DESIGN OF AN H_∞ OBSERVER FOR NONLINEAR PARAMETER-VARYING SYSTEMS WITH LIPSCHITZ NONLINEARITY AND MODEL UNCERTAINTY

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Abstract – This paper presents the design of an H_∞ observer for a class of Nonlinear Parameter-Varying (NLPV) systems featuring Lipschitz-continuous nonlinearities, bounded model uncertainties, and external disturbances. The proposed observer ensures robust state estimation while satisfying prescribed H_∞ performance criteria. A Lyapunov-based approach is adopted, utilizing a polytopic representation to capture the dependence of the system matrices on the scheduling parameters. Sufficient conditions for the existence of the observer are expressed in terms of Linear Matrix Inequalities (LMIs), which are evaluated at the vertices of the polytope. The observer gain matrices are obtained by solving these LMIs, thereby guaranteeing robustness against model uncertainties and external disturbances across the entire operating range. The effectiveness and robustness of the proposed observer design are demonstrated through a simulation study involving a 2-DOF robotic manipulator.

Keywords – Robust H_∞ observer; Linear Matrix Inequality; NonLinear Parameter-Varying system; Model uncertainty; 2-DOF Robotic Manipulator

1. Introduction

Many practical control systems, such as robotic manipulators and aerospace vehicles, exhibit inherently nonlinear dynamics and parameter variations resulting from changing operating conditions [1]. Modeling these systems within the NLPV framework provides a flexible and accurate representation, thereby facilitating the development of advanced control and state estimation strategies. For instance, the dynamic behavior of robotic manipulators can vary significantly due to changes in inertia matrices caused by varying configurations and payloads, leading to nonlinear parameter dependencies [2–4]. Likewise, aerospace vehicles experience pronounced nonlinear aerodynamic effects and system variations over a broad flight envelope, which are inadequately captured by fixed linear models but are effectively described using NLPV models [5].

The inherent complexities of such systems necessitate robust and adaptive state estimation techniques that can address both nonlinearities and time-varying parameters. Designing observers for NLPV systems poses significant challenges, particularly in the presence of model uncertainties and external disturbances. The H_∞ observer design framework offers a robust solution by minimizing the worst-case estimation error in the face of such disturbances. This paper focuses on the development of H_∞ observers for NLPV systems characterized by Lipschitz continuous nonlinearities and bounded parametric uncertainties [6–8].

To systematically address these challenges, this paper investigates the design of an H_∞ observer for NLPV systems with Lipschitz nonlinearities and norm-bounded uncertainties. The proposed approach is formulated through estimation error dynamics and cast as a convex optimization problem using LMI conditions to compute the observer gains. The effectiveness of the method is validated on a 2-DOF robotic manipulator system, where simulation results demonstrate that the proposed observer achieves accurate and robust state estimation, outperforming the Extended Kalman Filter (EKF). Furthermore, the findings highlight the potential for extending the approach to more complex NLPV systems and real-time experimental implementations.

2. Problem Formulation and Observer Design

This section formulates the robust observer design problem for NLPV systems and develops the H_∞ observer structure based on a Lyapunov framework. The synthesis conditions are derived using a polytopic representation of parameter dependence, leading to an LMI-based design method.

2.1. System Description and Uncertainty Modeling

Consider the following NLPV system subject to external disturbances and model uncertainties:

$$\dot{x}(t) = A(\rho(t))x(t) + \Delta A(t)x(t) + Bu(t) + \Phi(x(t)) + \omega(t)$$
$$y(t) = Cx(t) + v(t) \quad (1)$$

where:

$x(t) \in \mathbb{R}^n$ is the system state vector;

$u(t) \in \mathbb{R}^m$ is the control input;

$y(t) \in \mathbb{R}^p$ is the measured output;

$A(\rho(t)) \in \mathbb{R}^{n\times n}$ is a parameter-dependent system matrix with $\rho(t) \in \mathcal{P} \subset \mathbb{R}^s$ representing a time-varying scheduling parameter;

$\Phi(x(t))$ denotes a nonlinear function satisfying the global Lipschitz condition:

$$\|\Phi(x_1) - \Phi(x_2)\| \le L_f\|x_1 - x_2\|, \forall x_1, x_2 \in \mathbb{R}^n \quad (2)$$

for some known Lipschitz constant $L_f > 0$;

$\Delta A(t)$ represents the time-varying model uncertainty, bounded as:

$$\|\Delta A(t)\| \leq L_\phi \quad (3)$$

for some known constant $L_\phi > 0$;

$\omega(t) \in \mathbb{R}^n$ and $v(t) \in \mathbb{R}^p$ denote process and measurement disturbances, respectively, assumed to be bounded energy signals.

2.2. Observer Design

The proposed observer structure is as follows:

$$\dot{\hat{x}}(t) = A(\rho(t))\hat{x}(t) + \Phi(\hat{x}(t)) + Bu(t) + L(\rho(t))(y(t) - \hat{y}(t))$$
$$\hat{y}(t) = C\hat{x}(t) \quad (4)$$

where $\hat{x}(t) \in \mathbb{R}^n$ denotes the estimated state and $L(\rho(t)) \in \mathbb{R}^{n\times p}$ is the observer gain to be designed. Defining the estimation error $e(t) = x(t) - \hat{x}(t)$, the corresponding error dynamics are given by:

$$\dot{e}(t) = [A(\rho(t)) - L(\rho(t))C]e(t) + \delta(t) + \Delta A(t)x(t) + \omega(t) - L(\rho(t))v(t) \quad (5)$$

For notational convenience, we introduce:

$$\delta(t) = \Phi(x(t)) - \Phi(\hat{x}(t)) \quad (6)$$

and note that, by the Lipschitz continuity of $\Phi(\cdot)$, there exists a constant $L_f > 0$ such that:

$$\|\delta(t)\| \leq L_f \|e(t)\| \quad (7)$$

The aim is to design $L(\rho(t))$ such that the estimation error dynamics are globally asymptotically stable, and the influence of disturbances $\omega(t)$ and $v(t)$ on the error $e(t)$ is attenuated below a prescribed H_∞ performance level.

2.3. LMI-Based Observer Synthesis

To achieve the design objectives, a quadratic Lyapunov function is:

$$V(e(t)) = e(t)^T P e(t) \quad (8)$$

Differentiating the two bounds of (7), we get:

$$\dot{V}(e) = \dot{e}(t)^T P e(t) + e(t)^T P \dot{e}(t) \quad (9)$$

$$\dot{V}(e(t)) = e(t)^T \left\{[A(\rho(t)) - L(\rho(t))C]^T P + P[A(\rho(t)) - L(\rho(t))C]\right\} e(t) + 2e(t)^T P\delta(t) + 2e(t)^T P\Delta A(t)x(t) + 2e(t)^T P[\omega(t))\text{-}L(\rho(t)v(t)] \quad (10)$$

To guarantee H_∞ performance with level γ, we require that for all nonzero $\omega(t)$ and $v(t)$:

$$\dot{V}(e) + e(t)^T Q e(t) + \gamma^2[\omega(t)^T\omega(t) + v(t)^T v(t)] \leq 0 \quad (11)$$

for some symmetric positive definite matrix $Q = Q^T > 0$

Using the Lipschitz condition:

$$2e(t)^T P\delta(t) \leq 2\|P\|L_f\|e(t)\|^2 \quad (12)$$

For the parametric uncertainty:

$$2e(t)^T P\Delta A(t)x(t) \leq 2\|P\|L_\phi\|e(t)\|\|x(t)\| \quad (13)$$

Assuming bounded $x(t)$, the uncertainty term can be absorbed by introducing an auxiliary matrix $R > 0$ via the S-procedure:

$$2e(t)^T P\Delta A(t)x(t) \leq e(t)^T({L_\phi}^2 R)e(t) + x(t)^T(R^{-1})x(t) \quad (14)$$

Similarly, disturbances $\omega(t)$ and measurement noise $v(t)$ are handled by introducing weighting terms and applying Young's inequality.

Grouping terms, we obtain the following differential inequality:

$$\dot{V}(e(t)) + e(t)^T[Q + {L_f}^2 P + {L_\phi}^2 R]e(t) + 2e(t)^T[P\omega(t) - L(\rho(t)v(t)] + \omega(t)^T\omega(t) + v(t)^T\omega(t) \leq 0 \quad (15)$$

Further, we complete the squares for disturbance terms:

$$2e(t)^T P\omega(t) \leq 2e(t)^T P^T P e(t) + \omega(t)^T\omega(t)$$
$$2e(t)^T(-L(\rho(t)v(t)] - PL(\rho(t)v(t)] \leq 2e(t)^T L\rho(t)^T P^T P L(\rho)(e(t) + v(t)^T v(t) \quad (16)$$

and combine these into a matrix inequality involving the stacked vector ξ:

$$\xi = \begin{bmatrix} e(t) \\ \omega(t) \\ v(t) \end{bmatrix} \quad (17)$$

Consequently, the robust H_∞ performance condition reduces to requiring that the following matrix inequality [9] holds:

$$\begin{bmatrix} \Omega_1 + Q + {L_\phi}^2 R + {L_f}^2 P & P & P \\ * & -R & 0 \\ * & * & -\gamma_\infty I \end{bmatrix} < 0 \quad (18)$$

with

$$\Omega_1 = [A(\rho(t)) - L(\rho(t))C]^T P + P[A(\rho(t)) - L(\rho(t))C]$$

for all admissible *ρ(t)*. (19)

Since $A(\rho(t))$ belongs to the convex hull of the vertices $\{A_i\}$, it suffices to verify the condition at each vertex A_i with the corresponding observer gain L_i. To linearize the product terms between P and L_i, the changes in the variables are shown:

$$Y_i = -PL_i \quad (20)$$

Substituting and applying congruence transformations yield the final set of LMIs:

$$\begin{bmatrix} \Omega_i + Q + {L_\phi}^2 R + {L_f}^2 P & P & P \\ * & -R & 0 \\ * & * & -\gamma_\infty I \end{bmatrix} < 0 \quad (21)$$

where:

$$\Omega_i = {A_i}^T P + PA_i + C^T {Y_i}^T + Y_i C \quad (22)$$

3. Application to a 2-DOF Robot Manipulator System

3.1. Nonlinear Dynamics with Uncertainty and Disturbance

The proposed robust observation framework applied to a 2-DOF robot controller can be represented in either the standard Euler-Lagrange form or the system identification form. By reformulating the model into a state-space representation suitable for observer design, the system is modeled as an NLPV system with uncertainties and external disturbances as:

$$\dot{x}(t) = A(\rho(t))x(t) + \Delta A(t)x(t) + Bu(t) + \Phi(x(t)) + \omega(t)$$
$$y(t) = Cx(t) + v(t) \quad (23)$$

where: $x(t) = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$ is the state vector consisting of joint positions and velocities,

$A(\rho(t))$ and B are the state and input matrices depending on the scheduling parameters $\rho(t)$ (e.g., functions of joint angles or velocities),

$\Delta A(t)$ denotes norm-bounded time-varying uncertainty due to unmodeled dynamics or parametric variations,

$\Phi(x(t))$ captures the remaining nonlinear terms such as Coriolis, centrifugal, and gravitational effects,

$\omega(t)$ represents bounded external disturbances.

In particular, the nonlinear function $\Phi(x)$ is defined via the following expression derived from the dynamics:

$$\Phi(x) = \begin{bmatrix} \dot{\theta}_1 \\ f_2(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) \\ \dot{\theta}_2 \\ f_4(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2) \end{bmatrix} \tag{24}$$

where: $f_2(\cdot)$ and $f_4(\cdot)$ incorporate the dynamic coupling effects, such as Coriolis and centrifugal forces, and gravity-induced torques. These components are explicitly derived from the Euler-Lagrange formulation of a 2-DOF robotic manipulator or the system identification around multiple operating points. The expressions are constructed using the physical parameters (e.g., link masses, lengths, inertia tensors, and center-of-mass positions) and are computed numerically for simulation and observer design.

This modeling framework allows the nonlinear function $\Phi(x)$ to be evaluated directly for any given state vector $x(t) = \left[\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2\right]^T$, and its Lipschitz constant L_f can subsequently be estimated to facilitate observer synthesis under Lipschitz-type conditions.

3.2. Lipschitz Constant of the Nonlinear and Uncertainty

3.2.1. Lipschitz Constant of the Nonlinear

The nonlinear function $\Phi(x): \mathbb{R}^n \to \mathbb{R}^n$ is assumed to satisfy the Lipschitz condition:

$$\|\Phi(x_1) - \Phi(x_2)\| \leq L_f \|x_1 - x_2\|, \forall x_1, x_2 \in \mathcal{D} \tag{25}$$

where $L_f > 0$ is the Lipschitz constant of the function $\Phi(.)$, and $\mathcal{D} \subseteq \mathbb{R}^n$ is a compact domain containing the trajectory of the system. To estimate L_f, we employ a data-driven numerical method by sampling state vectors $\{x_i\}_{i=1}^N \subset \mathcal{D}$, computing the pairwise differences:

$$L_f \approx \max_{i \neq j} \frac{\|\Phi(x_i) - \Phi(x_j)\|}{\|x_i - x_j\| + \epsilon} \tag{26}$$

where $\epsilon > 0$ is a small scalar to avoid division by zero. This estimation captures the maximum slope (or gain) of the nonlinear function within the operating region.

3.2.2. Lipschitz Bound of the Structured Uncertainty $\Delta A(t)x(t)$

We assume the structured uncertainty $\Delta A(t)$ is norm-bounded and state-dependent, and satisfies the inequality:

$$\|\Delta A(t)\| \leq L_\phi, \forall t \geq 0 \tag{27}$$

This implies:

$$\|\Delta A(t)x(t)\| \leq L_\phi \|x(t)\| \tag{28}$$

Here, L_ϕ can be interpreted as the maximum gain induced by the difference between the real system matrix and its nominal approximation (e.g., from the vertices of a polytope). It can be numerically estimated using:

$$L_\phi \approx \max_{i=1,\dots,N} \frac{\|A(x_i) - A_0\|_F}{\|x_i\| + \epsilon} \tag{29}$$

where $A(x_i)$ is the system matrix evaluated at a sampled point x_i, A_0 is the nominal or linearization matrix (e.g., at origin or average), and $\|\cdot\|_F$ denotes the Frobenius norm [10]. This estimation is crucial in handling norm-bounded uncertainties in robust observer design, especially when incorporated into LMI conditions through quadratic inequalities.

3.3. Identified System Matrices for Simulation

To validate the proposed observer framework on a realistic robotic platform, system identification was performed on the 2-DOF manipulator to obtain approximate state-space matrices for a selected operating point [11]. At one operating point, the identified matrices are:

$A_1 =$

-1.3512	3.0907 -	1.3106	-1.8119
1.9060	-1.4636	-0.7185	-1.0548
-3.1181	21.5764	-8.6293	-13.5763
22.8287	3.1558	-21.4970	-54.5012

$B_1 =$

1.0e+03 *

0.0164	-0.0680
-0.0007	0.0041
0.0258	-0.0962
-0.3269	1.4481

$C = I_4$

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

These matrices serve as a local linear approximation (A_i, B_i) of the NLPV model in the polytopic framework, and were used in solving the LMIs for observer gain synthesis. The output matrix C was chosen as the identity to allow full-state observation.

The vertex matrix A_1 corresponds to a specific configuration of joint angles and velocities during normal operation. In simulation, this matrix is used as one of the polytopic bounds for robust observer design under model uncertainty and disturbance.

The constants L_f are estimated according to formula (26): $L_f \approx 1.289$, and L_ϕ according to formula (29): $L_\phi \approx 0.09$.

3.4. H_∞ Observer Synthesis LMI Condition

For each vertex $i \in \{1, 2\}$ of the polytope approximation of the NLPV system, the following LMI condition in formula (21) is solved to synthesize a robust H_∞ observer:

$$\begin{bmatrix} \Omega_i + Q + {L_\phi}^2 R + {L_f}^2 P & P & P \\ * & -R & 0 \\ * & * & -\gamma_\infty I \end{bmatrix} < 0$$

where:

$$\Omega_i = {A_i}^T P + P A_i + C^T {Y_i}^T + Y_i C$$

Then the matrix of observations at the vertices of the polyhedron from formula (20): $L_i = -P^{-1} Y_i$

Use the CVX tool [12] in Matlab to solve the optimization problem in the LMI condition to find the observer matrix at the two vertices of ρ, L_1 and L_2.

The results obtained for the following parameters:

$\boldsymbol{\gamma}_\infty = \mathbf{0.22512}$ and the observer at the two vertices of ρ are L_1 and L_2

$L_1=$

73.0090	2.0089
2.6515	73.1334
-19.3454	15.1274
27.3147	6.7082

$L_2=$

72.1719	1.7330
2.3943	73.0927
-17.1503	17.1588
26.7900	6.4177

In the 2-vertex polytope design, $L\rho$ is calculated by interpolation from two matrices L_1 and L_2

$$L\rho = (1 - \rho)L_1 + \rho L_2 \quad (30)$$

3.5 Extended Kalman Filter for NLPV Systems

To estimate the system states in formula (23), an EKF is designed for the NLPV system with a nonlinear perturbation $\Phi(x) = 0.5sin\,(x)$. The parameter-varying matrix is computed as $A(\rho(t)) = (1-\rho(t))A_1 + \rho(t)A_2$, where $\rho(t) = 0.5 + 0.5sin(t)$. The state prediction is obtained via Euler approximation:

$$\hat{x}_{k|k-1} = \hat{x}_k + \Delta t\big(A(\rho_k)\hat{x}_k + \Phi(\hat{x}_k)\big) \quad (31)$$

The Jacobian used for linearization is:

$$A_{\text{lin}} = A(\rho_k) + 0.1 \cdot \text{diag}\big(\cos\,(\hat{x}_k)\big) \quad (32)$$

The error covariance is updated as:

$$P_{k|k-1} = P_k + \Delta t(A_{\text{lin}}P_k + P_k A_{\text{lin}}^\top + Q) \quad (33)$$

and the Kalman gain is:

$$K_k = P_{k|k-1}C^\top(CP_{k|k-1}C^\top + R)^{-1} \quad (34)$$

The state and covariance are updated as usual. To prevent divergence, the values of $\hat{x}_k$ are clipped to $[-\pi, \pi]$, and the covariance matrix is bounded. The filter assumes that process and measurement noises are zero-mean Gaussian with covariances $Q = R = 0.005I$. This EKF implementation is later benchmarked against the proposed H_∞ observer using RMSE, NRMSE, and the coefficient of determination R^2.

4. Simulation Results and Discussion

4.1. Simulation Results

In this section, the observer will be simulated in the time domain with the initial values x_0 of the system and $\hat{x}_0$ of the observer as follows:

$$x_0 = [0, 0, 0, 0]^T;\ \hat{x}_0 = [1, 1, 1, 1]^T$$

Additive process noise and measurement noise were introduced as:

$$\omega(t) = 5\times10^{-3}\cdot G(0, I_n),\ v(t) = 3\times10^{-3}\cdot G(0, I_p),$$

where G (0, I) denotes zero-mean Gaussian noise with identity covariance.

The scheduling parameter $\rho(t) = 0.5 + 0.5sin(t)$ introduces time-varying dynamics via interpolation between two linear subsystems. The simulation was run for 10 seconds (s) with a sampling interval $\Delta t = 0.01s$.

To analyze the simulation results, in this section, the values of x and $\hat{x}$ are compared. By comparing x and $\hat{x}$, combining the comparison with the EKF, we can evaluate the state estimation performance of the proposed method. Simulation scenario when the input signal is a sinusoidal function.

4.1.1. Scenario 1: Nominal Case with Small Nonlinearity and Noise

In this scenario, the nonlinear parameter-varying system is considered under nominal operating conditions, with small bounded nonlinearity and low-intensity process and measurement noise. The input matrix is set as $B = I_n$, representing a full-state input. The system is excited by a smooth sinusoidal input signal: $u(t)=sin(2t)$, which allows the system to evolve with persistent excitation. The nonlinear perturbation is modeled as a bounded Lipschitz function: $\Phi(x) = 0.5sin(x)$, representing a small-amplitude nonlinearity. The simulation results are shown in Figure 1.

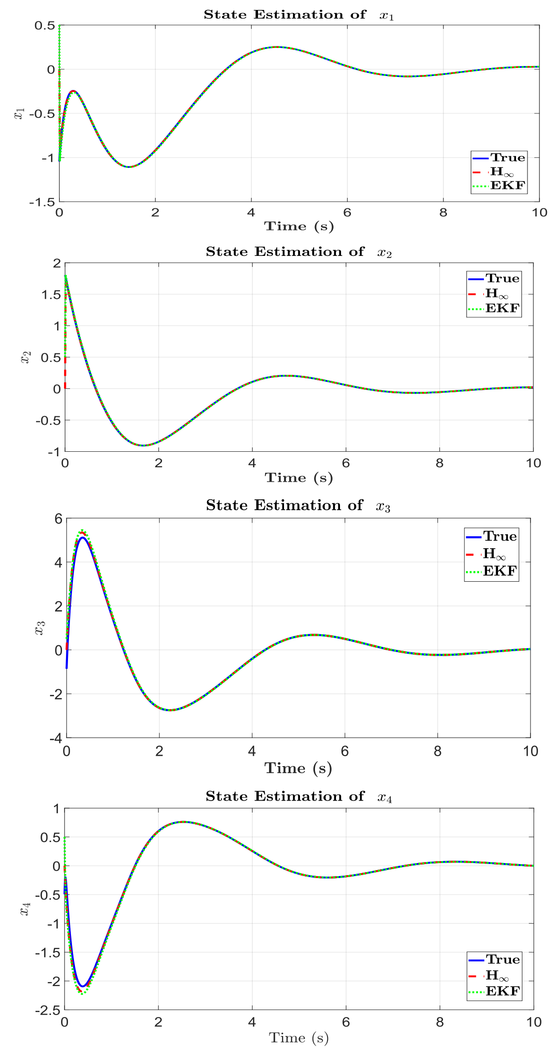

Figure 1. *Robot status observation results scenario 1*

Table 1. *State Deviation Index Table in scenario 1*

State	RMSE		NRMSE (%)		R^2	
	H_∞	EKF	H_∞	EKF	H_∞	EKF
x_1	0.0334	0.1525	2.46	3.67	0.9934	0.9841
x_2	0.0581	0.1457	2.15	1.54	0.9800	0.9901
x_3	0.0902	0.2270	1.15	1.59	0.9970	0.9940
x_4	0.0365	0.0778	1.28	1.97	0.9960	0.9920

4.1.2. Scenario 2: Input-Weighted States with Moderate Nonlinearity

In the second scenario, the system is tested under modified input influence and slightly increased nonlinearity to further evaluate the robustness of the proposed H_∞ observer. The input matrix is configured as:

B=diag(1, 1, 0.05,0.05). The input signal is designed as a higher-frequency sine function: *u(t)=sin(4t)*.

This scenario evaluates the observer's ability to handle partial observability, heterogeneous input effects, and more pronounced nonlinear dynamics while maintaining estimation accuracy and robustness. The simulation results are shown in Figure 2.

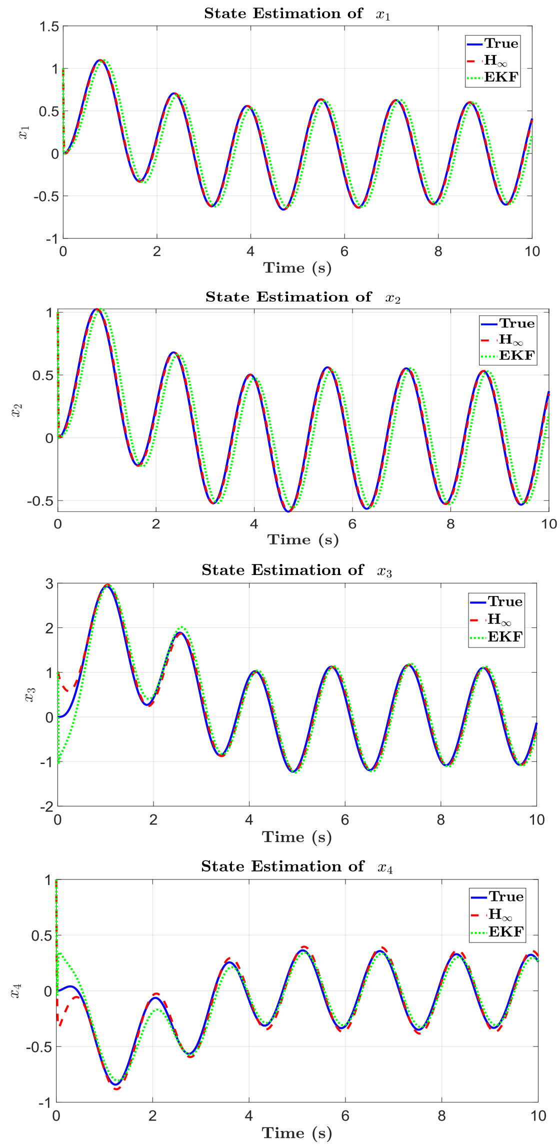

Figure 2. *Robot status observation results scenario 2*

Table 2. *State Deviation Index Table in scenario 2*

State	RMSE		NRMSE (%)		R^2	
	H_∞	EKF	H_∞	EKF	H_∞	EKF
x_1	0.0382	0.1525	2.17	8.68	0.9934	0.8951
x_2	0.0380	0.1457	2.35	9.03	0.9920	0.8823
x_3	0.1457	0.2270	3.51	5.46	0.9800	0.9524
x_4	0.0635	0.0778	5.26	6.46	0.9554	0.9332

4.2. Discussion of Simulation Results

The effectiveness of the proposed observer was evaluated using a simulated 4-state nonlinear system. As shown in Figure 1 and 2, the estimated states ($\hat{x}_i$, red dashed lines) closely track the true states (x_i, blue solid lines) over a period of 10 seconds for all state variables x_1 to x_4, and compare with the EKF ($\hat{x}_i$, green dotted lines). The proposed H_∞ observer demonstrates fast convergence and high estimation accuracy, even in the presence of model uncertainties and measurement noise. The negligible estimation error indicates the robustness and precision of the designed observer, making it suitable for NLPV systems.

Tables 1 and 2 provide comprehensive comparisons of state estimation performance between the proposed H_∞ observer and EKF. Across all state variables x_1 to x_4, the H_∞ observer consistently yields lower Root Mean Square Error (RMSE) and Normalized RMSE (NRMSE) values, along with a higher coefficient of determination (R^2), indicating superior estimation accuracy and model fit.

In Table 1, the results indicate that the H_∞ observer consistently achieves lower RMSE values across all four states. Specifically, for x_1, the RMSE is reduced from 0.1525 (EKF) to 0.0334. Similarly, x_3 shows a substantial error reduction from 0.2270 to 0.0902.

In terms of NRMSE, both methods perform within acceptable ranges (< 5%), but the H_∞ observer yields better normalized errors in three out of four states. Notably, for x_1, the NRMSE drops to 2.46% with the proposed method, compared to 3.67% using EKF.

Regarding the R^2 metric, both observers demonstrate high estimation accuracy (R^2 > 0.98), indicating good tracking capability. However, the H_∞ observer slightly outperforms EKF in most cases, particularly for x_1 and x_3, reflecting better model fit to the true state trajectory.

In Table 2, the improvements observed in x_1 and x_2 are notably significant, with the RMSE reduced by approximately 75% and the R^2 increased by over 10% compared to the EKF. These states often correspond to key system dynamics, implying that the H_∞ observer maintains better tracking capability for critical states even in the presence of model uncertainties and external disturbances.

The NRMSE values (for the H_∞ observer) remain below 5.5% for all states, whereas the EKF exhibits higher error percentages, reaching up to 9.03% for certain components. Furthermore, R^2 values above 0.99 (for the H_∞ observer) demonstrate its strong correlation with the true state trajectories, reinforcing its capability to approximate the true system behavior with high fidelity.

These quantitative results underscore the robustness and reliability of the proposed H_∞ observer, particularly in nonlinear and uncertain environments where traditional filtering methods like EKF may degrade in performance. Thus, the proposed observer emerges as a compelling alternative for high-accuracy state estimation in complex dynamic systems.

5. Conclusion

This paper proposes an H_∞ observer design for a class of Nonlinear Parameter-Varying (NLPV) systems under Lipschitz nonlinearities and norm-bounded uncertainties, applied to a 2-DOF robotic manipulator system. The observer synthesis was formulated as a convex optimization problem using linear matrix inequality (LMI) conditions derived from estimation error dynamics. A simulation study on the 2-DOF robotic manipulator demonstrated that the proposed observer provides accurate and robust state estimation, significantly outperforming the Extended Kalman Filter (EKF).

Future work will focus on experimental validation and real-time implementation on robotic hardware, as well as extensions to combined observer-controller structures for enhancing closed-loop performance.

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