A CONVOLUTION-BASED ASYMMETRY METRIC FOR INTERSEGMENTAL SYNERGISTIC COORDINATION IN SAGITTAL 2D GAIT ANALYSIS

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Abstract - This study proposes a novel method to evaluate gait symmetry based on the velocity patterns of body parts during walking. Unlike conventional approaches relying on electromyographic (EMG) or acceleration differences between sides, this method focuses on intersegmental coordination—the dynamic relationship between contralateral body parts. The coordination is modeled as a Linear Time-Invariant (LTI) system, reflecting how muscle contractions produce joint acceleration and how joint elasticity and viscosity affect velocity. Gait symmetry is then quantified by measuring the dissimilarity between the left and right LTI systems. The proposed metric was applied to video data from five healthy subjects walking under both normal and intentionally asymmetrical conditions. Results demonstrate that this velocity-based approach effectively captures gait asymmetries, providing a new framework for motion analysis that emphasizes the dynamic properties of musculoskeletal coordination.

Key words - Convolution-Based Dissimilarity Metric; Linear Time-Invariant (LTI) System; Inter-Segment Synergy Analysis; Gait Symmetry Evaluation

1. Introduction

Since the advent of Homo Sapiens, bipedal locomotion has been a fundamental human movement, characterized by alternating cyclic motion. This study focuses on the velocity patterns of various body parts during walking and explores a method for evaluating gait symmetry.

Previous studies evaluating gait symmetry have primarily relied on metrics such as maximum, minimum, and mean values or have visually assessed time-series patterns through graphical representations. For instance, Wang et al. [1] assessed asymmetry in knee flexion and extension during post-stroke gait using motion data and electromyography (EMG), focusing on the maximum and minimum knee joint angles and lower limb EMG signals. Rathore et al. [2] investigated asymmetrical gait in unilateral amputees using prosthetic limbs by employing a knee flexion angle potentiometer and a foot pressure sensor to analyze peak knee flexion angles and ground reaction forces. D'Arco et al. [3] evaluated gait symmetry based on plantar pressure parameters obtained from smart insoles. Similarly, Loiret et al. [4] utilized foot pressure sensors, as in Rathore et al. [2], but specifically examined the peak values of ground reaction forces at the thigh and hip level to assess symmetry. Willamson et al. [5] used accelerometers to evaluate symmetry based on the mean acceleration values. While these studies [1-5] incorporate temporal parameters such as stance duration and gait cycle time, they primarily focus on maximum, minimum, and average values of physical quantities. However, they do not perform detailed analyses of time-series patterns.

The studies conducted by Qin et al. and Yan et al. [6-9] analyzed gait symmetry by plotting simultaneously measured left and right electrostatic potential samples - generated through friction between the body, the floor, or clothing - on a plane and applying principal component analysis (PCA). However, these studies [6-9] do not take the sequential nature of the time-series data (i.e., that sample n is followed by sample $n+1$) or the nonlinear characteristics of time-series patterns into account.

Several studies have investigated gait symmetry through detailed analysis of time-series patterns. Yogev et al. [10] evaluated gait symmetry in patients with Parkinson's disease and elderly individuals at risk of falling using a foot pressure sensor system, considering swing time. Adamczyk et al. [11] estimated velocity from ground reaction forces and assessed gait symmetry through autocorrelation. Sant' Anna et al. [12] and Zhang et al. [13] employed wearable accelerometers to measure acceleration in both thighs and lower legs, using cross-correlation to evaluate symmetry. Gouwanda et al. [14] and Sheng et al. [15] proposed a symmetry evaluation method based on cross-correlation of angular velocity data obtained from wireless gyroscopes. Khoo et al. [16] assessed gait symmetry using time-series data of knee joint angles and joint moments. Arauz et al. [17] utilized motion capture technology to analyze changes in joint angles during treadmill walking (both normal-speed and high-speed conditions) to evaluate gait symmetry. Lena et al. [18] proposed a symmetry index by integrating a symmetry function derived from time-series data of foot joint angles over the gait cycle. Additionally, Diaz et al. [19] analyzed the vertical trajectory of the body's center of mass in lower-limb amputees (particularly transfemoral amputees) and introduced a frequency-domain gait symmetry evaluation method using Discrete Fourier Transform (DFT). These studies have compared left and right time-series patterns in both the time and frequency domains as symmetry evaluation metrics, employing analytical approaches similar to Proposed Method I in this study.

None of the existing methods evaluate the left-right difference in Time Series Pairs (TSPs), as proposed in Proposed Method II of this study. Additionally, these conventional studies rely on specialized equipment, making them less accessible for general use. This study aims to develop a simple and practical gait symmetry evaluation system that does not require special devices, utilizing smartphone video recordings and OpenPose. OpenPose extracts skeletal information from video footage captured by a standard camera. Since no specialized equipment is needed, this approach significantly reduces the burden of both preparation and measurement, making gait symmetry analysis more accessible and convenient.

The objective of this study is to propose a dissimilarity metric for evaluating the left-side and right-side intersegmental coordination systems underlying Time Series Pairs (TSPs) extracted from video recordings of gait, without the need for specialized equipment. Furthermore, this study aims to determine whether the proposed metric can effectively quantify left-right asymmetry in gait patterns.

2. Theoretical Background

In this study, we propose two indices of left-right symmetry for analyzing cyclic alternating movements such as walking. Proposed Method I: A method that compares left and right movement patterns by shifting half a walking cycle; Proposed Method II: A method that compares coordinated systems based on transfer functions.

Proposed Method I measures waveform similarity by shifting the movement velocity calculated from the left and right body coordinates by half cycle in total. Proposed Method II uses the transfer function between time-series signals of two body parts. Specifically, it evaluates left-right symmetry using a non-similarity index based on an LTI (Linear Time Invariant) system, where the movement of the wrist (horizontal or vertical) is analyzed in relation to the movement of the ankle (horizontal or vertical).

2.1. Left-Right Symmetry Index Based on half Cycle Shift Transformation

For a discrete-time series of period $N, \{x(0), \dots, x(N-1)\}$, Fourier transformation is applied, and the Fourier coefficient $X(k) \in \mathbb{C}$ of waveform k is given by: $X(k) = \sum_{n=0}^{N-1} x(n) \exp(-j\, 2\pi kn/N)$, where $k = -K, \dots, -1, 0, 1, \dots, K$, j is the imaginary unit, and K is the maximal integer less than or equal to $N/2$.

For a periodic function $x(t)$ with period T, the half cycle shift transformation is represented as:

$$x\left(t - \frac{T}{4}\right) = \sum_{k=-\infty}^{\infty} X(k) \exp\left(j \left(\frac{2\pi k}{T}\right)\left(t - \frac{T}{4}\right)\right)$$
$$= \sum_{k=-\infty}^{\infty} X(k) \exp\left(j \frac{2\pi kt}{T}\right) \exp(-j\, \pi k/2)$$
$$= \sum_{k=-\infty}^{\infty} (-j)^k X(k) \exp(j\, 2\pi kt/T).$$

This means that multiplying each Fourier coefficient $X(k)$ by $(-j)^k$ results in a $+1/4$ cycle shift, while multiplying by j^k results in a $-1/4$ cycle shift.

For observed discrete-time right-side and left-side data $x(n)$ and $y(n)$, respectively, with sample size M (including 3-6 strides), this transformation shifts half a cycle. The left-right movement symmetry is then evaluated using the correlation coefficient:

$$r = \rho_{X,Y} / (\sigma_X\, \sigma_Y),$$

where $\bar{x} = \sum_{n=0}^{M-1} x[n]$ and $\bar{y} = \sum_{n=0}^{M-1} y[n]$ are the means, $\sigma_X = (1/M) \sum_{n=0}^{M-1} (x[n] - \bar{x})^2$ and $\sigma_Y = (1/M) \sum_{n=0}^{M-1} (y[n] - \bar{y})^2$ are the standard deviations, and $\rho_{X,Y} == (1/M) \sum_{n=0}^{M-1} (x[n] - \bar{x})(y[n] - \bar{y})$ is the covariance.

2.2. Symmetry Index Based on Transfer Functions

The relationship between input and output time-series data $x[n], y[n], n = 0, \dots, N-1$ with N samples is modeled as an LTI system. Both sequences are transformed using the Z-transform as follows:

$$X(z) = x[0]z^0 + x[1]z^{-1} + \dots + x[N-1]\, z^{-N+1}.$$

The transfer function is defined as the rational function of z:

$$G(z) = \frac{Y(z)}{X(z)}.$$

To evaluate symmetry of the movement, two symmetrical pairs of body parts are chosen. If the transfer functions $G_1(z) = B(z)/A(z)$ for the right-side synergistic coordination system and $G_2(z) = Y(z)/X(z)$ for the left-side synergistic coordination system are similar, the movement is considered symmetric.

For this study, the speed of the right ankle is denoted as $a[n]$. The speed time-series of the right wrist is $b[n]$. And similarly, the speeds of the left ankle and wrist as $x[n]$ and $y[n]$, respectively. If the movement is symmetric, then:

$$A(z)Y(z) \approx X(z)B(z).$$

The coefficients of these equations form $(2N-1)$-dimensional vectors $u = a * y$ and $v = x * b$ where $*$ denotes convolution. The non-similarity measure is defined as:

$$\mathrm{Dis}((a, b), (x, y)) = \|u - v\|^2 / (\|u\| \|v\|).$$

This measure is used to evaluate movement symmetry. Figure 1 illustrates an example of the LTI system.

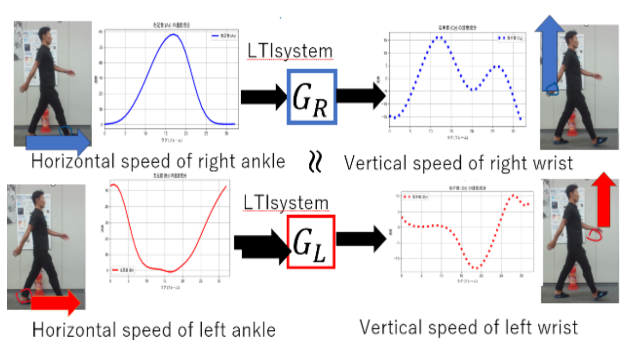

Figure 1. *Illustration of dissimilarity metric between LTI systems*

The left graph shows the time series data of horizontal ankle speed and the right graph shows the time series data of vertical wrist speed. These time series data are used to calculate the similarity between the system G_R and G_L, which is the linkage between ankle horizontal speed and wrist vertical speed.

3. Methodology and Experiments

We recorded the normal walking motion (referred to as Motion S) of five healthy male subjects. Immediately after, we asked them to perform an extremely asymmetrical walking motion (referred to as Motion A) and recorded it as well. Both motions were filmed from a lateral view at a distance of approximately 2 meters using a camera. The camera used was an LED light web camera (model NB-05) with a frame rate of 30 fps.

OpenPose was used to extract joint coordinates for both Motion S and Motion A. The tracked joints included both ankles, both knees, both hips, both wrists, and both shoulders. Simultaneously, the confidence score for each joint coordinate estimation was recorded for each frame. If a joint was occluded or difficult to estimate, its confidence score dropped below 0.5, and its coordinates were not recorded. In cases where the confidence score was 0.5 or lower, linear interpolation was applied using the coordinates from adjacent frames. Next, a moving average filter with a window size of 3 was applied to smooth the data.

Figure 2 illustrates a single frame of normal walking, while Figure 3 shows a single frame of asymmetric walking. Table 1 summarizes the stride count and the gait cycle (in frames) computed via autocorrelation analysis for both symmetric and asymmetric walking motions for each subject.

Table 1. *Recoded number of strides and walking cycle*

	Motion S		Motion A	
Subj.	Stride	Cycle [frame]	Stride	Cycle [frame]
1	3	33	7	34
2	3	27	3	35
3	3	35	4	41
4	3	45	6	66
5	4	33	4	31

Figure 2. *Illustration of symmetry evaluation using*

Figure 3. *A frame of deliberately asymmetrical gait*

Since the aim of this study is to verify the validity of the proposed method, we think that increasing the number of participants is not necessary for this aim. Furthermore, the purpose of this study is to examine whether different values emerge between symmetric and asymmetric gait patterns. We think that gait patterns from patients with actual gait impairments are not required for this purpose.

3.1. Left-Right Symmetry Index Based on half Cycle Shift Transformation

The movement velocity of each joint within the frame is calculated using the formula:

$$v = \sqrt{\Delta x^2 + \Delta y^2}$$

where $\Delta x, \Delta y$ represent the differences in horizontal and vertical coordinates between consecutive frames, respectively. The velocity is expressed in units of [Pixels/Frame].

For the left and right ankle speed time series, autocorrelation analysis is performed to determine the gait cycle. Then, a half shift transformation is applied to ankle speed pattern. This transformation is used to evaluate the left-right symmetry of ankle movement.

3.2. Symmetry Index Based on Transfer Functions

In the approach using transfer functions, the left-right symmetry of movement data is evaluated based on the horizontal or vertical motion of the ankles and wrists. This method assesses whether the movements exhibit symmetry between the left and right sides.

Figure 4 summarizes the horizontal and vertical velocity components used to evaluate the symmetry between the ankles and wrists.

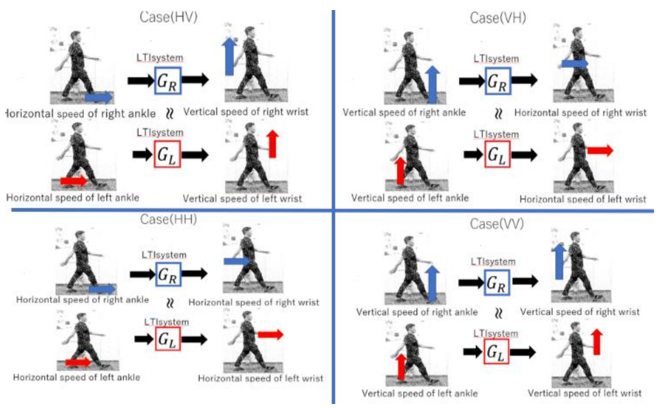

Figure 4. *Illustration of symmetry evaluation using horizontal and vertical ankle and wrist velocity components analyzed with LTI systems*

Top left: Case (HV) horizontal speed of the ankle as input and the vertical speed of the wrist as output. Top right: Case (VH) vertical speed of the ankle as input and the horizontal speed of the wrist as output. Bottom left: Case (HH) horizontal speed of the ankle as input and the horizontal speed of the wrist as output. Bottom right: Case (VV) vertical speed of the ankle as input and the vertical speed of the wrist as output.

4. Result analysis

4.1. Results of Left-Right Symmetry Index Based on half Cycle Shift Transformation

Figure 5 shows the speed of the left and right ankles for the symmetric motion S, shifted by a total of half a cycle using a half cycle shift transformation. Figure 6 illustrates the speed of the left and right ankles for the asymmetric motion A, also shifted by half a cycle. Table 2 presents the correlation results of the half cycle shift transformation for five individuals.

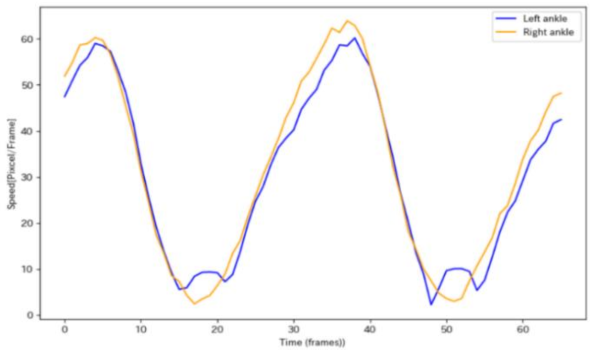

Figure 5. *Left and right ankle speed in motion S after half cycle shift transformation*

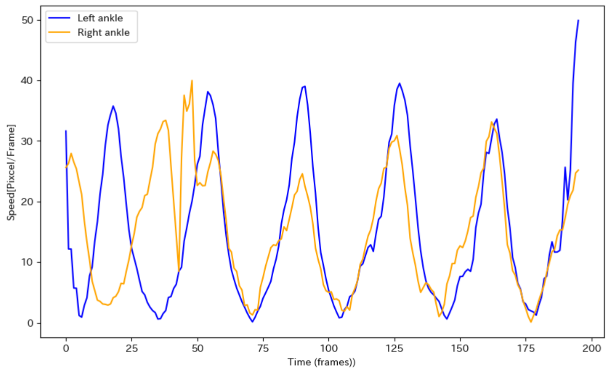

Figure 6. *Left and right ankle speed in motion A after half cycle shift transformation*

Table2*. Correlation coefficients between left and right ankle speed after half cycle shift transformation*

subject	Symmetric motion S	Asymmetric motion A
1	0.98	0.46
2	0.98	0.42
3	0.92	0.70
4	0.95	0.80
5	0.98	0.64

4.2. Results of Symmetry Index Based on Transfer Function

Figure 7 shows boxplot of dissimilarities in cases (HV) (VH) (HH) (VV) for Motion S and Motion A. It presents the asymmetry evaluation results for five walkers.

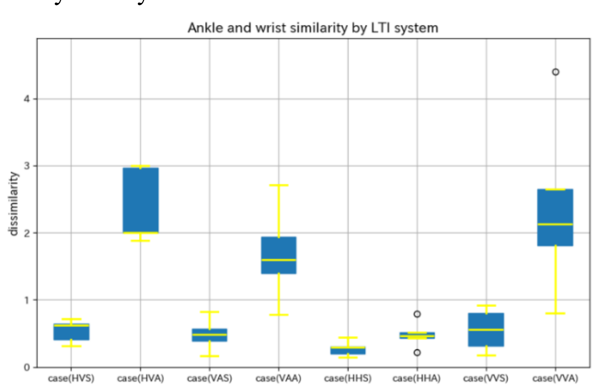

Figure 7. *Boxplot of dissimilarities in cases (HV) (VH) (HH) (VV) for Motion S and Motion A*

For motion S, the dissimilarity was less than 1 for all five individuals across all four cases, indicating a high level of symmetry. In contrast, for motion A, the dissimilarity values were 1 or greater in cases (HV), (VH), and (VV) for all five individuals, indicating low symmetry. However, in case (HH), the dissimilarity remained below 1 for all five individuals.

Figure 8 illustrates the horizontal and vertical speed of subject 1 during the symmetric motion S. The gait cycle was determined through autocorrelation analysis, and the data were segmented and overlaid based on the gait cycle. The horizontal axis represents frames, while the vertical axis represents horizontal speed components. Similarly, Figure 9 shows the horizontal and vertical speed of subject 1 during the asymmetric motion A. The different colours indicate:

- blue: right ankle;
- red: right wrist;
- green: left ankle;
- yellow: left wrist.

Figure 10 presents the convolution diagram for case (HH) of the symmetric motion S. Figure 11 shows the convolution diagram for case (HH) of the asymmetric motion A.

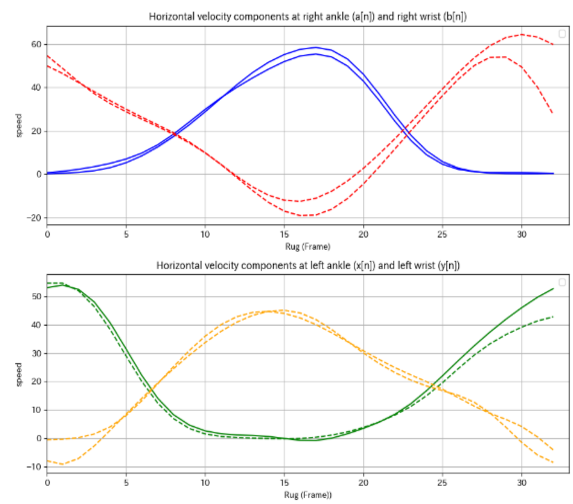

Figure 8-1. *Horizontal speed in motion S of subject 1. Blue: Right Ankle; Red: Right Wrist; Green: Left Ankle; Yellow: Left Wrist*

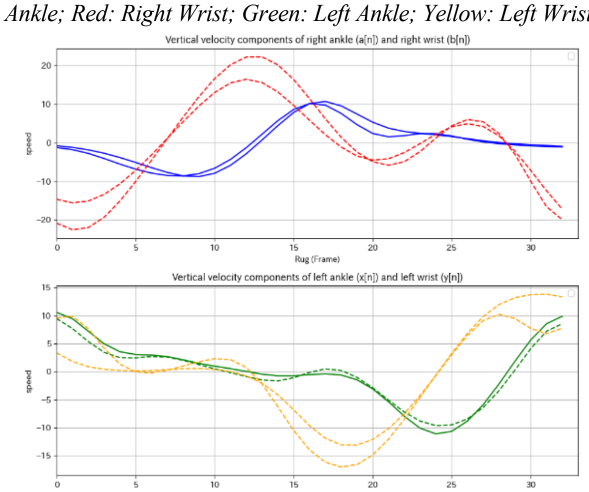

Figure 8-2. *Vertical speed in motion S of subject 1. Blue: Right Ankle; Red: Right Wrist; Green: Left Ankle; Yellow: Left Wrist*

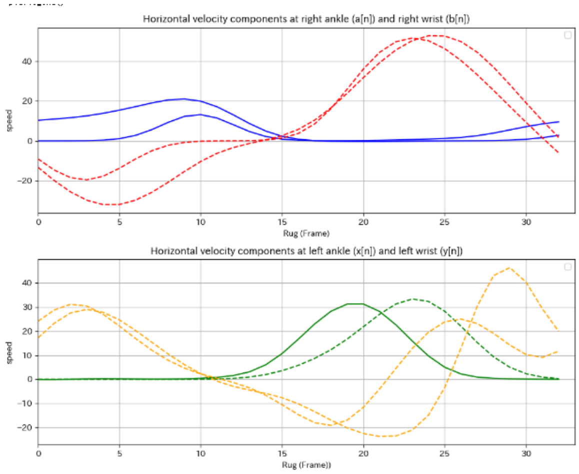

Figure 9-1. *Horizontal speed in motion A of subject 1. Blue: Right Ankle; Red: Right Wrist; Green: Left Ankle; Yellow: Left Wrist*

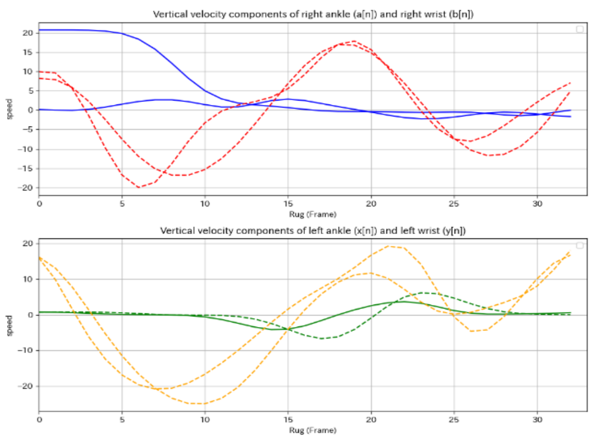

Figure 9-2. *Vertical speed in motion A of subject 1. Blue: Right Ankle; Red: Right Wrist; Green: Left Ankle; Yellow: Left Wrist*

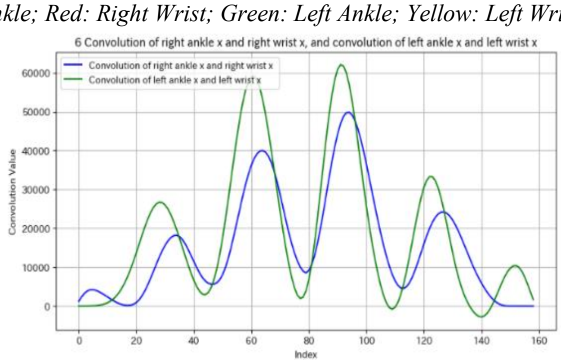

Figure 10. *Convolution computed case (HH) for motion S of subject 1*

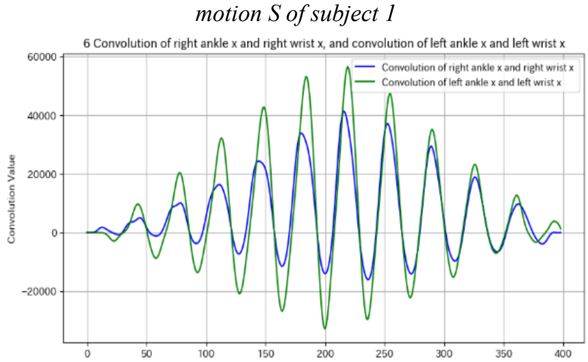

Figure 11. *Convolution computed case (HH) for Motion A of subject 1*

5. Discussion

The effectiveness of the symmetry index, which shifts the motion by a total of half a cycle using a half shift transformation for alternating periodic movements, was verified for the symmetric motion (S) and asymmetric motion (A) in five individuals. We think that the essence of conventional state-of-the-art methods is similar to that of our Proposed Method I. This paper does not compare directly our Proposed Methods I and II. However, our Proposed Methods I and II evaluate asymmetry from different perspectives, offering complementary insights beyond conventional approaches. When evaluating the dissimilarity of the transfer function from ankle movement to the movement of the ipsilateral wrist, excluding case (HH), the asymmetric movement A showed a higher value, indicating greater asymmetry compared to the symmetric movement S. Clearly, asymmetric movements exhibited a high degree of dissimilarity in the left and right transfer functions. On the other hand, in asymmetric performances, all five participants showed similar left and right transfer functions for the horizontal movement of the ankle to the horizontal movement of the ipsilateral wrist in case (HH). The reason for the low dissimilarity in case (HHA) is that, while asymmetric movement A is a performance where intentional postures and movements can be expressed, the dynamic characteristics of the participants' motor coordination are instinctive, making it difficult to completely conceal their inherent left-right symmetry.

6. Conclusion

In this study, we propose a dissimilarity measure for the left-side and right-side coupling systems underlying the time series pair (TSP) of two body points during walking, using video captured by a camera without the need for specialized equipment. We then examined whether the proposed measure can quantitatively assess left-right differences.When evaluating the left-right difference in transfer functions using an LTI system, the transfer function from the horizontal movement of the ankle to the ipsilateral wrist in asymmetric movements showed similarity between the left and right sides, revealing the walker’s inherent left-right symmetry.

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