\right)\right\| \le \varepsilon_2$$

Where, the values are set as $\varepsilon_1 = \varepsilon_2 = 10^{-4}$.

After determining the Most Probable Point **u*** the value of the limit state function g^F is calculated using the following expression:

$$g^F = g\left(u^*\right) \tag{10}$$

After determining g^F the RBDO problem turns into a deterministic optimization problem and the probabilistic constraints $P(g_j(\mathbf{X}, \mathbf{p}) \geq 0)) \geq R_j^*$ of the RBDO problem (8) become $g_j(\mathbf{X}, \mathbf{p}) \geq g^F$.

IRA is integrated into the optimization framework of V-belt drives to convert the reliability constraint associated with the belt strength limit state into a deterministic constraint. This allows the Genetic Algorithm (GA) to efficiently search for optimal design parameters while ensuring that the required reliability level (R* = 99.9%) is satisfied.

In this study, the reliability-based design optimization (RBDO) of V-belt drives is performed by integrating inverse reliability analysis (IRA) and Genetic Algorithm (GA). The overall RBDO framework is presented in Figure 5.

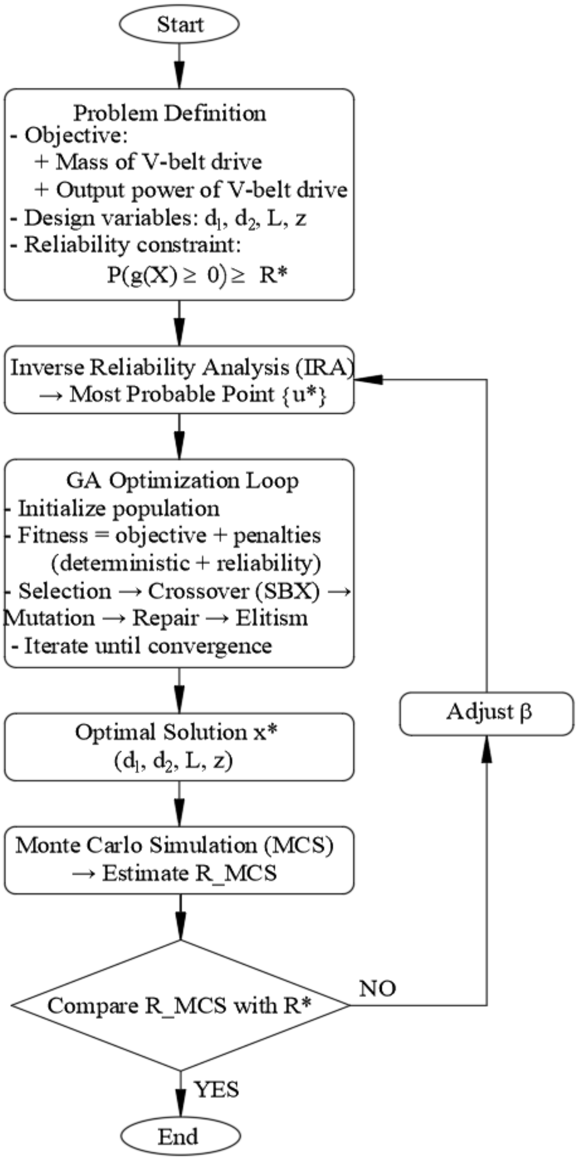

Figure 5. *Overall framework of the proposed reliability-based design optimization (RBDO) approach integrating inverse reliability analysis (IRA) and GA*

To solve the nonlinear RBDO problem with probabilistic constraints, the Genetic Algorithm (GA) is employed to find the optimal solution. The GA was configured with a population size of 100, a maximum of 200 generations, single-point crossover (p_c=0.8), uniform mutation (p_m=0.05), and elitism retaining the top 1–2 individuals per generation. The algorithm was terminated when no improvement in the best fitness was observed for 25 consecutive generations or when 200 generations were reached. To ensure reproducibility, a fixed random seed (seed = 2500) was used. To assess optimization stability, the GA was executed 30 independent times; the mean and standard deviation of the best fitness and design variables are reported.

2.3. Variance-based global sensitivity analysis

In this study, the variance-based global sensitivity analysis was performed using the Sobol method. This formulation follows the classical definitions introduced by Sobol [19] and subsequently refined by Saltelli et al. [20] to enable the practical computation of variance-based sensitivity indices.

Let $Y=f(X_1, X_2,\ldots, X_k)$ be the model response, where X_i are independent input variables with finite variance. The total variance of Y can be decomposed as:

$$Var(Y) = \sum_{1}^{k} V_i + \sum_{1}^{k}\sum_{i<j}^{k} V_{ij} + \cdots + V_{12\cdots k} \tag{11}$$

where $V_i = Var_{X_i}[E_{X_{\sim i}}(Y|X_i)]$ represents the contribution of the i^{th} variable alone, and higher-order terms V_{ij}, V_{ijk}, … represent interaction effects.

The first-order Sobol index quantifies the main effect of input X_i:

$$S_i = \frac{V_i}{Var(Y)} = \frac{Var_{X_i}\left[E_{X_{\sim i}}(Y|X_i)\right]}{Var(Y)} \tag{12}$$

where X_i is the i-th input factor and $X_{\sim i}$ denotes the matrix of all factors except X_i.

The total-effect Sobol index, representing the overall contribution of X_i including its interactions, is defined as:

$$S_{T_i} = 1 - \frac{Var_{X_{\sim i}}\left[E_{X_i}(Y|X_{\sim i})\right]}{Var(Y)} \tag{13}$$

In this work, both S_i and S_{Ti} were estimated using the Saltelli sampling scheme [20], which provides an efficient Monte Carlo estimator for both indices using N×(k+2) model evaluations. To ensure convergence, tests were conducted with N=5,000, 10,000 and 15,000 samples.

3. Numerical Example

A representative simulation example of a V-belt drive in a conveyor transmission system is presented in Figure 6 to illustrate the proposed optimization model. The system consists of a motor (1), V-belt drive (2), double-stage gear reducer (3), chain drive (4), and conveyor system (5). The V-belt drive has a rated power of P = 6.02 kW and input speed n1 = 968 rpm.

Objective function: The purpose of design optimization is to obtain either the minimum or maximum value of the objective function. For belt drive design optimization, the mass and the output power of the belt drive are selected as the objective functions:

$$\begin{cases} m = \rho_1 B \dfrac{\pi}{4}\left(d_1^2 + d_2^2\right) + \rho_2 z S L & \rightarrow \min \\ P_{out} = 2 v_b \left(F_0 - \rho v_b^2\right)\left(1-\xi\right)\left(\dfrac{1 - e^{\frac{f}{\sin\gamma}\alpha_1}}{1 + e^{\frac{f}{\sin\gamma}\alpha_1}}\right) & \rightarrow \max \end{cases} \quad (14)$$

Where: ρ_1 is the density of the pulley material; ρ_2 is the density of the belt material; ρ is the mass per unit length of the belt; B is the pulley width; S is the cross-sectional area of the belt; z is the number of belts.

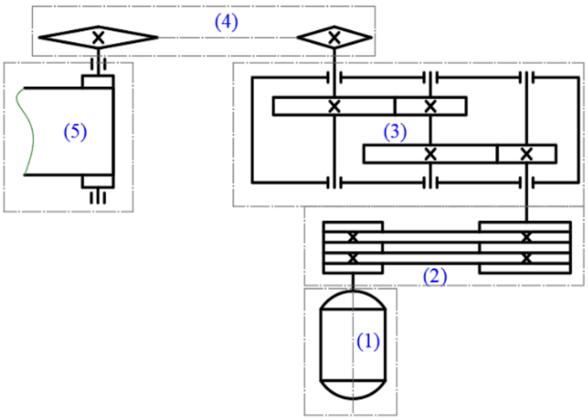

Figure 6. *Illustration of a conveyor transmission system used as a numerical example*

(1) motor, (2) V-belt drive, (3) gear reducer, (4) chain drive, and (5) conveyor belt.

Design variables: From the objective function of mass, four main design variables that directly affect the mass of the belt drive can be identified, including the diameter of the smaller pulley d_1, the diameter of the larger pulley d_2, the center distance between the two pulleys A, and the number of belts z. However, the belt length L is a discrete value selected from standard tables for each specific belt type. Therefore, to facilitate the optimization process and ensure consistency with practical conditions, the belt length L can be used as a substitute variable for the center distance A. With this approach, the set of design variables employed in the optimization problem includes:

$$X = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} d_1 & d_2 & L & z \end{bmatrix}^T$$

Subject to:

1. Driver pulley diameter: $140 \le d_1 \le 180$;

2. Driven pulley diameter: $180 \le d_2 \le 450$;

2. Belt length L: $1600 \le L \le 2800$;

3. Number of belts z: $1 \le z \le 6$;

4. The center distance between the two pulleys must satisfy the condition: $0.7(d_1 + d_2) \le A \le 2(d_1 + d_2)$.

Accordingly, the constraint function can be expressed as:

$$h_1(x) = 0.7(d_1 + d_2) - \left[\frac{1}{4}\left(L - \frac{\pi(d_1+d_2)}{2}\right) + \frac{1}{4}\sqrt{\left(L - \frac{\pi(d_1+d_2)}{2}\right)^2 - 2(d_2 - d_1)^2}\right] \le 0$$

$$h_2(x) = \frac{1}{4}\left(L - \frac{\pi(d_1+d_2)}{2}\right) + \frac{1}{4}\sqrt{\left(L - \frac{\pi(d_1+d_2)}{2}\right)^2 - 2(d_2 - d_1)^2} - 2(d_1 + d_2) \le 0$$

5. Belt speed: $5 \text{ m/s} \le V \le 35 \text{ m/s}$:

$$h_3(x) = \frac{\pi d_1 n}{60000} - 35 \le 0$$

$$h_4(x) = 5 - \frac{\pi d_1 n}{60000} \le 0$$

6. Minimum wrap angle: $\alpha_1(d_1, d_2) \ge \alpha_{min}$:

$$180 - 57\frac{(d_2 - d_1)}{A} \ge 120^\circ$$

7. Probability of the limit state function under the strength condition: $P(g(X) > 0) \ge R^*$

$$P(g(X) > 0) = $$

$$= P\left[\left(\sigma_{\lim 0}\sqrt[m]{\frac{N_0}{N}} - \left(\frac{F}{S}\frac{e^{\frac{f}{\sin\gamma}\alpha_1}}{e^{\frac{f}{\sin\gamma}\alpha_1} - 1} + \sigma_v + \sigma_u\right)\right) > 0\right] \ge R^*$$

To ensure that the belt drive does not fail during operation, its structure must achieve a minimum reliability level of R*=99.9%. With this objective, the design process is carried out based on the Reliability-Based Design Optimization (RBDO) approach, in which the required reliability R*=99.9% is imposed as the main constraint in the optimization process.

In summary, the reliability-based design optimization problem for the belt drive can be formulated as follows:

- Design variables: d_1, d_2, L, z

- Objective function:

$$\begin{cases} m = \rho_1 B \dfrac{\pi}{4}\left(d_1^2 + d_2^2\right) + \rho_2 z S L & \rightarrow \min \\ P_{out} = 2 v_b \left(F_0 - \rho v_b^2\right)\left(1-\xi\right)\left(\dfrac{1 - e^{\frac{f}{\sin\gamma}\alpha_1}}{1 + e^{\frac{f}{\sin\gamma}\alpha_1}}\right) & \rightarrow \max \end{cases}$$

(13)

- Subject to:

$$\begin{cases} 140 \le d_1 \le 180; \quad 180 \le d_2 \le 450; \quad 1600 \le L \le 2800 \\ 1 \le z \le 6 \\ h_1(x) \le 0; \quad h_2(x) \le 0; \quad h_3(x) \le 0; \quad h_4(x) \le 0 \\ h_5(x) = 180 - 57\dfrac{(d_2 - d_1)}{A} \ge 120^\circ \\ P(g(X) > 0) = P((\sigma_{\lim} - \sigma_{\max}) > 0) = \\ = P\left[\left(\sigma_{\lim 0}\sqrt[m]{\dfrac{N_0}{N}} - \left(\dfrac{F}{S}\dfrac{e^{\frac{f}{\sin\gamma}\alpha_1}}{e^{\frac{f}{\sin\gamma}\alpha_1} - 1} + \sigma_v + \sigma_u\right)\right) > 0\right] \ge R^* \end{cases}$$

Where: the calculated stress σ_v, the circumferential force F, the fatigue limit σ_{lim0}, … are random variables whose values are given in Table 1.

In the reliability-based design optimization (RBDO) process, several parameters of the V-belt drive are considered as random variables to capture the uncertainties arising from material variability, manufacturing tolerances, and operating conditions. These random variables include the belt fatigue limit, friction coefficient, belt tension, and geometric parameters, among others. Their statistical properties, namely mean values and standard deviations, are summarized in Table 1, which serves as the input data for the reliability analysis and subsequent optimization procedure.

Table 1. *Statistical properties of the design parameters*

Random variable	Unit	Mean value	Standard deviation
Belt tension force, F	N	660	66
Fatigue limit, σ_{lim0}	MPa	12	0.6
Elastic modulus of belt material, E	MPa	100	10
Centrifugal stress, σ_v	MPa	0.1	0.01
Fatigue strength exponent, m	–	8	0.8
Friction coefficient between belt and pulley, f	–	0.3	0.03
Belt cross-sectional area, S	mm²	138	13.8
Driver pulley diameter, d_1	mm	$\overline{d}_1$	$0.1\overline{d}_1$
Driven pulley diameter, d_2	mm	$\overline{d}_2$	$0.1\overline{d}_2$
Belt length, L	mm	$\overline{L}$	$0.167\overline{L}$
Reference number of cycles, N_0	cycle	10^7	-
Total number of operating cycles, N	cycles	$2.8\text{x}10^7$	-

In the reliability analysis, all random variables listed in Table 1 are assumed to follow normal probability distributions, consistent with previous studies [3, 5, 14]. Although possible correlations may exist among certain parameters, such as between the belt tension force and the friction coefficient, these correlations are neglected in this study to simplify the model. The random variables are therefore considered statistically independent, which is a reasonable assumption since each parameter is primarily influenced by distinct physical sources of uncertainty.

4. Results and Discussion

4.1. Optimal design results

The bi-objective optimization problem is reformulated using a weighted-sum method to balance system mass and transmitted power. The equivalent scalar objective function is expressed as

$$\min f(X) = w_1 \frac{M(X)}{M_0} - w_2 \frac{P(X)}{P_0}$$

Where, M_0 and P_0 are the reference values obtained from the deterministic baseline design, and w_1, w_2 are the weighting coefficients satisfying $w_1+w_2=1$.

In this study, M_0 =37.82 kg and P_0 = 4.75 kW correspond to the mass and transmitted power obtained from the deterministic optimization solution, which are used as reference values for normalization. This allows both objectives to be dimensionless and comparable in the weighted-sum formulation.

All optimization and reliability analyses were implemented in MATLAB R2023b using a combination of the Global Optimization Toolbox and user-defined MATLAB functions.

Table 2 presents the optimal design results of the V-belt drive obtained by two approaches: deterministic optimization (DO) and reliability-based design optimization (RBDO). The design variables include the driver pulley diameter (d_1), driven pulley diameter (d_2), belt length (L), and number of belts (z). The results indicate that the RBDO design achieves a significantly lower system mass (30.93 kg) compared to the deterministic design (37.82 kg), corresponding to a reduction of approximately 18.2%. In terms of reliability, the DO model yields R_{D0}= 0.99625 ± 0.00012, which does not satisfy the target reliability level of R* = 0.999. In contrast, the RBDO approach ensures R_{RBDO} = 0.99975 ± 0.000031, confirming that the reliability constraint is fully satisfied. These results indicate that the RBDO framework not only guarantees the desired reliability but also produces a lighter and more efficient belt drive design.

Specifically, the RBDO design delivers 5.65 kW compared to 4.75 kW in DO, highlighting that accounting for uncertainties not only improves safety but also enhances performance.

Table 2. *Optimal design results of the V-belt drive*

Design variables	Unit	Method	
		DO	RBDO (R* = 99.9%)
d_1	mm	149.06	168.62
d_2	mm	349.07	298.25
L	mm	1625.6	1614.3
z	-	2	2
Mass	kg	37.82	30.93
Output power	kW	4.75	5.65
Reliability	%	99.627	99.975

A sensitivity study was performed by varying w_1 in the range [0.3, 0.7], while maintaining $w_2=1-w_1$. The optimal solutions obtained for different weight combinations exhibit smooth variation in both mass and transmitted power, indicating stable convergence of the Genetic Algorithm (GA). The selected RBDO solution ($w_1 = w_2 = 0.5$) lies near the center of the Pareto front, confirming its robustness and balanced performance between lightweight design and high transmission power. Representative Pareto-optimal designs are summarized in Table 3 to illustrate the trade-off between these competing objectives.

The Pareto front clearly demonstrates the trade-off between mass and transmitted power under the reliability constraint, as shown in Table 3. The lightweight-favored design (Solution A) achieves the minimum mass but at the cost of reduced power transmission, while the power-favored design (Solution C) increases power output at the expense of a heavier system.

***Table 3.** Pareto-optimal designs obtained from GA*

Design / Parameters	Solution A	Solution B (RBDO)	Solution C
w_1	0.7	0.5	0.3
w_2	0.3	0.5	0.7
Mass (kg)	28.85	30.93	33.50
Power (kW)	5.45	5.65	6.15
Reliability	0.99975	0.99975	0.99975
Trade-off Type	Lightweight-favored	Balanced	Power-favored

One notable advantage of the RBDO approach is the reduction in the total mass of the V-belt drive while maintaining the required reliability. Compared to the initial design, the mass is reduced by approximately 18.2%, which contributes to a lighter transmission system, reduced load on bearings, material savings, and improved economic efficiency. Furthermore, the increase in transmitted power indicates higher operational effectiveness. Thus, RBDO not only provides a safe design but also optimizes both performance and cost.

4.2. Reliability Verification using Monte Carlo Simulation

Table 4 summarizes the key statistical descriptors of the limit-state function g(X) obtained from Monte Carlo Simulation with N = 10^6 samples. The mean value $m_{g(X)} = 2.378$ and standard deviation $S_{g(X)} = 0.647$ indicate a positive and relatively stable safety margin. The estimated reliability from MCS reaches $R_{RBDO} = 0.99975$, which is almost identical to the target reliability R*=99.9%. This confirms that the RBDO solution strictly satisfies the probabilistic constraint and validates the effectiveness of IRA as an efficient surrogate for direct MCS during optimization.

***Table 4.** Result of Monte Carlo simulation*

Simulation samples N	Mean value $m_{g(X)}$	Standard deviation $S_{g(X)}$	Reliability R_{RDBO}
10^6	2.378	0.647	0.99975

Figure 7 provides a visual illustration of the random distribution of fatigue limit σ_{lim}, maximum stress σ_{max} and the limit-state function g(X)). The results show that for the RBDO design, nearly all samples remain in the safe domain (g(X) > 0). The histogram of g(X) exhibits a near-Gaussian distribution with skewness close to zero and kurtosis around three, indicating negligible tail effects and a very low risk of extreme unsafe scenarios.

Monte Carlo validation was performed with N=10^6 samples. The estimated reliability for the RBDO design was $R_{RDBO} = 0.99975$ with a 95% confidence interval of [0.99972, 0.99978] (normal approximation). Convergence of the MCS estimate was assessed by plotting R(N) and its 95% CI for increasing sample sizes N={10^3, 5×10^3, 10^4, 5×10^4, 10^5, 5×10^5, 10^6}, as presented in Figure 8.

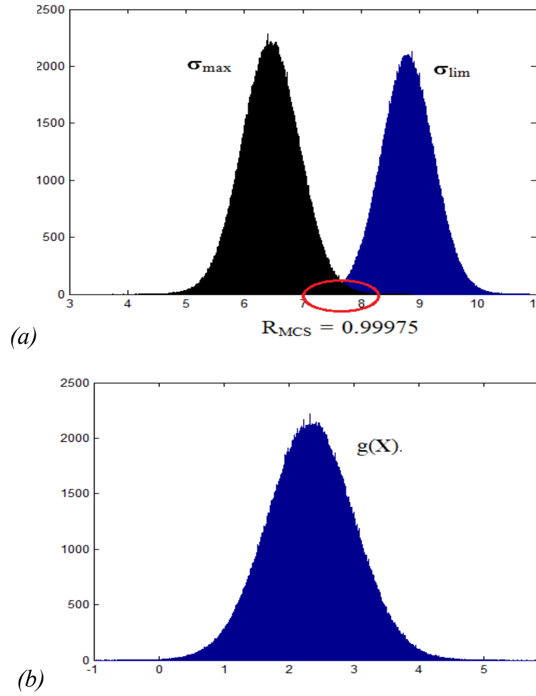

***Figure 7.** Monte Carlo simulation: a) Fatigue limit σ_{lim} and Stress σ_{max}; b) Limit-state function g(X)*

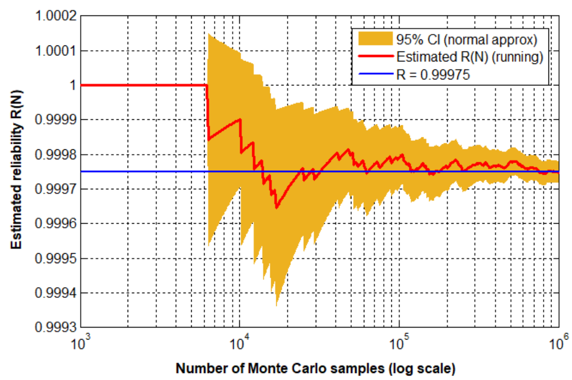

***Figure 8.** Convergence of the estimated reliability R(N)*

4.3. Sensitivity Analysis

To further evaluate the reliability of the V-belt drive under uncertainty, a global sensitivity analysis (GSA) was performed using Sobol indices. The analysis considered key random design variables, including pulley diameter (d_i), belt tension (F), fatigue limit (σ_{lim}), bending stress (σ_u), centrifugal stress (σ_v), belt length (L), friction coefficient (f) and elastic modulus (E).

The Sobol indices were estimated using the extended sampling scheme of Saltelli, which allows simultaneous estimation of both the first-order indices (S_i) and the total-effect indices (S_{Ti}). The base sample size was set to N = 10^4, resulting in a total of $N \times (k + 2) = 10^4 \times (9 + 2) = 1.1\times10^5$ model evaluations, where k = 9 random variables were considered: σ_{lim}, f, L, d_1, d_2, F, S, σ_v, and E.

To verify convergence, the S_i and S_{Ti} results were compared for sample sizes of N = 5×10^5 and N = 15×10^5. The indices showed only minor variations when N was increased or decreased (absolute change ≤ 0.02 and relative change ≤ 3% for most variables), indicating that the Sobol indices had sufficiently converged. Therefore, the conclusions regarding the relative importance of the random variables are considered robust with respect to the chosen sampling configuration. The results are summarized in Table 5 and Figure 9.

Table 5. *Global sensitivity analysis results of random variables using Sobol indices.*

Random Variable	S_i (First-order)	S_{Ti} (Total effect)	Remarks
Fatigue limit (σ_{lim})	0.72	0.81	Dominant factor
Friction coefficient (f)	0.09	0.14	Moderate effect, interaction with belt tension
Belt length (L)	0.07	0.11	Influences wrap angle and stress distribution
Driver pulley diameter (d_1)	0.01	0.02	Minor effect, mainly through wrap angle
Driven pulley diameter (d_2)	0.03	0.05	Slightly higher effect due to transmission ratio
Belt tension (F)	0.05	0.09	Small effect, increases when interacting with ff
Belt cross-sectional area (S)	0.04	0.06	Relatively small influence
Centrifugal stress (σ_v)	0.02	0.03	Limited impact
Elastic modulus (E)	0.01	0.02	Very minor influence

The GSA results provide deeper insights into the reliability behavior of V-belt drives. As shown in Table 4, the fatigue limit of the belt material (σ_{lim}) is the most critical parameter, accounting for approximately 72% of the first-order effect and more than 80% of the total effect. This confirms that variability in fatigue properties governs the system's service life and reliability. Accordingly, material quality control and enhancement of fatigue performance are the primary strategies for improving system reliability.

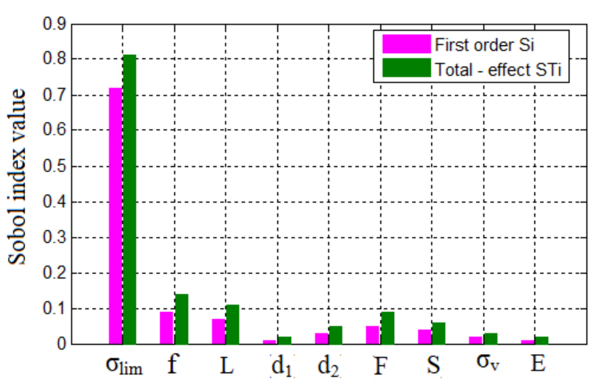

Figure 9. *Global sensitivity analysis results using Sobol indices*

The friction coefficient (f) and belt length (L) also contribute significantly, particularly through their interaction with belt tension (F). This finding underscores the importance of maintaining stable frictional contact conditions and ensuring proper belt installation to achieve reliable power transmission. In contrast, the geometric parameters (d_1, d_2) and elastic modulus (E) have relatively minor effects, suggesting that variations within their design ranges do not substantially compromise safety. Nevertheless, their inclusion in the optimization process remains essential to ensure robustness under practical design constraints.

Overall, GSA complements the local sensitivity analysis by identifying not only the most influential random variables but also their interaction effects. These insights offer valuable guidelines for material selection, design improvement, and preventive maintenance, thereby enhancing the robustness of the proposed RBDO framework.

This analysis also reflects the inherent uncertainty in the mechanical properties of the belt material, which is the primary load-bearing component of the transmission system. To further illustrate this point, the influence of the standard deviation of the fatigue limit on the failure probability was investigated. As presented in Figure 10, increasing the standard deviation of the fatigue limit leads to a substantial rise in failure probability, even when the mean value remains constant. This emphasizes the dominant role of material variability in determining system reliability.

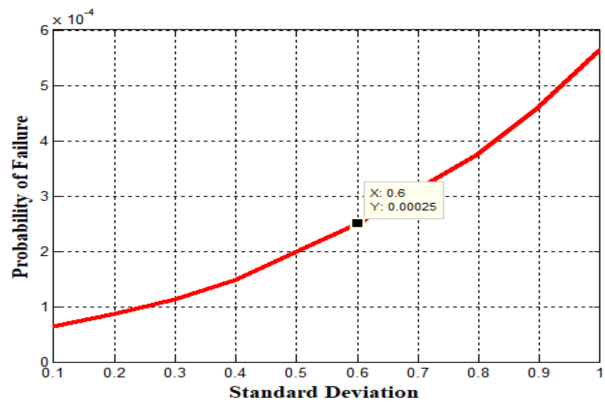

Figure 10. *Effect of the standard deviation of the fatigue limit on the failure probability*

In summary, the fatigue limit of the belt material (σ_{lim}) exhibits the most dominant effect on failure probability, followed by the friction coefficient (f) and belt length (L). This result is physically reasonable since the belt is subjected to repeated bending and frictional contact, making its fatigue properties the governing factor in overall reliability. From a practical perspective, enhancing reliability requires reducing the scatter in fatigue strength by: (i) selecting materials with higher and more consistent fatigue performance, (ii) optimizing manufacturing and surface treatment processes to minimize defects, and (iii) strengthening batch quality control. Additionally, maintaining proper surface conditions (pulley surface treatment, lubrication) and accurately controlling initial belt tension are essential, while selecting an appropriate belt length helps stabilize stress distribution.

Therefore, sensitivity analysis not only identifies the most critical random variables but also provides actionable guidelines for improving material selection, design robustness, and maintenance strategies, thereby ensuring high reliability in V-belt drive systems.

5. Conclusions

This study developed a Reliability-Based Design Optimization (RBDO) framework for V-belt drives by integrating Inverse Reliability Analysis (IRA) with a Genetic Algorithm (GA) and validating the results through Monte Carlo Simulation (MCS). The findings reveal that the RBDO approach not only ensures the required reliability level ($R^* = 99.9\%$) but also provides significant advantages over deterministic optimization. Specifically, it reduces system mass by approximately 18.2%, increases transmitted power from 4.75 kW to 5.65 kW, and improves overall operational efficiency.

The sensitivity analysis highlights that the fatigue limit of the belt material is the most critical factor influencing reliability, underlining the importance of material properties in the safe design and operation of belt drives.

Overall, the proposed RBDO framework demonstrates its feasibility and effectiveness in designing lightweight, reliable, and high-performance V-belt drives. Moreover, it offers a promising basis for extending reliability-based optimization to other mechanical power transmission systems, enabling a better balance between performance, safety, and economic efficiency.

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