DYNAMIC ANALYSIS AND FORCED RESPONSE OF A ROTOR-BEARING SYSTEM USING THE FINITE ELEMENT METHOD

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Abstract - This paper presents a comprehensive dynamic investigation of a rotor-bearing system using the finite element method (FEM). The modeled system comprises a uniform shaft, two disk masses, an elastic coupling, and two journal bearings. A numerical model is developed using beam elements with eight degrees of freedom (DOFs) per element, leading to the assembly of global mass, damping, gyroscopic, and stiffness matrices. Modal analysis is performed to extract the natural frequencies and associated mode shapes. In addition, the study includes static deflection analysis, Campbell diagram construction, and forced vibration simulations under unbalanced excitation. The results highlight critical rotational speeds and key dynamic behaviors essential for ensuring the reliable and efficient operation of high-speed rotating machinery. These findings provide valuable insights for the design, balancing, and predictive maintenance of rotor-bearing systems.

Key words - Rotor dynamics; Finite element method; Modal analysis; Campbell diagram; Forced vibration; Unbalance response

1. Introduction

Rotor-bearing systems are essential components in a wide range of industrial machines such as turbines, compressors, generators, and electric motors. These systems often operate at high speeds, where vibration and dynamic instability can significantly impact performance, safety, and longevity.

To model and predict dynamic behavior, the finite element method (FEM) has been widely employed. Recent advances have demonstrated its effectiveness in capturing the vibration characteristics of rotor-bearing systems under various excitation conditions, including unbalance and misalignment. For example, a recent study used FEM to investigate a rotor system with time-dependent misalignment, illustrating the method's capacity for nonlinear dynamic analysis [1]. Another approach developed a second-order FEM model for analyzing flexible rotor–bearing structures with improved accuracy [2].

Experimental rotor test rigs also play a crucial role in validating numerical models and exploring dynamic responses. A virtual rotor test rig was introduced to support balancing experiments and simulate operational conditions [3], while a separate study constructed a full-scale rotor test setup to measure dynamic behavior using force and displacement sensors [4]. These platforms enable researchers to derive key parameters such as stiffness, damping, and critical speeds, and are particularly useful for benchmarking FEM simulations.

The integration of FEM with empirical methods has gained traction. In [5], researchers combined FEM with experimental modal testing to investigate the response of an elastic Jeffcott rotor supported by fluid film bearings. The resulting hybrid methodology provided accurate insight into mode shapes and resonance behavior. Similarly, another study analyzed time-varying system parameters in a rotor rig to highlight the influence of transient dynamics [6].

In hydropower and large-scale systems, FEM has been employed to model rotor–bearing–stator interactions with realistic boundary conditions. For example, [7] presented a full 3D finite element study on the rotor dynamics of vertical hydropower units, accounting for structural flexibility and bearing behavior. These efforts support system-level diagnostics and design optimization.

Advanced sub structuring and component-level modeling techniques are also being developed. A recent contribution applied experimental dynamic sub structuring to isolate and characterize specific subsystems within a rotor-bearing assembly [8], offering new avenues for health monitoring and fault detection.

Comprehensive numerical models can also inform predictive maintenance strategies. A study in [9] developed a FEM framework to assess critical speeds, mode shapes, and stress distributions in rotor systems. Additionally, FEM-based vibration analysis has been applied to evaluate dynamic stability and optimize the design of high-speed rotating machinery [10].

Building upon these advancements, the present study aims to develop and validate a finite element model of a rotor-bearing system subjected to static gravitational loading and unbalanced excitation. The analysis includes modal extraction, Campbell diagram construction, and forced vibration response, providing a complete assessment of the system's dynamic characteristics

2. System Configuration

The modeled system consists of a uniform shaft supported by two journal bearings, with two disks mounted along its length and connected to a motor through an elastic coupling. The geometric layout and element positions, including the locations of bearings and mass disks, are described in Figure 1 and specification of this system are listed in Table 1.

***Table 1.** Specification of the modeled system*

Component	Parameter	Value	Unit
Shaft	Material	Steel	–
	Diameter	10	mm
	Length	580	mm
	Young's Modulus	205	GPa
	Density	7800	kg/m³
Disk	Material	Steel	–
	Diameter	75	mm
	Thickness	25	mm
	Mass	0.8675	kg
Bearing	Mass	0.150	kg
	Stiffness (X-direction)	$1,25\times10^5$	N/m
	Stiffness (Y-direction)	$3,83\times10^8$	N/m

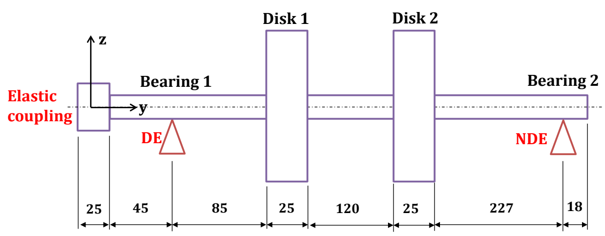

***Figure 1.** Geometry and element configuration of the modeled rotor-bearing system*

3. Numerical model

A finite element approach is employed using beam elements with 8 DOFs per element. The total number of degrees of freedom is defined as 4 times the number of nodes. The governing equation of motion is:

$$[\boldsymbol{M}]\ddot{\boldsymbol{x}} + ([\boldsymbol{C}] + \Omega[\boldsymbol{G}])\dot{\boldsymbol{x}} + [\boldsymbol{K}]\boldsymbol{x} = \boldsymbol{W} + \boldsymbol{F}(t)$$

where $[\boldsymbol{M}]$, $[\boldsymbol{C}]$, $[\mathbf{G}]$, and $[\mathbf{K}]$ are the global mass, damping, gyroscopic, and stiffness matrices, respectively.

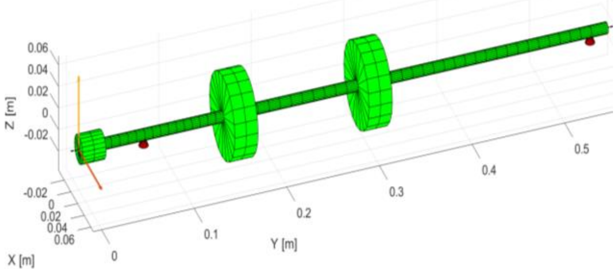

***Figure 2.** 3D finite element model of rotor system*

To numerically analyze the rotor-bearing system using the finite element method (FEM), all structural components must be represented in the model. The system is composed of the following elements:

- One rotating shaft;
- One elastic coupling;
- Two rigid disk masses mounted along the shaft;
- Two supporting bearings.

The shaft is discretized into beam elements. Each element connects two nodes and includes both translational and rotational degrees of freedom. The finite element formulation is defined as follows:

- Number of elements: Nel;
- Number of nodes: N_{node} = Nel+1;
- Degrees of freedom per element: 8 (4 DOFs per node × 2 nodes);
- Total number of degrees of freedom: $N_{dof} = 4 \times N_{node}$.

The nodal displacement vector of a generic element ***j*** is expressed as:

$$\boldsymbol{x}_j^{(r)} = \left\{u_j^{(r)} \quad w_j^{(r)} \quad \theta_{x_j}^{(r)} \quad \psi_{z_j}^{(r)} u_{j+1}^{(r)} \quad w_{j+1}^{(r)} \quad \theta_{x_{j+1}}^{(r)} \quad \psi_{z_{j+1}}^{(r)}\right\}^T$$

Where:

- u and w represent translational displacements in the X and Z directions, respectively.
- θ_{x_j} and ψ_{z_j} denote the rotations around the X and Z axes, respectively.

This formulation allows for the accurate representation of shaft bending in both vertical and horizontal planes and is suitable for modeling the coupled dynamic behavior of rotor-bearing systems

4. Results and discussion

4.1. Modal analysis and mode shapes

Modal analysis is a fundamental step in understanding the dynamic characteristics of a rotor-bearing system. By solving the eigenvalue problem derived from the finite element formulation, the system's natural frequencies and corresponding mode shapes are obtained. These mode shapes describe how the shaft deforms at each frequency, providing insight into potential resonance conditions and dynamic behavior under excitation.

The first six mode shapes of the system are extracted and visualized in 3D (see in Figure 3), revealing a range of bending patterns in both the horizontal (X) and vertical (Z) planes. The computed natural frequencies are as follows:

- Mode #1: 28.8 Hz – predominantly vertical bending;
- Mode #2: 28.8 Hz – predominantly horizontal bending;
- Mode #3: 125.4 Hz – vertical bending with additional inflection points;
- Mode #4: 125.8 Hz – horizontal bending with similar dynamic behavior;
- Mode #5: 347.1 Hz – higher-order vertical bending;
- Mode #6: 357.5 Hz – higher-order horizontal bending.

It is noteworthy that each pair of modes (1 & 2, 3 & 4, 5 & 6) exhibit very close natural frequencies. This is typical for rotor systems with symmetric properties in the X and Z directions, where modal shapes manifest as coupled pairs of orthogonal bending vibrations.

From the mode shapes:

- Modes 1 and 2 indicate the fundamental flexural behavior of the shaft, where maximum displacement occurs near the mid-span. These modes are the most critical, as they are often associated with low-speed resonances.
- Modes 3 and 4 show increased curvature, signifying

higher stiffness regions due to disk influence and shaft dynamics. These modes are sensitive to unbalance-induced excitations in the mid-frequency range.

- Modes 5 and 6 display complex deformations with multiple inflection points, highlighting the shaft's response at high frequencies. They typically relate to structural resonances that could occur at high rotational speeds.

Understanding these modal patterns is essential for predicting the behavior of the rotor under operating conditions. Matching the mode shapes with physical insights - such as disk locations, bearing positions, and coupling stiffness - allows for accurate diagnostic interpretation and the design of appropriate vibration control strategies.

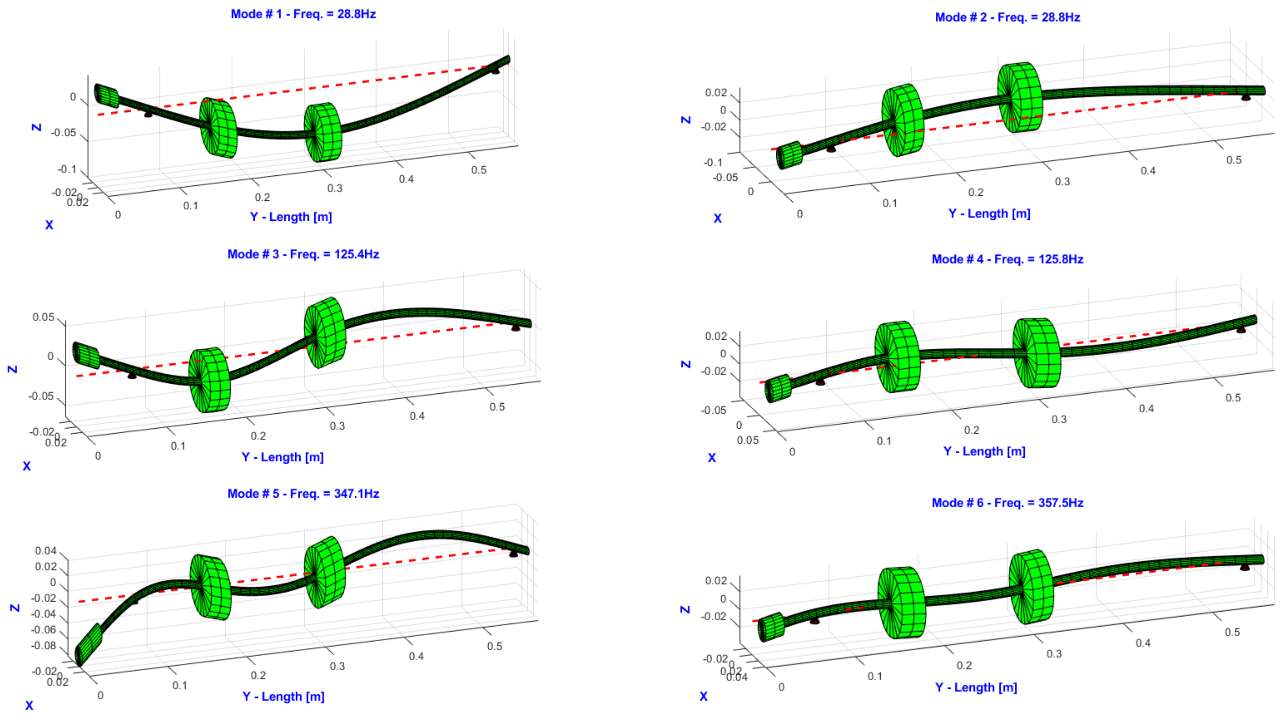

Figure 3. *First six mode shapes of the rotor in 3D (X–Z planes)*

4.2. Campbell diagram

The Campbell diagram is a powerful tool used to visualize the evolution of a rotor system's natural frequencies with respect to shaft rotational speed. It is constructed by plotting the computed natural frequencies (modal frequencies) on the vertical axis against the shaft speed on the horizontal axis. Superimposed on this diagram are excitation lines, typically at 1× and 2× the rotational frequency (shown as red dashed lines), which represent the frequencies at which unbalance or other periodic forces act on the rotor.

It can be seen in Figure 4, the Campbell diagram reveals multiple natural frequency branches, with both forward and backward whirl components evident in the higher modes. The slopes of these frequency curves reflect the gyroscopic effects introduced by the rotating disks. As shaft speed increases:

- Some branches (particularly the backward whirl modes) slightly decrease in frequency.

- Others increase linearly, showing typical forward whirl behavior.

The critical speeds correspond to the intersection points of the natural frequency curves with the 1X and 2X excitation lines:

- The first intersection with the 1X line (near ~1700 RPM) corresponds to the fundamental critical speed.

- Higher-order intersections represent additional resonance conditions and should be considered carefully in rotor design and operational planning.

These intersections indicate regions where the system may experience resonance, potentially leading to excessive vibration amplitudes, increased dynamic loading on bearings, and structural fatigue. Therefore, they must be avoided during continuous machine operation.

The identification of critical speeds offers direct insights into potential resonance zones. These critical speeds can serve as reference thresholds in real-time condition monitoring systems. By tracking the operational speed in relation to these thresholds, engineers can schedule preventive maintenance or perform rotor balancing procedures to proactively avoid resonance and extend machinery lifespan.

The Campbell diagram also provides valuable insight into the system's dynamic stiffness and stability margins, aiding in:

- Identifying safe operating speed ranges;

- Evaluating the influence of gyroscopic effects;

- Supporting decisions related to design modifications or the inclusion of damping mechanisms.

Overall, this diagram serves as an essential reference for ensuring safe and reliable operation of the rotor-bearing system under variable-speed conditions.

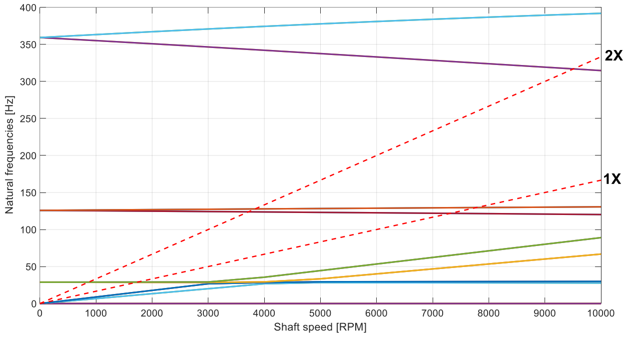

Figure 4. *Campbell diagram of the system*

4.3. Static deflection

To evaluate the structural deformation under steady-state conditions, a static deflection analysis was performed by applying gravitational loading on the rotor-bearing system. The results are presented in terms of displacement amplitudes along both the vertical and horizontal directions over the shaft span.

As illustrated in the Figure 5, the shaft undergoes significant deflection in the vertical direction (blue curve), while the displacement in the horizontal direction (orange curve) remains negligible throughout the entire length. This is consistent with expectations, as gravity acts vertically and induces bending in the Z-direction. The static deflection curve follows a parabolic shape, with maximum deflection occurring near the mid-span between the two disks. This behavior is in agreement with classical Euler–Bernoulli beam theory, where the deflection of a simply supported beam under uniform distributed load is greatest at the center.

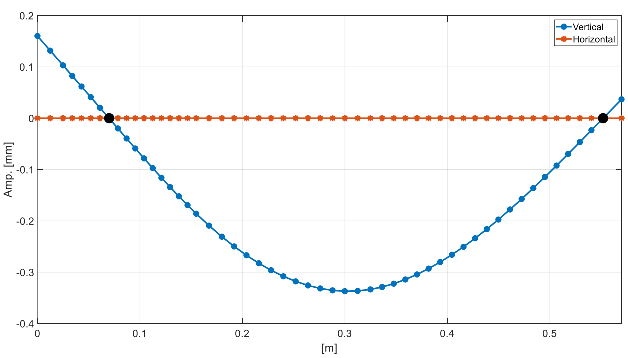

Figure 5. *Static deflection of rotor*

4.4. Forced displacement response

In rotating machinery, unbalance is one of the most common and critical sources of excitation, leading to significant dynamic responses. To simulate this condition, an unbalanced mass is introduced on the rotor, creating a centrifugal force that acts radially outward during rotation.

4.4.1. Modeling Unbalance Force

As illustrated in the Figure 6, the unbalance is modeled as a small mass $m = 0.4\ g$ placed at an eccentricity $OG = 30\ mm$ from the rotor's geometric center. This results in a harmonic excitation force acting at the rotation frequency Ω defined as:

$$F = m \times OG \times \Omega^2$$

This force induces lateral vibrations that are proportional to the square of the rotational speed and vary in both the horizontal (X) and vertical (Y) directions.

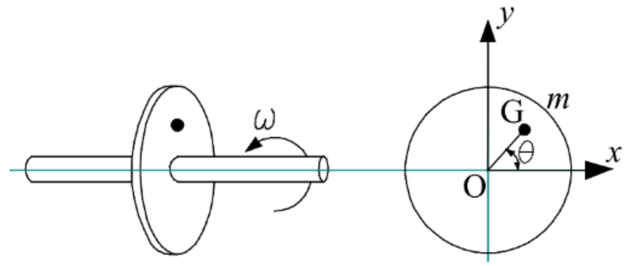

Figure 6. *Schematic of a rotating disk with unbalance mass*

4.4.2. Excitation setup and observation nodes

To investigate the rotor's forced response, the unbalance is applied at a specific location along the shaft, and the system response is monitored at four key nodes (see Figure 7):

- **Node #5**: near the coupling;
- **Node #15**: near the first disk;
- **Node #20**: between two disks;
- **Node #30**: near the free end.

These positions allow for observing the spatial distribution of vibration amplitudes and how proximity to the unbalanced mass affects the response

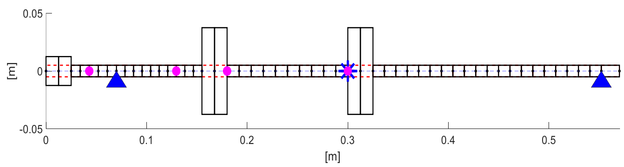

Figure 7. *Finite element mesh with boundary conditions, disk positions, and unbalance location*

4.4.3. Response Analysis

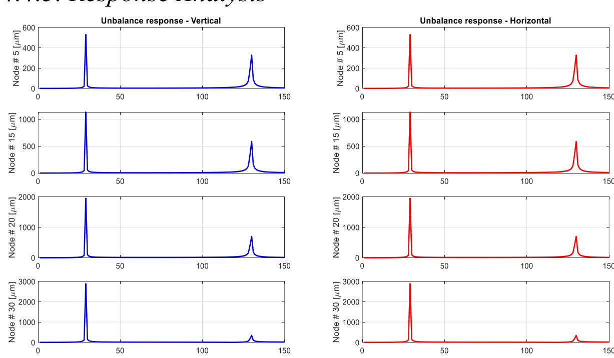

Figure 8. *Frequency response functions (FRFs) at key nodes under unbalance excitation, for both X and Z directions*

The frequency response functions (FRFs) at these nodes are illustrated in the displacement versus frequency plots shown in Figure 8, covering both vertical and horizontal directions. Key observations can be concluded:

- Prominent resonance peaks appear at the natural frequencies identified in modal analysis, indicating that the system is highly sensitive to excitation near those speeds.

- The first critical speed occurs around 28.8 Hz, matching the first mode. This peak is dominant in all nodes, with particularly high amplitudes observed at Node #30.

- A second peak appears around 125 Hz, correlating with the second mode shape.

- Higher amplitude responses are recorded at nodes closer to the unbalance location, due to the direct transmission of centrifugal force.

The maximum displacement recorded at Node #30 exceeds 3000 µm, indicating that high-speed operation in the resonance region can lead to dangerous shaft excursions and potential mechanical failure. These findings underscore the importance of rotor balancing and provide quantitative guidelines for identifying safe speed ranges in real-world applications.

5. Conclusion

This research delivers an in-depth investigation into the dynamic behavior of a rotor-bearing assembly through the application of the finite element method (FEM). By systematically modeling critical components—including the shaft, disk masses, elastic coupling, and journal bearings—the developed numerical model effectively characterizes both static deformations and dynamic responses of the system.

The modal analysis yields essential insights into the system's vibrational behavior by extracting both fundamental and higher-order natural frequencies along with their respective mode shapes. These findings are corroborated through the construction of a Campbell diagram, which not only confirms the computed eigenfrequencies but also pinpoints critical rotational speeds and potential resonance regions influenced by gyroscopic forces and harmonic excitations.

Under gravitational loading, the static deflection analysis reveals the shaft's bending profile and offers a basis for understanding preload conditions at the bearings. Additionally, a forced response analysis under unbalanced excitation highlights localized amplification of vibration amplitudes, particularly at nodes situated near the disturbance. Resonant peaks aligned with the system's natural frequencies underscore the need for precise speed management and rotor mass balancing.

The simulation results presented in this work provide a practical foundation for:

- Engineering rotor systems with enhanced stability and durability,

- Avoiding resonance-prone speed ranges during operation,

- Facilitating condition-based maintenance through targeted vibration diagnostics.

Furthermore, the developed FEM model can be integrated into existing industrial simulation environments such as ANSYS or MATLAB/Simulink by exporting stiffness and damping matrices. These allow real-time monitoring algorithms or predictive maintenance modules to be deployed effectively, enabling continuous assessment of critical vibration indicators during operation.

Ultimately, the proposed modeling strategy and analytical outcomes set the stage for future explorations involving nonlinear dynamics, bearing clearance modeling, thermal effects, and experimental validation using a rotor test bench.

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