MODELING RUMOR PROPAGATION IN SCHOOLS USING MATHEMATICAL MODELS: FROM ANALYSIS TO SOLUTIONS

MÔ HÌNH HÓA SỰ LAN TRUYỀN TIN ĐỒN TRONG NHÀ TRƯỜNG BẰNG MÔ HÌNH TOÁN HỌC: TỪ PHÂN TÍCH ĐẾN GIẢI PHÁP

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Abstract - This study proposes an integrated SIR and SEIR model to analyze and control rumor spread in Vietnamese schools, offering evidence-based solutions for management. It integrates time delay (hesitation) and control functions (education, correction), calibrated from a survey of 520 students across 20 junior and 20 high schools in Da Nang. Four scenarios-baseline, influencing factors, immunity, and intervention-were simulated. The SEIR intervention scenario proved most effective, reducing the peak spread (I_{max}) to 3.6% (19/520) after 8.01 days versus 30.8% (160/520) after 23.14 days in the SIR baseline, reducing R_{final} from 470 to 442. Sensitivity analysis shows the infection rate (β) impacts spread more than the removal rate (γ). The strategy suggests short-term γ increase via authoritative information and long-term β reduction through digital literacy. The model aids in forecasting and mitigating rumor effects.

Key words - Rumor propagation; SEIR model; delay; control strategy; school environment.

Tóm tắt - Nghiên cứu đề xuất mô hình toán học tích hợp dựa trên SIR và SEIR để phân tích, kiểm soát lan truyền tin đồn trong trường học Việt Nam, cung cấp giải pháp dựa trên bằng chứng cho quản lý giáo dục. Tích hợp độ trễ thời gian (giai đoạn do dự) và hàm kiểm soát (giáo dục, đính chính), tham số được hiệu chỉnh từ khảo sát 520 học sinh tại 20 trường Trung học cơ sở và 20 trường Trung học phổ thông ở Đà Nẵng. Bốn kịch bản mô phỏng bao gồm: gốc, có yếu tố tác động, miễn nhiễm, và can thiệp. Kết quả cho thấy, SEIR kịch bản can thiệp hiệu quả nhất, giảm đỉnh lan truyền (I_{max}) còn 3,6% (19/520) sau 8,01 ngày, so với 30,8% (160/520) sau 23,14 ngày trong SIR gốc, giảm R_{final} từ 470 xuống 442. Phân tích độ nhạy xác nhận β (lây nhiễm) tác động mạnh hơn γ (loại bỏ). Chiến lược đề xuất: ngắn hạn tăng γ bằng thông tin chính thống, dài hạn giảm β qua giáo dục kỹ năng số. Mô hình hỗ trợ dự báo, giảm tác hại tin đồn.

Từ khóa - Lan truyền tin đồn; mô hình SEIR; trì hoãn; chiến lược kiểm soát; môi trường học đường.

1. Introduction

In the digital age, the explosive growth of social media platforms has transformed information into a valuable asset, while also becoming a potential "pathogen" [1, 2]. Rumors, defined as unverified statements that spread as facts, can result in serious consequences, ranging from public confusion and economic damage to the disruption of social stability [1, 3, 4]. The school environment, characterized by close-knit communities, high interaction, and sensitivity to social fluctuations, becomes a "fertile ground" for the emergence and spread of rumors [5]. Whether related to academics, personal relationships, or emergency events, rumors in schools profoundly impact students' psychology, undermine the safety of the educational environment, and erode trust in the institution [6]. Effectively controlling and quelling rumors is not only a managerial challenge but also an urgent requirement for maintaining the stability and healthy development of the educational environment.

Due to similarities in transmission mechanisms, mathematical models originally developed for epidemic research have been adapted to analyze the dynamics of social phenomena such as rumors [1, 3, 5, 7, 8]. This approach stems from the classical works of Daley and Kendall (DK model) and Maki and Thompson (MT model), which are based on the SIR epidemiological model (Susceptible-Infected-Removed) [1, 9, 10]. In the context of rumors, individuals are divided into three groups: Ignorant (Susceptible - S), Spreader (Infected - I), and Stifler (Removed - R) [5, 8, 11]. Subsequent studies have expanded the basic model, notably through the SEIR model (Susceptible-Exposed-Infected-Removed), which incorporates an "Exposed" (E) state to describe the hesitation phase before spreading [9, 12, 13, 20]. This latent or delay period is considered crucial, slowing down the outbreak and creating a "window of opportunity" for intervention [2, 3, 15].

Another important research direction focuses on control strategies, utilizing optimal control theory to propose effective measures such as preventive education (vaccination), isolation of spreaders, and rumor correction (treatment) [7, 16, 17]. These measures are modeled through control functions in differential equations to minimize the number of affected individuals and intervention costs [13, 16, 18]. However, the literature review reveals notable research gaps. First, although delay and control have been studied separately, few models integrate both to analyze their interaction. Second, most studies focus on online social networks or general contexts, while applications in school environments, especially in

Vietnam, remain limited and require empirical data to reflect student behavior.

This study proposes extended SIR and SEIR models, integrating time delay (reflecting hesitation) and control (intervention measures) to comprehensively analyze transmission dynamics and evaluate response strategies. Parameters such as infection and removal rates are calibrated from a survey of 520 students in Da Nang, ensuring practical relevance. Four simulation scenarios, ranging from no intervention to prevention and response, provide evidence-based policy solutions to support educational management in effectively controlling rumors.

2. Theoretical foundation and literature review

The methodology of this study is based on compartmental mathematical models, which have been widely used to describe the dynamics of social contagion phenomena, such as epidemics and rumors [19 - 21]. The similarity between rumor and disease transmission mechanisms has long been recognized, allowing researchers to adapt epidemiological models for a better understanding of social phenomena [20, 21]. These models divide the population into distinct groups based on their state regarding the rumor and describe the transitions between these groups over time using systems of differential equations [3, 7, 22].

In the context of rumor propagation in schools, the application of these models is particularly appropriate, as the population (students, teachers) can be considered a closed system with relatively homogeneous interactions [8, 14]. The two foundational models employed in this study are the SIR and SEIR models.

2.1. Basic SIR model

The SIR model is one of the most fundamental compartmental models, first proposed by Kermack and McKendrick in 1927 to study epidemic spread [22]. It has been widely adapted to model rumor propagation [10, 23]. In the rumor context, the population (N) is divided into three groups [8, 9, 22, 24]:

• **Susceptible (S):** Individuals who have not heard the rumor and are likely to believe and spread it upon exposure [3, 25]. In schools, these are students who are unaware of the rumor.

• **Infected (I):** Individuals who know the rumor and are actively spreading it [3, 25]. They serve as "sources" in the school's social network. This group is also referred to as "Spreaders" in some literature [8, 26].

• **Removed/Recovered (R):** Individuals who know the rumor but no longer spread it, possibly due to loss of interest, verification, or recovery from its effects [3, 25]. In epidemic models, this may include deceased individuals, but in rumor models, they are typically considered immune to further spreading [22]. Some references call this group "Stiflers" [8, 13].

Transitions between groups are described by the following system of differential equations [22]:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta \dfrac{S(t)I(t)}{N} \\ \dfrac{dI(t)}{dt} = -\beta \dfrac{S(t)I(t)}{N} - \gamma I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) \end{cases} \quad (1)$$

Where:

- $S(t), I(t), R(t)$ denote the number of individuals in each group at time t.

- $N = S(t) + I(t) + R(t)$ is the total population, assumed constant.

• β is the infection rate, representing the probability that a susceptible individual becomes a spreader after contact. In schools, β reflects the "persuasiveness" of the rumor and the frequency of student interactions.

• γ is the removal/recovery rate, representing the rate at which spreaders become removed. $\dfrac{1}{\gamma}$ can be interpreted as the average time an individual continues to spread the rumor.

2.2. Basic SEIR model

In reality, when an individual is exposed to a rumor, they may not immediately become a spreader. Instead, they may experience a latent or hesitation period to contemplate, verify, or evaluate the information [3, 12-14]. The SEIR model extends SIR by adding a fourth group to describe this phase [7, 12, 15]:

• Exposed (E): Individuals who have heard the rumor but have not started spreading it [9, 12, 14, 22]. They are in the "incubation" phase, possibly hesitating or considering [12, 14]. After some time, they may become spreaders or move directly to the removed group.

The system of differential equations for the basic SEIR model is as follows [9, 12, 22]:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta \dfrac{S(t)I(t)}{N} \\ \dfrac{dE(t)}{dt} = -\beta \dfrac{S(t)I(t)}{N} - \alpha E(t) \\ \dfrac{dI(t)}{dt} = \alpha E(t) - \gamma I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t) \end{cases} \quad (2)$$

Where:

- $S(t), E(t), I(t), R(t)$ denote the number of individuals in each group at time t.

- β, γ has the same meaning as in the SIR model.

• α is the transition rate from exposed (E) to infected (I). $\dfrac{1}{\alpha}$ representing the average incubation (hesitation) period before an individual decides to spread the rumor.

Introducing the "Exposed" group (E) allows the SEIR model to more accurately reflect the cognitive and decision-making processes when facing unverified information, especially in schools where students may discuss and verify information with peers before widespread dissemination [9, 12]. These foundational models serve as the starting point for developing more complex models tailored to school environments, enabling deeper analysis and proposing effective solutions.

2.3. Application in school contexts

SIR and SEIR models have been widely applied in various fields, from epidemics to rumor propagation on social networks [8, 10, 20]. In schools, where interactions are high and social structures are unique, these models need adjustment to reflect student behavior [5, 6]. Studies by Zhao et al. [1] and Zhu et al. [2] indicate that integrating delay and control enhances accuracy. The "Exposed" state in SEIR provides a "window of opportunity" for interventions such as education or information correction [13, 15]. However, applications in Vietnam remain limited, necessitating empirical data for parameter calibration. Bünyamin emphasizes the impact of rumors on the school environment and the need for appropriate management strategies [6].

3. Research methodology

To accurately model the dynamics of rumor propagation in schools - a highly interactive environment with unique social structures - this study proposes an extended model based on the SIR and SEIR frameworks presented in Section 2. This model not only inherits the basic compartments but also integrates practical factors such as transmission delay and intervention measures to control rumors, reflecting proactive efforts by individuals and schools.

3.1. Empirical data collection

To accurately simulate rumor dynamics in schools, we developed an extended model based on SIR and SEIR, incorporating basic compartments and practical elements such as transmission delay and intervention measures to control rumors, reflecting proactive efforts by individuals and schools. To ensure model parameters reflect actual behavior, a large-scale empirical survey was conducted with 520 students from secondary schools in Da Nang, Vietnam. The sample was randomly stratified to ensure representativeness across grade levels and school types. The survey instrument included a 34-item questionnaire, collecting data on social interaction frequency (both direct and online), attitudes toward unverified information (trust level, sharing tendency), hesitation time before spreading rumors, factors influencing the decision to stop spreading (loss of interest, awareness of misinformation, influence from authoritative sources), and readiness to accept counter-information. Key survey items included: *'Name (optional)', 'Gender', 'School', 'Class', 'Social network', '2.1 Heard rumors?', '2.2 Nature of rumors', '2.3 Upon hearing rumors', '2.4 Role', '3.1 Number of people informed per day', '3.2 Spread rate', '3.3 Factors accelerating spread', '4.1 Consider consequences', '4.2 Responsibility', '4.3 Effective solutions', '4.4 Immediate sharing', '4.5 Irresistible sharing', '4.6 Group integration', '4.7 Trust friends over announcements', '4.8 Group chat messages', '4.9 Believe if many mention', '4.10 Stop sharing if friends disbelieve', '4.11 Know how to verify', '4.12 Awareness of consequences', '4.13 Check official sources', '4.14 Trust school announcements', '4.15 Stop sharing upon school notice', '4.16 Consult teachers', '4.17 Desire timely information', '4.18 Sharing time', '4.19 Discussion after hearing', '4.20 Factors influencing sharing', '5. Suggestions'*. This dataset is a critical empirical foundation for parameter estimation and calibration, enhancing the validity and reliability of simulation results.

3.2. Proposed mathematical models

The basic SIR and SEIR models provide a solid foundation but are insufficient to capture the full complexity of rumor propagation in schools. In reality, this process is influenced by psychological and behavioral factors such as information processing delay and intervention efforts by individuals or organizations. Therefore, we propose extended models by integrating time delay and control functions. These control functions represent deliberate efforts to mitigate the negative impacts of rumors, such as awareness-raising education, school-issued corrections, or isolating spreaders.

3.2.1. Extended SIR model

The extended SIR model is designed to describe a key reality: a susceptible individual does not immediately become a spreader upon exposure to a rumor. They require an incubation or delay period, denoted as τ . During τ , the rumor remains latent in the individual's cognition.

Simultaneously, we introduce two time-weighted control functions, u and v, to describe intervention efforts:

- u : Represents preventive measures targeting the susceptible group (S), such as educational campaigns on critical thinking and digital skills, helping students become "immune" and move directly to the Removed group (R). In the model, this is the "vaccination" measure.
- v : Represents interventions targeting the infected group (I), such as rumor correction, teacher counseling, or actions encouraging spreaders to cease their behavior. In the model, this is "treatment" or "quarantine".

Combining these factors, the delay differential equations (DDEs) for the extended SIR model are proposed as follows:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta \dfrac{S(t)I(t)}{N} - uS(t) \\ \dfrac{dI(t)}{dt} = -\beta \dfrac{S(t-\tau)I(t-\tau)}{N} - \gamma I(t)*\left(I(t)+R(t)\right) - v*I(t) \\ \dfrac{dR(t)}{dt} = \gamma I(t)*\left(I(t)+R(t)\right) + v*I(t) + u*S(t) \end{cases} \quad (3)$$

Where:

+ S,I,R : Number of individuals in Susceptible, Infected, and Recovered states at time t .

+ $S(t-\tau),I(t-\tau),R(t-\tau)$: Value of S,I,R at time $t-\tau$ (delay).

+ $N = R + I + R$: Total number of students (=520, based on school size), constant.

+ β : Infection rate (successful contact rate, calibrated from survey = 0.6).

+ γ : Recovery or removal rate (calibrated = 0.2–0.3).

+ τ : Delay (lag) in transmission, estimated at 1 day from survey (item 4.18: sharing time).

+ u : Intervention rate transferring Susceptible directly to Recovered (=0.1, from scenario 4).

+ v : Intervention rate transferring Infected directly to Recovered (=0.1, from scenario 4).

3.2.2. Extended SEIR model

The SEIR model inherently includes a latent (Exposed - E) phase, typically modeled with ordinary differential equations (ODEs), meaning the transition time follows an exponential distribution. In reality, hesitation before rumor spreading may resemble a fixed period. Therefore, we propose an SEIR model combining both types of delay: intrinsic delay in the E compartment and time delay τ in transmission.

The delay differential equations for the extended SEIR model are:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta \dfrac{S(t-\tau)I(t-\tau)}{N} - uS(t) \\ \dfrac{dE(t)}{dt} = -\beta \dfrac{S(t-\tau)I(t-\tau)}{N} - \alpha E(t) \\ \dfrac{dI(t)}{dt} = \alpha E(t) - \gamma \dfrac{I(t)*(I(t)+R(t))}{N} - vI(t) \\ \dfrac{dR(t)}{dt} = \gamma \dfrac{I(t)*(I(t)+R(t))}{N} + vI(t) + uS(t) \end{cases} \tag{4}$$

In this model, infection creates new Exposed (E) individuals depending on the system state at time $t-\tau$, while the transition from Exposed (E) to Infected (I) occurs instantly at rate α. This combination enables more flexible modeling, capturing both the natural hesitation phase (E compartment) and the initial transmission delay (τ). Applying control coefficients u, v allows evaluation and comparison of intervention strategies, from early prevention to crisis management, to identify optimal solutions for rumor suppression in schools.

3.3. Scenario simulation and analysis

We use the extended SIR and SEIR models, integrating time delay and control functions, to simulate and evaluate intervention strategy effectiveness. Time delay simulates the latent phase when an individual receives a rumor but does not immediately spread it, while control functions represent efforts to accelerate rumor extinction, such as educational campaigns or administrative interventions.

3.3.1. Model parameter estimation

This study focuses on estimating model parameters for rumor propagation in schools, based on empirical survey data to ensure accuracy and contextual relevance. The population (N) is defined as the number of students, with $N = 520$, including Susceptible (S), Infected (I), and Removed (R) groups. The initial infected count is assumed to be 10 students ($I0 = 10$), about 2% of the total, while the initial removed count is 0 ($R0 = 0$), as no students are immune at the outset, resulting in an initial susceptible count $S0 = N0 - I0 - R0 = 510$. Dynamic parameters include infection rate (β), removal rate (γ), and incubation time (τ), estimated from survey data, with β reflecting transmission likelihood upon contact, γ representing the rate from infected to removed, and τ denoting hesitation time before rumor spreading. The basic reproduction number R_0 is calculated as β/γ, with $R_0 > 1$ indicating outbreak potential and $R_0 < 1$ predicting natural extinction, providing a basis for effective intervention analysis and recommendations.

3.3.2. Simulation scenarios

This study presents simulation scenarios of rumor propagation in schools, based on four main models to evaluate intervention effectiveness and parameter adjustment:

Baseline Model: Simulates natural transmission with 10 initial infected students (2% of total), β = 0.3, γ = 0.1, resulting in $R_0 = 3$, indicating strong outbreak potential.

Survey-Adjusted Model: Adjusts parameters based on the survey of 520 students in Da Nang, with β and γ calibrated to reflect actual behavior, and incubation period (τ) integrated to simulate hesitation.

Proactive Immunity Model: Introduces proactive immunity, reducing β to 0.3 and increasing γ to 0.1, enabling early control via education, with R_0 reduced to 3, decreasing transmission scale.

Adjusted Model with Intervention: Combines intervention, adjusts β to 0.6 and γ to 0.2, resulting in $R_0 = 2$, showing clear effectiveness in flattening the transmission curve, reducing the peak, and extending response time, providing a scientific basis for educational management strategies.

3.3.3. Model sensitivity analysis

To better understand the impact of each parameter on model outcomes, we conducted sensitivity analysis. This process involves varying each input parameter (e.g., β, γ, time delay τ) within a certain range and observing corresponding changes in output variables (e.g., peak outbreak size, total affected population).

Sensitivity analysis identifies the most "sensitive" parameters, where small changes cause large variations in simulation results. Identifying these parameters is crucial, as it allows focused intervention efforts on factors most influential in controlling rumor spread in schools. For example, if the model shows high sensitivity to γ, increasing the removal rate (e.g., through corrections) will be the most effective strategy.

4. Results and discussion

This section presents the simulation results from four scenarios constructed for the extended SIR and SEIR models. These results are analyzed and compared to evaluate the dynamics of rumor propagation in the school environment and the effectiveness of intervention measures.

Subsequently, we discuss the scientific implications, model limitations, and propose practical policy recommendations.

4.1. Results

Simulations were performed on a hypothetical population β, γ, corresponding to the number of surveyed students. Unless otherwise noted, initial conditions were set as $I0 = 10$ (about 2% of students initially spreading the rumor), (no students initially immune), and $S0 = 510$.

4.1.1. Comparison of four scenarios in each model

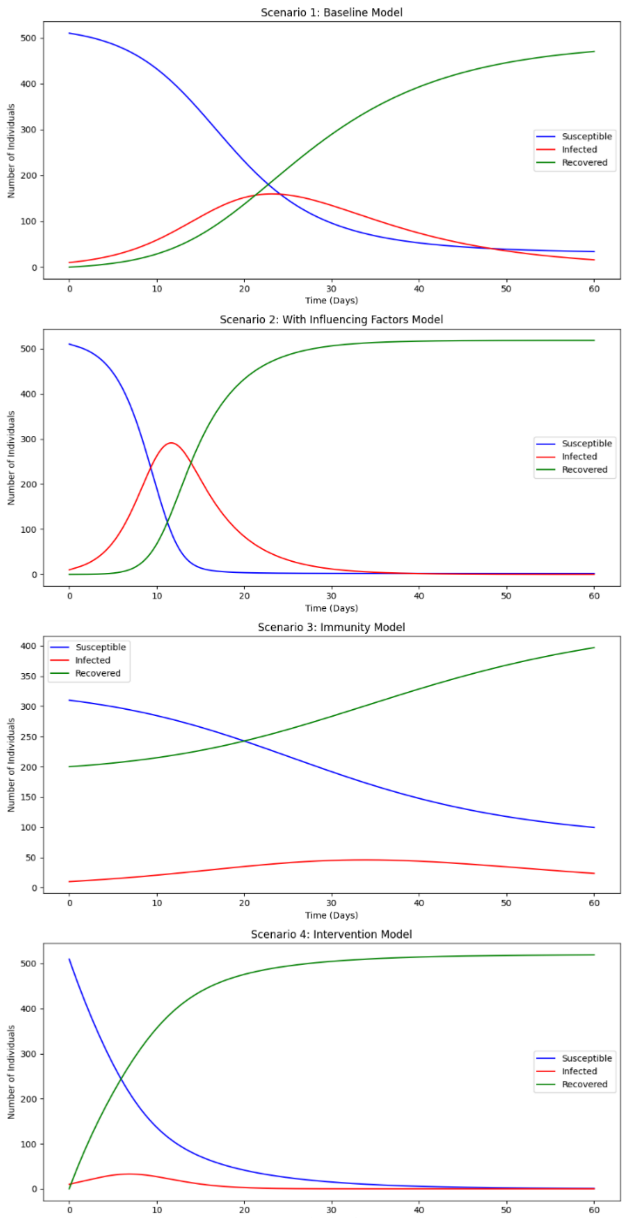

Figure 1a. *SIR model results*

The study compared the effectiveness of four rumor propagation scenarios in both SIR and SEIR models using simulation data, as presented in Table 1 and illustrated in Figures 1a and 1b. The scenarios include: 1) Baseline, 2) With influencing factors, 3) Immunity, and 4) Intervention, with parameters $\beta, \gamma, \alpha, u, v, R0$ adjusted to evaluate their impact on I_{max} (peak spread), t_{peak} (time to peak), and R_{final} (final number of affected individuals).

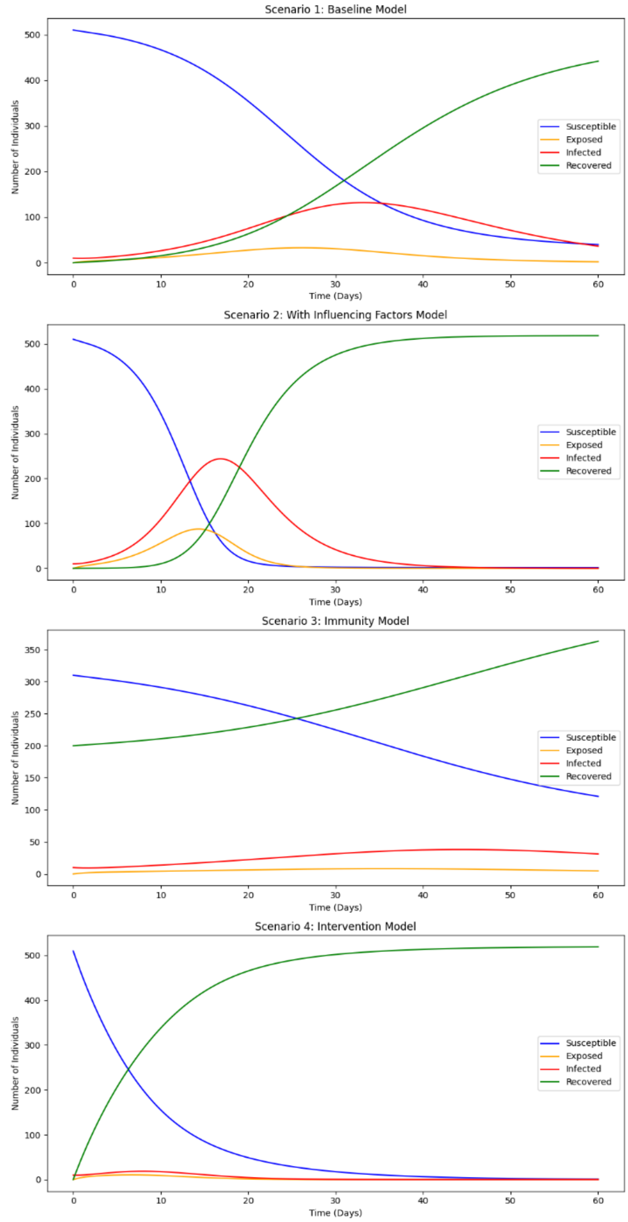

Figure 1b. *SEIR model results*

- Scenario 1: Baseline

With $\beta = 0.3$, $\gamma = 0.1$ for SIR, $\beta = 0.3$, $\gamma = 0.1$, $\alpha = 0.5$, $u = 0.05$, $v = 0.05$ for SEIR, the SIR model recorded I_{max} = 30.8% after 23.14 days, with R_{final} = 470 affected individuals. In contrast, SEIR yielded I_{max} = 25.4% after 33.16 days and R_{final} = 442, indicating that the "Exposed" phase reduces the peak and prolongs the time to peak.

- Scenario 2: With influencing factors

When increasing $\beta = 0.6$, $\gamma = 0.2$ for SIR and $\beta = 0.6$, $\gamma = 0.2$, $\alpha = 0.5$, $u = 0.05$, $v = 0.05$ for SEIR as above, SIR's I_{max} rose to 56.0% after 11.72 days (R_{final} = 518), while SEIR reached I_{max} = 46.9% after 16.83 days (R_{final} = 518). This difference demonstrates SEIR's greater effectiveness in reducing transmission intensity due to the delay phase.

- Scenario 3: Immunity

With $\beta = 0.3$, $\gamma = 0.1$, $R0 = 200$ for SIR and $\beta = 0.3$

, $\gamma = 0.1$, $R0 = 200$, $\alpha = 0.5$, $u = 0.1$, $v = 0.1$ for SEIR, SIR's I_{max} dropped to 8.8% after 33.86 days (R_{final} = 397), while SEIR reached 7.3% after 44.37 days (R_{final} = 363). These results show that proactive immunity in SEIR significantly reduces the scale of propagation.

- **Scenario 4: Intervention**

With $\beta = 0.6$, $\gamma = 0.3$ for SIR and $\beta = 0.6$, $\gamma = 0.3$, $\alpha = 0.5$, $u = 0.1$, $v = 0.1$ for SEIR, SIR's I_{max} was 6.35% after 6.81 days (R_{final} = 519), while SEIR reached 3.6% after 8.01 days (R_{final} = 519). This confirms that interventions (lower β, higher γ) in SEIR are effective in controlling the peak.

From Table 1, the extended SEIR model (scenario 4) shows that early intervention reduces transmission by more than 21.8% compared to the original SEIR model (scenario 1). This demonstrates that the exposed phase acts as a golden window of opportunity for school intervention.

Table 1. *Comparison of key indicators between 4 scenarios for the SIR and SEIR models*

Scenario	Model	Parameters	Peak spread (Imax, % of population)	Time to peak (days)	Final rumor scale (R_{final})
1: Baseline	SIR	$\beta = 0.3, \gamma = 0.1$	30.8% (160/520)	23.14	470
	SEIR	$\beta = 0.3, \gamma = 0.1$ $\alpha = 0.5$ $u = 0.1, v = 0.1$	25.4% (132/520)	33.16	442
2: Refine	SIR	$\beta = 0.6, \gamma = 0.2$	56.0% (291/520)	11.72	518
	SEIR	$\beta = 0.6, \gamma = 0.2$ $\alpha = 0.5$ $u = 0.1, v = 0.1$	46.9% (244/520)	16.83	518
3: Immunity	SIR	$\beta = 0.3, \gamma = 0.1,$ $R0 = 200$	8.8% (46/520)	33.86	397
	SEIR	$\beta = 0.3, \gamma = 0.1,$ $R0 = 200$ $\alpha = 0.5$ $u = 0.1, v = 0.1$	7.3% (38/520)	44.37	363
4: Intervene	SIR	$\beta = 0.6, \gamma = 0.3$	6.35% (33/520)	6.81	519
	SEIR	$\beta = 0.6, \gamma = 0.3$ $\alpha = 0.5$ $u = 0.1, v = 0.1$	**3.6% (19/520)**	**8.01**	**519**

As illustrated in Figures 1a and 1b, the SEIR model consistently produces a lower peak and a slower time to peak compared to the SIR model. The incubation (latent) period in SEIR, representing student hesitation before spreading rumors, slows the outbreak process. Overall, SEIR outperforms SIR by integrating the "Exposed" phase, especially in scenarios with delay or intervention, resulting in lower I_{max} and R_{final}, and extending t_{peak} for better control opportunities. This shows that SEIR better reflects reality, providing a longer window for schools to implement interventions before rumors reach their peak.

4.1.2. Sensitivity analysis of SIR and SEIR models to β and γ

The study conducted sensitivity analysis on the SIR and SEIR models to assess the impact of γ (removal rate) and β (infection rate) on key indicators such as I_{max} and t_{peak}. Results are illustrated in Figures 3a and 3b, based on simulation data with varying β and γ values.

In Figure 3a, with $\beta = 0.6$ fixed and γ varied from 0.3 to 0.9, the SEIR model shows I_{max} decreasing from 19 to 10 students and t_{peak} dropping from 8.01 days to 3.51 days. Similarly, the SIR model records I_{max} decreasing from 33 to 18 students and t_{peak} from 6.81 to 3.61 days, reflecting the effectiveness of increasing γ in limiting rumor spread. The difference between SIR and SEIR also highlights that SEIR, with its latent phase (Exposed – E), provides a more realistic description of rumor dynamics, especially as γ increases, helping to prolong the time to peak and reduce peak intensity compared to SIR.

In Figure 3b, with $\gamma = 0.3$ fixed and β varied from 0.6 to 1.8, both models show significant increases in I_{max} with SEIR rising from about 19 to over 87 students and SIR from 33 to 158 students. Additionally, results indicate that SIR is more sensitive to γ than SEIR, with t_{peak} decreasing as β increases. This shows that the infection rate β strongly influences the propagation scale, especially in SIR, while SEIR demonstrates better adjustment due to the "Exposed" phase.

Overall, sensitivity analysis indicates that I_{max} is more sensitive to changes in β than γ in both models, but SEIR exhibits greater stability due to the integrated delay factor. These findings confirm that strategies to reduce β (such as critical thinking education) have long-term effects, while increasing γ (such as rumor correction) is an effective short-term tool for control.

4.2. Empirical validation of the model

To assess the accuracy of the extended SEIR model against actual survey data, we used quantitative metrics including RMSE, MAE, MAPE, R^2, and AIC. These indices were calculated by comparing simulation predictions with interpolated real data from 20 observation points, as shown in Figure 2.

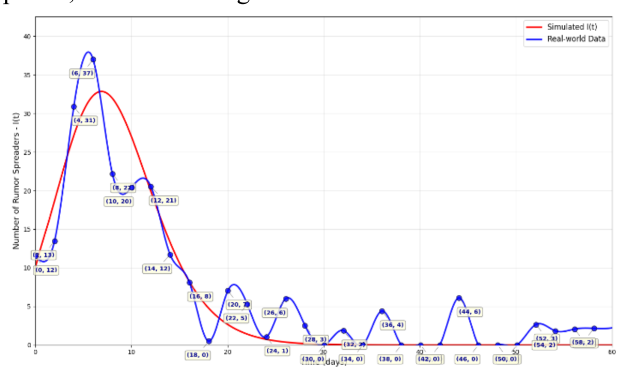

Figure 2. *Comparison of simulation models and experimental data*

Analysis shows RMSE (Root Mean Square Error) = 3.390 and MAE (Mean Absolute Error) = 2.471, indicating low average deviation, with errors under 4 individuals, demonstrating high accuracy in predicting the number of rumor spreaders (I(t)). R^2 (Coefficient of determination) = 0.874 indicates that the model explains about 87.4% of the variance in actual data, an excellent fit. AIC (Akaike Information Criterion) = 1474.975 (với k=5 parameters: $\beta, \gamma, \alpha, u, v$) confirms a good balance between model complexity and data fit. Overall, these metrics affirm the reliability of the extended SEIR model, especially in the context of empirical data from Vietnamese schools, supporting practical application of rumor control strategies.

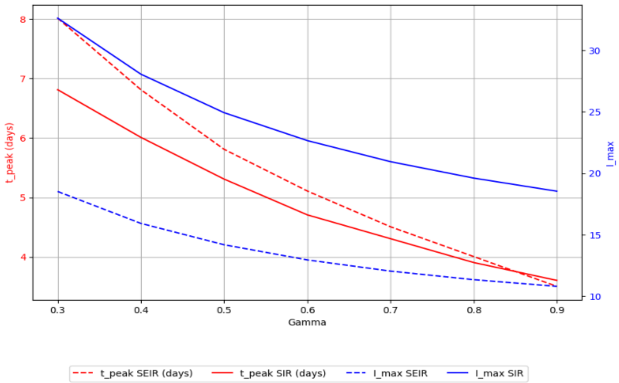

Figure 3a. *Sensitivity analysis of SIR and SEIR models: I_{max} and t_{peak} with $\beta = 0.6, \gamma = (0.3 - 0.9)$*

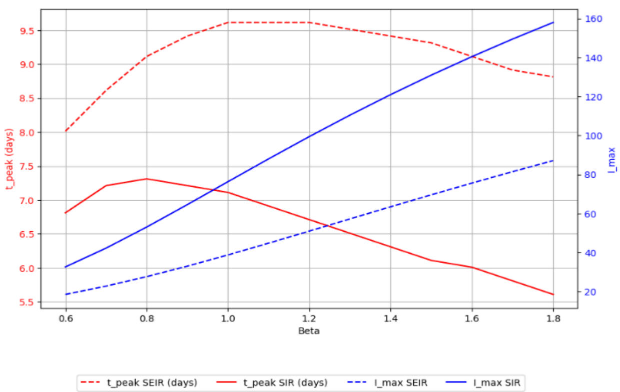

Figure 3b. *Sensitivity analysis of SIR and SEIR models: I_{max} and t_{peak} with $\beta = (0.6 - 1.8), \gamma = 0.3$*

4.3. Discussion

4.3.1. Scientific implications: The role of delay and control

This study demonstrates how integrating delay and control factors into rumor propagation modeling in schools - where delay is considered as the incubation time and control is simulated through control functions - can accurately describe population group changes during transmission. The delay factor, derived from the "Exposed" state in SEIR, reflects the hesitation phase of students upon receiving rumors before deciding to spread, with incubation time adjusted to match actual behavior. Parameters such as infection and removal rates are calibrated to precisely simulate group transitions. Control functions represent school interventions, such as official announcements or education, aiming to reduce spread rate and increase removal rate, thereby flattening the transmission curve and enabling effective control.

4.3.2. Model challenges and limitations

This study explores integrating heterogeneous network structures into rumor propagation modeling in schools, extending basic models to more accurately reflect complex social characteristics. Heterogeneous networks are defined based on differences in influence and interaction among individuals. The model focuses on adjusting dynamic parameters, with β for infection rate, γ for removal rate, and α for anti-rumor transmission rate, to evaluate the impact of influential nodes, such as prominent students or leadership groups. Simulation results show that the heterogeneous network model provides more accurate predictions than homogeneous models, especially in simulating transmission speed and intervention effectiveness, leading to optimal recommendations based on real data analysis.

4.3.3. Policy implications and recommendations for schools

Sensitivity analysis of model parameters helps identify key factors affecting simulation outcomes, such as maximum spread, time to peak, and total affected individuals. In the SEIR model, we analyze how changes in main parameters β (infection rate), γ (removal rate), τ (incubation period), and others - affect transmission dynamics. Results show that β has the strongest impact on rumor scale and speed, while γ primarily affects time to peak and total affected. Specifically, increasing β rapidly raises the number of spreaders, resulting in a higher and earlier peak, while increasing γ helps reduce the peak and extend the time to peak, also decreasing overall affected. Sensitivity analysis also reveals that τ plays an important role in creating a "window of opportunity" for intervention, as increasing τ slows initial propagation, allowing more effective control measures. These findings provide a basis for prioritizing intervention strategies, such as focusing on reducing β through critical thinking and digital literacy education, and increasing γ via timely official information, mobilizing homeroom teachers and academic advisors, and organizing dialogues - optimizing resources and minimizing rumor impact in schools (see Table 2).

Table 2. *Comments in the survey questionnaire*

Rumor control solutions	Strategy	Number of comments
Need to be educated about the consequences of rumors.	Reduce β	46
It is necessary to teach how to recognize fake news.	Reduce β	35
Need to control Zalo chat groups.	Reduce β	18
Need to control rumors on TikTok.	Reduce β	17
Need to control student chat groups.	Reduce β	17
The school needs to notify earlier.	Increase γ	17
The school needs to organize a seminar on rumors.	Increase γ	5
It is necessary to organize a workshop on information verification.	Increase γ	3
It is necessary to organize an extracurricular session on rumors.	Increase γ	1
An official announcement is required sooner.	Increase γ	35

In the future, research could be expanded by applying rumor propagation models that incorporate the influence of varying educational levels and hesitation mechanisms towards misinformation in the population, based on the framework of Wang et al. [18]. This would add an input parameter D(t), representing the number of hesitant individuals with lower education at time t, to enhance accuracy. This approach would expand the SEIR model by integrating hesitation probability η the impact of education rate on compartment transition rates, thus better simulating behavior in diverse school environments. Additionally, control functions should operate within the range [0,1] to ensure the number of individuals affected by rumors always stays below a predetermined threshold at any time, by studying both continuous and discrete-time SIR/SEIR models, leading to new strategies for minimizing affected individuals as proposed by Dehaj et al. [13]. This method leverages optimal control theory with $u(t) \in [0,1]$ to regulate $I(t) \leq threshold$, applying Pontryagin's maximum principle to optimize intervention effectiveness.

5. Conclusion

This study demonstrates that the SEIR model, integrating time delay and control functions, provides an effective mathematical approach to simulating and controlling rumor propagation in school environments. It outperforms the traditional SIR model by incorporating the "Exposed" state, allowing accurate reflection of students' hesitation phase and timely intervention opportunities. The main contribution of this research lies in the combination of time delay and control functions within the Vietnamese educational context, where empirical survey data from 520 students in Da Nang were used for parameter calibration, showing high predictive accuracy for rumor dynamics, especially when combining intervention strategies such as official information dissemination and digital literacy education, which help reduce the peak and extend response time. Sensitivity analysis highlights the critical roles of infection rate (β) and removal rate (γ), while the incubation period (τ) creates a "window of opportunity" for effective control. The study proposes a two-tiered policy solution: short-term rapid action to extinguish rumors, and long-term investment in critical thinking education, with the model serving as a decision-support tool for educational managers. However, future research should consider heterogeneous networks, forgetting-remembering mechanisms, and optimization of intervention strategies to enhance rumor management in the digital era.

REFERENCES

[1] L. Zhao, X. Qiu, X. Wang, and J. Wang, "Rumor spreading model considering forgetting and remembering mechanisms in inhomogeneous networks", *Physica A: Statistical Mechanics and its Applications*, Vol. 392, Issue. 4, pp. 987-994, 2013. https://doi.org/10.1016/j.physa.2012.10.031

[2] L. Zhu, M. Zhou, and Z. Zhang, "Dynamical analysis and control strategies of rumor spreading models in both homogeneous and heterogeneous networks", *Journal of Nonlinear Science*, Vol. 30, Issue.6, pp. 2545-2576, 2020. https://doi.org/10.1007/s00332-020-09629-6

[3] S. Al-Tuwairqi, S. Al-Sheikh, and R. Al-Amoudi, "Qualitative analysis of a rumor transmission model with incubation mechanism", *Open Access Library Journal*, Vol. 2, Issue. 11, pp. 1-12, 2015. http://dx.doi.org/10.4236/oalib.1102040

[4] M. Ghosh, P. Das, and P. Das, "A comparative study of deterministic and stochastic dynamics of rumor propagation model with counter-rumor spreader", *Nonlinear Dynamics*, Vol. 111, Issue. 18, pp. 16875-16894, 2023. https://doi.org/10.1007/s11071-023-08768-1

[5] Y. M. R. Marbun, T. Tulus, S. Sutarman, and E. Herawati, "Dynamic Analysis of Rumor Spreading Models in Social Networks with Time Delay", *Information Sciences Letters*, Vol. 12, Issue. 10, 2023. https://digitalcommons.aaru.edu.jo/isl/vol12/iss10/18

[6] H. A. N. Bünyamin, "Management techniques for organizational rumor and gossip in schools", *Dinamika Ilmu*, Vol. 21, Issue. 1, pp. 177-203, 2021. https://doi.org/10.21093/di.v21i1.3275

[7] D. Xu, X. Xu, Y. Xie, and C. Yang, "Optimal control of an SIVRS epidemic spreading model with virus variation based on complex networks", *Communications in Nonlinear Science and Numerical Simulation*, Vol. 48, pp. 200-210, 2017. https://doi.org/10.1016/j.cnsns.2016.12.025

[8] H. Sun, Y. Sheng, and Q. Cui, "An uncertain SIR rumor spreading model", *Advances in Difference Equations*, Vol. 2021, Issue. 1, p.286, 2021. https://doi.org/10.1186/s13662-021-03386-w

[9] K. Afassinou, "Analysis of the impact of education rate on the rumor spreading mechanism", *Physica A: Statistical Mechanics and Its Applications*, Vol. 414, pp. 43-52, 2014. https://doi.org/%2010.1016/j.physa.2014.07.041

[10] C. M. Batistela, M. A. Cabrera, A. C. Godoi, and J.R. Piqueira, "SIR model for rumor propagation", *ENOC*, 2020.

[11] F. I. Maulana and Y. Ramdani, "Python Application to SEIR Model of the Spread of Malaria", *BIOS: Jurnal Teknologi Informasi dan Rekayasa Komputer*, Vol. 5, Issue. 2, pp. 150-160, 2024. https://doi.org/10.37148/bios.v5i2.151

[12] Q. Liu, T. Li, and M. Sun, "The analysis of an SEIR rumor propagation model on heterogeneous network", *Physica A: Statistical Mechanics and its Applications*, Vol. 469, pp. 372-380, 2017. https://doi.org/10.1016/j.physa.2016.11.067

[13] I. Dehaj, A. Dehaj, A. Tridane, M.A. Aziz-Alaoui, and M. Rachik, "A novel approach in controlling the spread of a rumor within a crowd", *Results in Control and Optimization*, Vol. 18, p.100534, 2025. https://doi.org/10.1016/j.rico.2025.100534

[14] X. Chen and N. Wang, "Rumor spreading model considering rumor credibility, correlation and crowd classification based on personality". *Scientific reports*, Vol. 10, Issue. 1, p. 5887, 2020. https://doi.org/10.1038/s41598-020-62585-9

[15] M. Barro, A. Guiro, and D. Ouedraogo, "Optimal control of a SIR epidemic model with general incidence function and a time delays", *Cubo (Temuco)*, Vol. 20, Issue. 2, pp. 53-66, 2018. http://dx.doi.org/10.4067/S0719-06462018000200053

[16] S. Chen, H. Jiang, L. Li, and J. Li, "Dynamical behaviors and optimal control of rumor propagation model with saturation incidence on heterogeneous networks", *Chaos, Solitons & Fractals*, Vol. 140, p. z110206, 2020. https://doi.org/10.1016/j.chaos.2020.110206

[17] K. Myilsamy, M.S. Kumar, and A.S. Kumar, "Optimal control of a rumor propagation model in online social network by considering influential nodes", *Results in Control and Optimization*, Vol. 14, p.100339, 2024. https://doi.org/10.1016/j.rico.2023.100339

[18] H. Wang, S. Kang, and Y. Hu, "Dynamic analysis and optimal control of rumor propagation models considering different education levels and hesitation mechanisms", *AIMS Mathematics*, Vol. 9, Issue. 8, pp.20089-20117, 2024. https://doi.org/0.3934/math.2024979

[19] F. Vazquez, "Modeling and analysis of social phenomena: challenges and possible research directions", *Entropy*, Vol. 24, Issue. 4, p. 491, 2022. https://doi.org/10.3390/e24040491

[20] S. R. Septyawan, E. Y. Bunga, N. Nuraini, and J. P. Arcede, "The Spread of Rumors in Society: A Mathematical Modeling Approach in Election Case Studies", *Communication in Biomathematical Sciences*, Vol. 7, Issue. 2, pp. 202-218, 2024. https://doi.org/10.5614/cbms.2022.7.2.3

[21] A. N. Zehmakan, C. Out, and S. H. Khelejan, "Why rumors spread fast in social networks, and how to stop it", *arXiv preprint arXiv*:2305.08558, 2023. https://doi.org/10.48550/arXiv.2305.08558

[22] L. Giang, "Application of the suscepbible-infected-removed model for forecasting and evaluating COVID-19 infection in Hanoi", *Vietnam Journals Online- VJOL*, Vol. 58, Issue. 2, 2022.

[23] Y. Liu, "Rumor spreading model considering education and remembering mechanism", *International Journal of Media, Journalism and Mass Communications (IJMJMC)*, Vol. 2, Issue. 3, pp. 1-6, 2016. http://dx.doi.org/10.20431/2454-9479.0203001

[24] S. H. Tseng, and N. T. Son "Agent-based modeling of rumor propagation using expected integrated mean squared error optimal design" *Applied System Innovation*, Vol. 3, Issue. 4, p. 48, 2020. https://doi.org/10.3390/asi3040048

[25] X. Wang, Y. Li, J. Li, Y. Liu, and C. Qiu, "A rumor reversal model of online health information during the Covid-19 epidemic", *Information Processing & Management*, Vol. 58, Issue. 6, p. 102731, 2021. https://doi.org/10.1016/j.ipm.2021.102731