OPTIMIZING PRODUCTION AND STORAGE DECISIONS FOR DETERIORATING ITEMS IN ENERGY-CONSTRAINED SYSTEMS

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Abstract - This study presents an enhanced Economic Production Quantity (EPQ) model that incorporates energy consumption and product deterioration to promote sustainable operations. The model simultaneously considers energy use during production and storage phases, deterioration behavior modeled by exponential decay, and a full backordering policy. It captures the interactions between inventory levels, energy efficiency, and product quality loss to better reflect real industrial conditions. Numerical analysis shows that integrating energy-related costs significantly reduces total expenses and minimizes deterioration losses. The findings underscore the importance of setup and energy costs in production planning and highlight energy efficiency as a key technological innovation for sustainable manufacturing. The proposed model offers practical guidance for industries striving to align economic objectives with environmental goals.

Key words - Inventory model; Energy-efficient production; Deteriorating inventory; Sustainable manufacturing; Backorder policy; Cold supply chain

1. Introduction and Literature review

The global energy crisis, exacerbated by geopolitical instability, has intensified the urgent need to optimize energy usage and reduce greenhouse gas emissions across supply chains. In 2022, the industrial sector accounted for 26% of global CO_2 emissions (~9 Gt CO_2) and remained the largest energy consumer [1]. By 2023, global energy consumption rose by 2.2%, with manufacturing and construction contributing 57% of total emissions [2]. Emissions from fossil fuels and industrial activities are projected to reach a record 37.41 Gt CO_2 by 2024 [3]. Despite ongoing government mitigation efforts, industrial demand continues to drive 75% of post-COVID energy growth and is expected to account for two-thirds of the 2.5% increase in global gas demand in 2024 [4].

A large share of industrial energy is used in cold supply chain for storing temperature-sensitive products, with refrigeration alone accounting for up to 30% of global energy use [5]. At the same time, product deterioration leads to resource loss and environmental harm. Solid waste from the global food industry is expected to rise by 70%, from 2.01 billion tons in 2016 to 3.4 billion tons by 2050 [6]. Improper disposal of spoiled goods can release harmful substances, threatening ecosystems, public health, and contributing further to emissions.

In response, many enterprises are shifting toward sustainable production by optimizing energy consumption and reducing CO_2 emissions throughout the product lifecycle. Recent research has incorporated energy considerations into the Economic Order Quantity (EOQ) model, as proposed by F.W. Harris [7]. Research by S. Zanoni et al. [8] developed a system where a single product is manufactured on two machines with three buffers. Then, the study is extended to consider energy usage [9-10] and carbon emission [11] in both production and idle phases. Several authors continued to analyze energy impacts in warehouses, demonstrating that energy costs can constitute a considerable portion of total inventory expenses [12-13]. More recently, some studies proposed an EPQ-based model that accounts for energy consumption in both warehousing and production [14-15]. These studies underscore a growing academic and practical focus on minimizing energy usage and carbon emissions in inventory management.

Most models focus on general items and ignore time-based deterioration, which is crucial for perishable goods like food, pharmaceuticals, and chemicals. Effective inventory control for such items reduces waste and boosts profitability. The deterioration modeling stems from the foundational work of R. P. Covert and G. C. Philip [16], who applied the distribution to describe time-based quality decay. Their framework has since been extended in many EOQ/EPQ models for perishable products with constant or variable deterioration rates as well as in several real-life case studies [17-21]. In the context of rising emissions, sustainability-oriented models for deteriorating products have also emerged. The necessity of incorporating carbon management into deteriorating inventory systems was emphasized [22-23]. However, most of these models focus primarily on economic or carbon costs, while neglecting energy consumption, a core driver in sustainable production.

This highlights a significant research gap: existing EPQ models fail to simultaneously incorporate energy consumption and product deterioration within a unified framework. This lack of integration hinders the formulation of comprehensive inventory strategies that can jointly minimize costs and advance sustainability objectives. To address this gap, this study proposes a novel inventory model with the following contributions: (1) Develop an EPQ model for deteriorating items that integrates energy consumption and exponential product decay, promoting sustainable inventory control. (2) Evaluate the impact of input costs under full backordering to identify key cost drivers in real-world production. (3) Deliver actionable insights from sensitivity analysis to support long-term, sustainable decision-making.

These contributions enrich inventory theory and offer practical tools for businesses pursuing technological innovation for sustainable development.

2. Model development

This study proposes an enhanced EPQ model for deteriorating items in a single-product, single-machine setting with energy considerations. After production ends, the machine enters a standby mode that still consumes energy. The model extends the classical EPQ by incorporating product deterioration, modeled via exponential decay, and allowing for shortages. It captures energy usage across production, idle, and storage phases for a comprehensive assessment of energy impacts. The production rate is adjustable within specified bounds and remains higher than the constant demand. The objective is to minimize the total average cost by optimizing cycle time, production rate, machine utilization, and energy consumption in all phases.

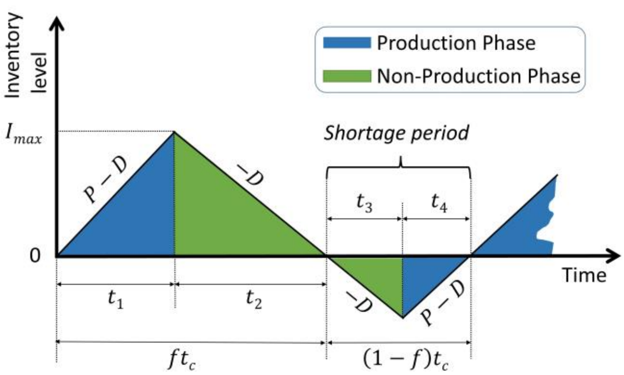

Figure 1. Inventory level over the production cycle

2.1. Notations and assumptions

The following notation is used to describe the model:

Parameters

D: demand rate (unit/h);

S: setup cost ($/setup);

H: holding cost per unit ($/(unit.h));

C: unit production cost ($/unit.h);

B: backorder cost ($/(unit.h));

W: idle energy consumption (kW);

K: energy consumption for one unit producing (kWh/unit);

E: energy cost ($/kWh);

α: deterioration rate, assumed to remain constant rate;

T_w: expected warehouse temperature (°C);

T_r: referenced warehouse temperature (°C);

T_{hot}: outside warehouse temperature (°C);

ρ: coefficient linking SEC to various storage temperatures;

λ, μ: positive coefficients dependent on the characteristics of the warehouse, where $\mu \in (0, 1)$;

δ, γ: positive coefficients dependent on the filling level of the warehouse;

Dependent variable

t_1: time of production time (h);

t_2: time of non-production time (h);

t_3: time of consumption sub-time in shortage period (h);

t_4: time of production sub-time in shortage period (h);

I: inventory level at time t (unit);

I_{max}: maximum storage capacity of the warehouse (unit);

I_b: stockout demand (units).

Decision variable

P: production rate (unit/h);

t_c: cycle time (h);

f : fraction of period length with positive inventory level, $f \in (0, 1]$.

The assumption of an inventory model for product life cycle are as follows: Demand is known and has a constant rate; The production rate P is bounded within [P_{min}, P_{max}], with $P > D$ at the beginning of each cycle; Stortages are allowed with complete backlogging; Lead time is negligible, and items are available for immedia use after production; Items begin to deteriorate immediately upon storage, following an exponential decay with a constant deterioration rate α; No replacement or repair is performed for deteriorated items during the cycle; The cost of a deteriorated item equals the unit production cost C, including any salvage value; The machine remains idle during non-production phases, consuming energy.

2.2. Mathematical modeling

As shown in Fig. 1, the inventory level starts at zero at $t = 0$ and rises steadily to a maximum , I_{max} , at $t = t_1$. After production stops at t_1, the inventory declines due to constant demand and exponential deterioration. By $t = t_2$, the inventory is exhausted. During t_3, demand persists but is unmet, resulting in backorder accumulation. Production resumes at At $t = t_4$, fulfilling both existing backorders and ongoing demand. Let α denote the constant deterioration rate, the inventory level over the entire cycle $[0; t_c]$ is governed by the following system of differential equations.

$$\frac{dI_1}{dt} + \alpha I_1 = P - D, \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_2}{dt} + \alpha I_2 = -D, \qquad 0 \le t \le t_2 \qquad (2)$$

$$\frac{dI_3}{dt} = -D, \qquad 0 \le t \le t_3 \qquad (3)$$

$$\frac{dI_4}{dt} = P - D, \qquad 0 \le t \le t_4 \qquad (4)$$

The differential equations are solved using Spiegel's method [24] under the following boundary conditions $I_1 = 0$, the initial inventory, and at $t = t_1$, $I_2 = I_{max}$; at $t = t_2$, $I_3 = 0$ and at $t = t_3$, $I_4 = I_b$, are

$$I_1 = \frac{P-D}{\alpha}[1-e^{-\alpha t}], \qquad 0 \le t \le t_1 \tag{5}$$

$$I_2 = \frac{D}{\alpha}\left[\frac{e^{\alpha t_2} - e^{\alpha t}}{e^{\alpha t}}\right], \qquad 0 \le t \le t_2 \tag{6}$$

$$I_3 = -Dt, \qquad 0 \le t \le t_3 \tag{7}$$

$$I_4 = (P-D)t, \qquad 0 \le t \le t_4 \tag{8}$$

Similarly, $t = t_1$ or $t_2 = 0$, I_{max} can be write as:

$$I_{max} = \frac{P-D}{\alpha}[1-e^{-\alpha t_1}] \tag{16}$$

$$I_{max} = \frac{D}{\alpha}[e^{\alpha t_2} - 1] \tag{17}$$

As shown in Fig. 1, the time points t_1, t_2, t_3, t_4 can be approximated using linear-cycle assumptions to simplify calculations. This approach, originally proposed by [16], addresses the complexity of exact expressions. It was later refined through the correction method of Newton, which has been widely adopted in deterioration-related inventory models. These time variables can then be expressed in terms of f and t_c as follows:

$$t_1 = \frac{D}{P} f t_c \tag{18}$$

$$t_2 = \frac{P-D}{P} f t_c \tag{19}$$

$$t_3 = \frac{P-D}{P}(1-f)t_c \tag{20}$$

$$t_4 = \frac{D}{P}(1-f)t_c \tag{21}$$

Setup cost. Similar to the classical EPQ model, the setup cost in this model is calculated using the standard formulation as follows:

$$SC = \frac{S}{t_c} \tag{22}$$

Holding cost. The holding cost for deteriorating items in inventory models is determined by calculating the inventory level over the time intervals during which stock is available, specifically t_1 and t_2, and then multiplying this by the average holding cost per cycle. By substituting t_1 and t_2 as defined in Eqs. (18) and (19), the holding cost can be expressed as follows:

$$HC = \frac{HDf^2 t_c (P-D)}{2P}\left(1 + f t_c \alpha \frac{(P-2D)}{3P}\right) \tag{23}$$

Deterioration cost. Deterioration occurs in t_1 and t_2, so the cost is calculated as the number of deteriorated units multiplied by the average unit value per cycle, as given in Equation (24).

$$DC = \frac{\alpha f^2 t_c CD(P-D)}{2P}\left(1 + f t_c \alpha \frac{(P-2D)}{3P}\right) \tag{24}$$

Backorder cost. Similar to the calculation of deterioration cost, the backorder cost is defined over the shortage period $(t_3 + t_4)$. The backorder cost function is derived as follows:

$$BC = \frac{B(1-f)^2 t_c}{2} D\left(1 - \frac{D}{P}\right) \tag{25}$$

Average related-production energy consumption cost. The system's energy consumption comes from two sources: the production phases (t_1 and t_4) and the non-production phases (t_2 and t_3).

By using SEC, the average energy consumption cost during production time (t_1 and t_4) is formulated as (26).

$$EC_{prod.} = \left(\frac{W}{P} + K\right) DE \tag{26}$$

The energy cost of keeping the machine active during idle periods, non-production phases, is expressed in Eq. (27).

$$EC_{nonp.} = \frac{WE}{t_c}(t_2 + t_3) = WE\left(1 - \frac{D}{P}\right) \tag{27}$$

Average related-warehousing energy consumption cost. The SEC function for warehousing, introduced by Zanoni et al. [25], depends on the storage level. Marchi et al.[12] later extended it by considering the impact of ambient temperature. Base on this, the warehousing energy cost is define as follows

$$EC_{ware.} = \frac{E}{t_c}\int_0^{t_c} SEC(T_w I(t)) I_{max}\, dt \tag{28}$$

The first term, $\lambda I_{max}^{-\mu}$, represents the baseline SEC which depends on the reserved storage volume. Here, λ and $\mu \in (0;1)$ are positive coefficients, reflecting the influence of ambient conditions, required temperature, warehouse design, and operation. As the maximum volume of the warehouse increases, SEC decreases. The second term, $\delta\left(1 - \frac{I(t)}{I_{max}}\right)^{\gamma}$, captures the energy inefficiency cause by underutilized storage space. The coefficients δ and γ determine this effect, indicating that energy consumption is lower when the warehouse is fully utilized.

The final component, ρ, is the ratio of coefficients of performance (COP) for refrigeration systems, defined as:

$$\rho = \frac{COP_{Tr}}{COP_{Tw}} = \frac{T_r}{T_{hot} - T_r}\frac{T_{hot} - T_w}{T_w} \tag{29}$$

The maximum inventory level is defined in Eq. (16) and (17), and the inventory levels at time t, as mentioned in Eq. (5), (6), (7), and (8), are recorded as follows

$$I(t) = \begin{cases} \frac{P-D}{\alpha}[1-e^{-\alpha t}], & if\ 0 \le t \le t_1 \\ \frac{D}{\alpha}\left[\frac{e^{\alpha t_2} - e^{\alpha t}}{e^{\alpha t}}\right], & if\ 0 \le t \le t_2 \\ 0, & if\ t_1 + t_2 < t_3, t_4 \le t_c \end{cases} \tag{30}$$

The elements of (28) can be separated and developed as follows

$$EC_{ware.} = \frac{E}{t_c}\left[\int_0^{t_c} \lambda I_{max}^{-\mu+1} \rho\, dt + \int_0^{t_c} \delta\left((1 - \frac{I(t)}{I_{max}}\right)^{\gamma} \rho I_{max}\, dt\right]$$

$$= \frac{E}{t_c}[A(t) + \delta\rho I_{max} B(t)] \tag{31}$$

Where

$$A(t) \quad = \lambda\rho t_c\left[D\left(\frac{P-D}{P}\right) f t_c\right]^{-\mu+1} \tag{32}$$

$$B(t) \quad = B_1(t) + B_2(t) + B_{3,4}(t) \tag{33}$$

$$B_1(t) = \frac{f t_c}{(\gamma+1)}\frac{D}{P}$$
$$B_2(t) = \frac{f t_c}{(\gamma+1)}\left(\frac{P-D}{P}\right)$$
$$B_{3,4}(t) = (1-f)t_c$$

After the simplification, (31) can be yielded as

$$EC_{ware.} = \lambda\rho E\left[D\left(\frac{P-D}{P}\right)f t_c\right]^{-\mu+1} + \delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right) \quad (34)$$

The average total cost of the system in this case is calculated as the sum of the Setup Cost, Holding Cost, Deterioration Cost, Backorder Cost, Energy Consumption Cost, and, as expressed in Eq. (35).

$$ATC\ (t_c, f, P) = \frac{S}{t_c} + \frac{HDf^2 t_c(P-D)\rho}{2P}\left(1 + f t_c \alpha\frac{(P-2D)}{3P}\right) + \frac{\alpha f^2 t_c CD(P-D)}{2P}\left(1 + f t_c\alpha\frac{(P-2D)}{3P}\right) + \frac{B(1-f)^2 t_c}{2}D\left(1-\frac{D}{P}\right) + \left(\frac{W}{P}+K\right)DE + WE\left(1-\frac{D}{P}\right) + \lambda\rho E\left[D\left(\frac{P-D}{P}\right)f t_c\right]^{-\mu+1} + \delta\rho t_c ED\left(\frac{P-D}{P}\right)\left(1-\frac{\gamma f}{\gamma+1}\right) \quad (35)$$

3. Resolution approach

The objective of the model is to minimize the average total cost function (35) by determining the optimum values of the production rate (P), the cycle time (t_c), and fraction of period length (f). The problem can be summarized as follows:

$$Minimize\ ATC(t_c, P, f\ )$$

Subject to

$$P_{min} \le P \le P_{max}, \qquad 0 < f < 1$$

We can use commercial solvers for NLP problems (e.g., LINGO, MATLAB) to find the two optimal values: $ATC(t_c, P_{min}, f)$ and $ATC\ (t_c, P_{max}, f)$. The optimal solution is the smallest value among these two values. Finally, we have the optimal decision values for t_c^*, P^*, f^*.

4. Numerical Analysis

A numerical analysis has been carried out to demonstrate the model's properties. The process includes: (1) Implementing the resolution procedure for a specific case using existing research data. (2) Analyzing the impact of energy components on the production inventory model by comparing the energy-based EPQ model with deterioration items to the traditional EPQ model with deterioration items. (3) Conducting a sensitivity analysis to examine how input parameters influence the total cost and decision variables.

4.1. Numerical examples

The input parameters related to the manufacturing process ($i.e. D, S, P_{min}, P_{max}, W$) and the input parameters related to warehousing ($i.e. \lambda, \mu, \delta, \gamma$) have been obtained from Nguyen et al. [14] and the parameter of deterioration (α, C) from Misra [26]. Table 1 summarizes the data set used in all the examples presented in this section.

Table 1. *Input parameters for the numerical examples*

D	100	$unit/h$	α	0.02	
S	300	$\$/h$	λ	$445 \cdot 10^{-4}$	
H	$1 \cdot 10^{-5}$	$\$/unit \cdot h$	μ	0.23	
W	100	kW	δ	$171.2 \cdot 10^{-4}$	
K	0.05	$kWh/unit$	γ	0.5	
E	0.2	$\$/(kWh)$	T_w	-18	$°C$
B	0.01	$\$/unit \cdot h$	T_r	6	$°C$
C	3	$\$/unit \cdot h$	T_{hot}	16	$°C$
P	[200:500]	$unit/h$			

The optimal solution based on the given input parameters is shown in Fig. 2, which also demonstrates the concave nature of the average total cost function with respect to P. Based on the solution approach presented in the previous section, the function $ATC(t_c, P, f\ )$ attains its minimum at $t_c^* = 18.451\ h,\ P^* = P_{min}, f^* = 0.057$ with the optimal value $ATC^*(t_c, P, f) = 53.678\ (\$/h)$. The optimal solution obtained will be used to compare with the traditional EPQ model considering deterioration, in order to evaluate its effectiveness.

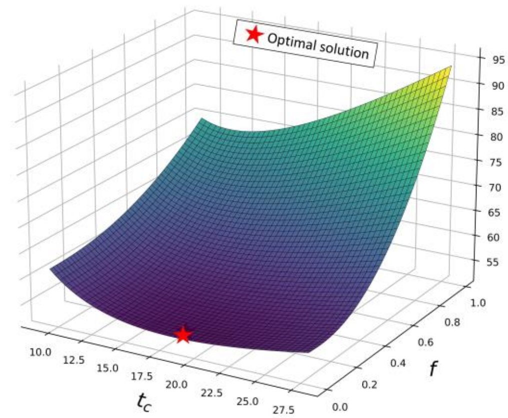

Figure 2. *Average total cost with the variation f and t_c*

4.2. Impact of Energy-Related Factors on the Traditional EPQ Model with Deterioration

To examine the impact of energy factors on production and inventory storage, two models are analyzed: (1) the traditional EPQ model with deterioration (EPQ-D) and (2) an extended EPQ model that includes energy consumption in both production and storage (EEPQ-D). This comparison evaluates inventory policies and economic performance, serving as a basis for further discussion.

In the EPQ-D model, the average total cost is formulated based on traditional cost components, as defined by Eq. (22) – (25). The comparison results are presented in Table 2, with all input parameters specified in Table 1. The sum of setup cost and holding cost is labeled as Traditional cost ($C_{trad.}$), whereas the total energy-related costs in production and storage are denoted as Energy cost (EC) and Deterioration cost (DC).

Table 2. *Optimal solutions and component costs of two model*

	P^*	f^*	t_c^*	ATC	$C_{trad.}$	EC	DC
EEPQ-D	200	0.057	18.451	53.678	16.260	33.224	0.089
EPQ-D	200	0.143	37.412	62.174	8.026	46.136	1.142

Base on the result in Table 2, both models achieve the same optimal production quantity ($P^* = 200$). However, the EEPQ-D model has a shorter production cycle (t_c) and a lower production time ratio (f) compared to the EPQ-D model. This results in more frequent setups but shorter machine operating periods, leading to notable energy savings. Notably, the deterioration cost in the EEPQ-D model is only 0.089 (\$/h) − 92.24% lower than the 1.142 (\$/h) observed in the EPQ-D model. The reduced value of f shortens the deterioration period, thereby lowering losses and preserving inventory value. Although the traditional cost in the EEPQ-D model is higher (16.260 (\$/h) vs 8.026(\$/h), its energy cost is substantially lower at 33.224 (\$/h) compared to 46.136 (\$/h) in the EPQ-D model (a reduction of 27.98%). As a result, the EEPQ-D model achieves a lower total average cost of 53.678 (\$/h), which is 13.66% less than the 62.174 (\$/h) in the EPQ-D. These results highlight that incorporating energy considerations not only reduces total cost but also minimizes deterioration, making the system more efficient and sustainable – especially in energy-intensive settings.

4.3. Sensitivity analysis

Changes in parameter values can significantly affect system performance. To assess their impact, this study conducts a sensitivity analysis by varying one parameter at a time while keeping others fixed, based on the numerical results presented earlier. Parameters are grouped into three categories: group 1 includes general production parameters (S, H, C, B, D); group 2 includes energy consumption and deterioration factors (K, W, E, α); group 3 includes inventory storage energy parameters ($T_w, \rho, \mu, \lambda, \delta, \gamma$)

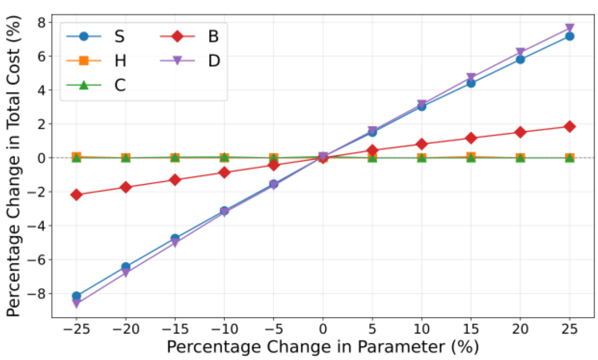

Figure 3. *Impact of varying Group 1 on the total cost*

The analysis shows that demand (D) is the most influential factor affecting total cost, followed by setup cost (S). Specifically, increasing D and S by 25% leads to a 6% increase in total cost, while reducing them by 25% results in an 8% decrease. This highlights the significant impact of market demand on total cost, which is consistent with real-world production environments. Backorder cost (B) also shows a noticeable effect, with a 25% increase in B causing total cost to rise by approximately 2%. In contrast, holding cost (H) and unit production cost (C) have minimal influence, contributing less than 1% to total cost variation under similar changes.

In group 2, energy cost (E) has the greatest impact, increasing total cost by 15%, confirming the critical role of energy in the model. Combined with the influence of setup cost, this underscores the need to choose energy-efficient machines. Idle energy consumption (W) is also important, highlighting the value of managing standby energy. In contrast, the deterioration rate (α) has minimal effect, as the model already minimizes time-related losses.

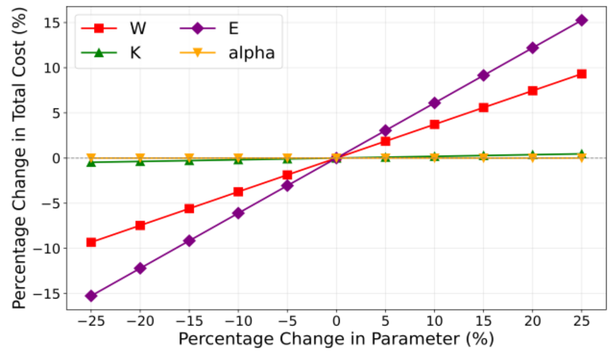

Figure 4. *Impact of varying Group 2 on the total cost*

In group 3 energy efficiency (ρ), warehouse temperature (T_w), and storage utilization factors (δ, γ) significantly influence total cost approximately 4 - 6%, while μ and λ have minimal impact. Therefore, optimizing warehouse temperature, energy efficiency, and storage utilization is key to reducing costs.

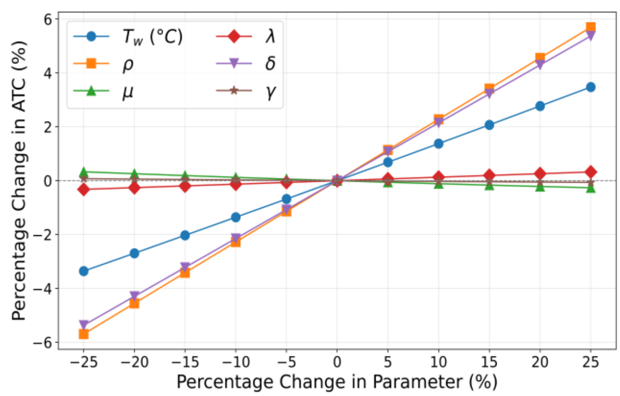

Figure 5. *Impact of varying Group 3 on the total cost*

5. Conclusion

This study proposes an enhanced EPQ model that integrates energy consumption, product deterioration, and backordering to better reflect real-world production environments. The model systematically considers energy usage during both production and storage stages and applies an exponential decay function to represent product deterioration, thereby improving the accuracy and practicality of inventory decision-making. Numerical results indicate that integrating energy costs into the deterioration inventory model results in significant cost savings and improved operational efficiency. From a managerial perspective, the findings highlight the importance of accurate demand forecasting and responsive production planning to manage demand variability. The important factors according to the study are energy cost and setup costs thus emphasizing the importance of using energy-efficient machinery and scheduling methods along with energy-efficient operational practices. In addition, maintaining optimal storage conditions and inventory levels is essential for minimizing deterioration and reducing energy waste in warehousing operations.

Future research may extend this model by incorporating variable deterioration rates for greater realism, expanding to multi-stage supply chains for coordinated decisions, and including transportation energy use to capture total system energy costs. Further integration of carbon emissions and renewable energy could strengthen the model's sustainability focus.

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