GENERAL METHODOLOGY FOR PROBLEM SOLVING

PHƯƠNG PHÁP LUẬN TỔNG QUÁT GIẢI CÁC BÀI TOÁN

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Abstract - The article presents a general methodology for problem solving in finding and proving problems based on known knowledge, especially for difficult and intractable problems. Regarding the theoretical basis, the author builds a comprehensive model for thinking and problem-solving that includes: problems with the focus on "What needs to be found/proved"; the psychology and emotions of the mathematical agent under the influence of the environment; thinking processes such as thinking activities to reach solutions, data exploitation activities, logical reasoning, equivalent transformations, and creativity; in addition, the work inherits, to a certain extent, the views of previous theorists on problem-solving methods (such as Polya, Terence Tao, and Arthur Engel). The method includes two stages: preparation and implementation, with five steps: (1) understanding and evaluating the feasibility of the problem; (2) analyzing data; (3) determining the solution direction and forming ideas; (4) perfecting the solution; (5) checking and drawing conclusions.

Key words - General methodology for problem solving; Structure of thinking for solving problems; Equivalent transformation and logical reasoning; Creativity in solving problems; exploiting data for problems

Tóm tắt - Bài báo trình bày Phương pháp luận tổng quát giải các bài toán tìm/chứng minh dựa trên những tri thức đã biết nhất là hướng tới những bài toán khó và nan giải. Về cơ sở lý luận, tác giả xây dựng mô hình cấu trúc tư duy, giải toán mang tính toàn diện gồm: bài toán với trọng tâm là "Điều cần tìm/chứng minh"; tâm lý – cảm xúc của chủ thể dưới ảnh hưởng của môi trường; quá trình tư duy như hoạt động tư duy đi tới lời giải, hoạt động khai thác dữ kiện, lập luận logic, biến đổi tương đương và sáng tạo; ngoài ra, công trình có sự kế thừa nhất định của các quan điểm của các nhà lý luận về phương pháp giải toán trước đó (như Polya, Terence Tao, Arthur Engel…). Phương pháp bao gồm 2 khâu: chuẩn bị và thực hiện với 5 bước: (1) hiểu và đánh giá tính khả thi của bài toán; (2) phân tích dữ kiện; (3) xác định hướng giải – hình thành ý tưởng; (4) hoàn thiện lời giải; (5) kiểm tra, đúc kết.

Từ khóa - Phương pháp luận tổng quát giải toán; Cấu trúc tư duy giải toán; Biến đổi tương đương và suy luận logic; Tính sáng tạo trong giải toán; khai thác dữ kiện bài toán

1. Introduction

Mathematics today plays a major role in all fields of science and technology. It has become the foundation and pillar of modern science and technology, helping human beings transform the world. Moreover, modern mathematics is not only a computational tool but also a foundation of thinking for the formation and development of science and technology.

In the history of the development of mathematics, in addition to works of an inventive nature and those concerned with the construction of mathematical theories, there are also problems based on known knowledge. The solutions to these problems demonstrate the foundational role of mathematics in science and technology. Solving this type of problem reflects learners' mathematical thinking ability and helps them cultivate logical thinking, analytical ability, as well as the application of knowledge to real life.

However, in the current context of the strong development of computers and artificial intelligence (AI), these problems can now be solved by computers and AI, which are able to provide solutions quickly and accurately and can solve a great many problems within a very short time. This raises the following question: Is it still necessary to study methods of thinking for solving problems based on available theory?

In response to this question, the following observations may be made. First, not every type of problem can be solved by computers or AI; for problems for which no algorithm has been developed, or problems that are ambiguous or contain "implicit" data that must be understood in context, they may produce incorrect results. Second, they cannot evaluate the practical significance of the results. Third, direct human problem solving helps develop logical thinking, creativity, and generalization, whereas AI cannot. Fourth, without a theoretical foundation for methods of applied problem solving, the learning of mathematics would be reduced to the mere operation of entering data into machines, thereby losing its value in intellectual development, which is one of the discipline's essential values. Fifth, problem solving by computers and AI is based on checking all possible cases or on formal reasoning according to mechanical logic without intuition, leading to "inelegant" solutions that lack creativity, pursue the shortest solution, and fail to provide a general perspective. Therefore, developing a universal methodological framework for problem-solving based on available theory is meaningful for teaching, for the development of thinking, and for clarifying the boundary between human problem-solving methods and the computational power of computers and AI.

However, within the scope of this article, the author presents a hypothesis on the construction of a general methodology for solving problems based on known knowledge.

2. Overview of representative works on the methodology of problem solving

The foundational work in 1945 was the book *How to Solve It* by George Pólya. This was the first classic work on the methodology of problem solving. The mathematician proposed four basic steps in solving mathematical problems, including: first, understanding the problem; second, devising a plan; third, carrying out the plan; and fourth, looking back at the solution. The author provided readers with strategies such as drawing diagrams, experimenting, and working backward to help them develop problem-solving thinking. This work laid the foundation and influenced the thinking of later authors [1].

Continuing the Pólya tradition (1970–1990), the book *Problem Solving: A Handbook for Teachers*, published in 1980 by Stephen Krulik and Jesse A. Rudnick - leading researchers in mathematics education - provided a theoretical foundation inherited from Pólya and proposed specific techniques to help teachers improve their professional competence. The book emphasized strategic thinking and creative thinking rather than memorizing formulas. Moreover, it served as a resource for helping teachers develop a student-centered approach, first and foremost in mathematics teaching and problem solving. A notable feature of the book is its introduction of mathematical heuristics such as drawing diagrams, trying simple cases, working backward, and analogous reasoning, etc. [2].

The book *Mathematical Problem Solving* by Alan H. Schoenfeld was published in 1985. Inheriting Pólya's four-step method of problem solving, the author argued that success in problem solving depends not only on knowledge but also on thinking strategies, process management, and the beliefs of the mathematical agent. Therefore, he added a framework consisting of: Resources (knowledge resources) – Heuristics (strategies/techniques) – Control (process control) – Beliefs (beliefs and attitudes). With this framework, the author advised teachers not only to teach formulas together with illustrative examples, but also to help learners build problem-solving strategies, encourage metacognitive thinking through self-questioning, self-checking, and self-evaluation, and create an environment that changes the belief that mistakes are normal and experimentation is necessary [3].

Serving Olympiad and advanced training, *Problem-Solving Strategies* by Arthur Engel, published in 1997, is a classic book for gifted mathematics students and teachers. The author presented strategies and methods of thinking commonly used in Olympiad and advanced problems. For example, there is the strategy of reducing a complex problem to a simpler one, or transforming it into an equivalent form for easier handling; or the technique of dividing the solution space according to conditions in order to solve the problem thoroughly, etc. The emphasis is that the author not only provided "solution formulas" but also focused on creative and flexible thinking so that users could apply them to many different types of problems. The book helps readers develop skills in analysis, idea finding, experimentation, and the combination of multiple techniques [4].

The book *The Art and Craft of Problem Solving* by Paul Zeitz, published in 1999 (reprinted in 2007), is a well-known work for learners and teachers in the context of Olympiad training and the cultivation of creative problem-solving thinking. The author argued that problem solving is not only a technique but also an art and a practical skill, and that problem solving is a process of training thinking rather than merely finding the final answer. The book focuses on principles, techniques, and strategies commonly encountered in problem solving [5].

From the perspective of a leading mathematician, *Solving Mathematical Problems: A Personal Perspective* by Terence Tao - a Fields Medalist - was published in 2006 to serve pupils, students, and participants in international mathematics competitions in approaching problem solving in a creative and systematic way. A notable point of the book is that it presents the way a mathematician thinks when solving problems, including experiences, mistakes, and the ways in which the author drew lessons from the process of doing mathematics. He proposed general principles for problem solving, namely that one must understand the problem clearly, check special cases, try small examples, generalize, use symmetry, and especially always ask "why" rather than only "how." Throughout the work, many techniques are presented, such as reasoning with symmetry and transformations, the use of extremal methods, etc. [6].

A modern approach is found in the book *Thinking Mathematically* (2nd edition, 2010) by John Mason, Leone Burton, and Kaye Stacey. This work serves pupils, students, and teachers by helping readers develop mathematical thinking through problem-solving methods. The work consists of 11 chapters, each focusing on a different aspect of the problem-solving thinking process. In it, the authors encourage everyone to begin solving mathematical problems through specialization and generalization. In particular, the work describes three stages in the problem-solving process: the Entry stage, the "Attack" stage, and the Review stage [7].

Thus, the above studies on the methodology of problem solving have provided useful approaches and strategies for developing creative thinking and problem-solving skills in mathematics. Methods such as understanding the problem, devising a plan, and experimenting are all highly suitable for helping learners apply them in problem solving. However, what needs to be supplemented and clarified here is: (1) it is necessary to construct a methodology from a general perspective, considering the fundamental roots from a holistic and exhaustive perspective; only in this way can intractable problems be solved; and (2) in the context of the development of information technology and AI, it is necessary to integrate digital support tools and technological applications to support the solution of complex problems in education and research.

3. General methodology for problem solving

3.1. Basic methods

To construct a general methodology for problem solving, the author employs the following basic methods.

First, the logical method, which examines the fundamental elements and the necessary relationships of thought in the process of solving a problem leading to a result, and considers the specific characteristics of problem-solving thinking. Second, the principle of comprehensiveness: in order to form a general methodology, it is necessary to examine fully and comprehensively the problem-solving process, from the approach to the problem, the elements involved in problem-solving thinking, and extrinsic factors (cognitive psychology and environmental contexts), to consideration of the levels of difficulty of problems (from easy problems to intractable ones), and to consideration of the effectiveness of problem-solving thinking when leveraging existing heuristics and domain-specific methods as well as accumulated problem-solving experience. In addition to the two basic methods above, the study also relies on other methods such as the integrated method of analysis and synthesis, and the systems thinking approach.

3.2. Theoretical basis

3.2.1. Concepts

A problem is an issue containing the object of mathematics. In the book *A Philosophical Analysis of the Nature of Mathematical Knowledge*, the object of mathematics is clearly identified as follows: "the object of mathematics: the characteristic of mathematics as a science lies in the fact that it analyzes quantitative relations and spatial forms existing in all forms of motion…", and "In modern mathematics, its direct object is abstract mathematical structures" [8, p. 62].

Thus, the objects of mathematics are quantitative relations and spatial forms, or abstract mathematical structures. A solution is the detailed explanation and reasoning by the agent that satisfies the requirement set by the problem in a logical manner, or affirms in a logical manner that the requirement set cannot be satisfied. A solution must be clear, rigorous, and logical, and must convincingly show that the stated requirement is either satisfied or not satisfied.

An answer is the direct response that satisfies the requirement posed by the problem. The solution must contain the answer. The answer only tells us the result to be obtained, without revealing by what path that result is reached or whether it is logically valid. A problem may have multiple solutions but only one unique correct answer. It is like there being only one destination, but many paths leading to it.

The Givens of a problem (hereinafter referred to as "Givens") are the set of information derived from the object, the assumptions, and the conditions of the problem. The data must be collected fully and accurately; this is a critical condition for the agent to determine a solution The structure of a problem includes the following. First, the object of the problem (hereinafter referred to as the "object") is the object toward which the agent directs the processing of quantitative relations and forms in order to satisfy the requirement posed by the problem. In solving the equation $x^3 + 5x^2 - 8 = 0$, "$x^3 + 5x^2 - 8 = 0$" is the object of the problem. This is an essential component that every problem must have. Second, the assumptions of the problem (hereinafter referred to as "assumptions") represent the given data of the object that serve the requirements posed by the problem. Some problems have no assumptions, while others do. Third, the requirement of the problem (hereinafter referred to as the "requirement") is what the problem poses and what the mathematical agent must seek a solution for. This is an essential component that every problem must have. This requirement may take the form of "what needs to be found/proved," as seen in problems to find and problems to prove. Fourth, the conditions of the problem (hereinafter referred to as the "conditions") are the limitations and constraints on the object or on the agent's problem-solving activity. The conditions of the problem are divided into two types: Intrinsic conditions and operational constraints. Intrinsic conditions are those arising from the object of the problem itself; they may be given in advance or must be discovered by the agent (for example, the requirement that a denominator must be nonzero, or that a square-root expression must be non-negative). Operational constraints pertain to the agent's problem-solving activity, including the time available for solving (as prescribed by the problem or self-imposed by the agent) and the essential tools the agent must utilize, such as paper, pencil, ruler, and compass, etc. Note that among the Intrinsic conditions, there may be cases where the time is predetermined in the statement itself. Intrinsic conditions may or may not exist, but operational constraints always exist because, first of all, the time available to the mathematical agent is not infinite and must be limited.

The structure of the problem is modeled as in Figure 1.

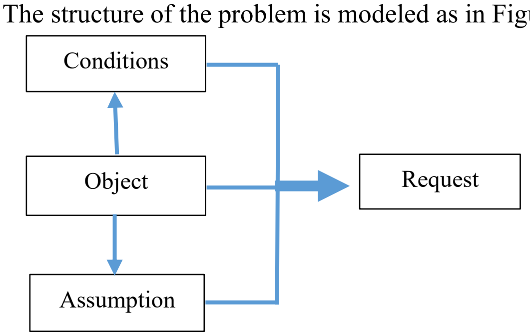

Figure 1. *Structure of the problem*

Problem classification: in mathematics there are innumerable types of problems, but in general they may be reduced to the following basic and comprehensive categories.

Based on the requirement of the problem, problems are divided into proof problems and finding problems. A problem to prove is one that requires the agent to reason and clarify a thesis, affirming or refuting a proposition on a logical basis. A problem to find is one that requires the agent to determine an object or value satisfying the intrinsic conditions (such as finding extrema, simplifying, evaluating an expression, or solving an equation, etc). Based on the source of knowledge mobilized, problems are divided into problems based on known knowledge and problems requiring the development of theory. A problem based on known knowledge is one that can be solved by currently available theorems and standard knowledge. A problem requiring the development of theory is one that requires the addition of new concepts, propositions, or methods within the scope of the problem (on the basis of existing knowledge).

Combining the two criteria above, we obtain four groups of problems: finding/proof problems based on known knowledge, and finding/proof problems requiring the development of theory.

Based on the possibility of drawing a conclusion that satisfies the requirement of the problem statement, problems may be classified into solvable problems, unsolved problems, and unsolvable problems. A solvable problem is one for which, from the given data and existing knowledge, we can draw a logically valid conclusion; the conclusion may be either "has a solution" or "has no solution". An unsolved problem is one whose statement is clear, but humanity has not yet found a solution to it. There are a small number of rare problems for which no solution can be given because human mathematical knowledge is not yet sufficient to solve them. For example, the Collatz Conjecture problem (also called the $3x + 1$ Conjecture). This is a problem in number theory. It requires us to begin with any positive integer n. If n is even, divide n by 2; if $n$is odd, multiply it by 3 and then add 1. Continue repeating this process until the number reaches the value 1. The Collatz Conjecture asks whether, for any positive integer, after a certain number of steps, we can always reach the value 1. To date, no one has solved this problem.

An unsolvable problem is one that falls into the case of insufficient data or contradiction. For instance, there are problems that do not provide enough data to satisfy the requirement of the problem statement, such as the following simple problem: A man buys 100 chickens and dogs. After 5 years, excluding those that died and those that were born, how many chickens and how many dogs does he have? For this problem, the agent is unable to provide a definitive answer due to insufficient data. There are also problems containing contradictions in the statement. For example, a problem requires finding a natural number X such that X is divisible by 4 and 5, but at the same time requires that $0 < X < 4$. This is contradictory, because there does not exist a nonzero natural number less than 4 that is divisible by both 4 and 5.

3.2.2. Basic theoretical views

In addition to defining the concepts in the section above, especially the structure of the problem, the following are the theoretical views serving as the basis for the general methodology of problem solving.

a. The structure of problem-solving thinking – a comprehensive approach

In order to construct a general methodology for problem solving, it is necessary to adopt a comprehensive approach. The author defines the comprehensive framework of fundamental components and factors involved in the agent's problem-solving activity as follows. First, the problem itself. This is the starting point and an important basis for thought to produce a solution. Fully understanding and thoroughly analyzing, without omitting any component in the structure of the problem, is a principled requirement in constructing a general methodology. The mathematical agent needs and must always direct attention to the components of the problem such as the object, the requirement, the assumptions, and the conditions. Within the problem, there is a very important component for orienting thought toward a solution, namely the requirement of the problem, or in the form of what needs to be found (in finding problems) or what needs to be proved (in proof problems), abbreviated as "what needs to be found/proved." What needs to be found/proved is what thought must aim toward, exploiting information and data as a basis for orientation and for narrowing the search space for a solution. Therefore, this is an important component that is separated out and prioritized for clarification.

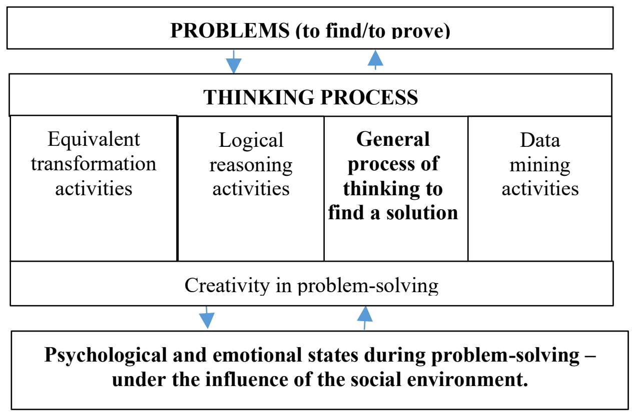

Figure 2. *General diagram of the structure of problem-solving thinking*

Second, psychology and emotions in the solving process - under the impact of the social environment. The influence of the agent's cognitive psychology and emotions on his or her own thinking is unavoidable. The challenge lies in cultivating positive emotional and psychological states, transforming them into factors that enable the agent's thinking to reach its maximum cognitive capacity Moreover, such psychology and emotions are placed under the influence of the external environment, first and foremost and most importantly the social-organizational environment in which the agent participates in carrying out the problem-solving activity. If that influence causes inhibition or discomfort to the agent, it will lead to negative psychology in problem solving; conversely, if that environment provides conditions and encouragement, it will make the agent psychologically stable, confident, and stronger in solving difficult and intractable problems.

Third, the agent's thinking is regarded as the central factor governing the entire problem-solving process, from the stage of analyzing data, to forming ideas, to checking and refining the solution. It includes the following constituent elements: 1/ the general activity of thinking toward the solution; 2/ the activity of exploiting data (related knowledge) by thinking; 3/ the activity of logical reasoning; 4/ the activity of equivalent transformation; 5/ the activity of creativity in problem solving. These elements are analyzed as follows.

First, the general activity of thinking toward the solution. This is the general overarching factor of the thinking process. It is manifested in the steps and stages of thinking from understanding the problem statement to completing and checking the solution. In order to achieve effectiveness, from the perspective of a general methodology for solving all problems from easy to difficult and intractable ones, the general process of thinking must proceed according to the following necessary steps: Step 1: understand the problem / identify the correct problem to be solved and consider its feasibility; Step 2: analyze the problem; Step 3: determine the solution direction and form ideas for the problem; Step 4: refine the ideas into a solution; Step 5: check again and draw conclusions (if necessary). Among these five steps, Step 2 and Step 3 are the most important.

Second, the activity of exploiting data. This is an important activity for thought to obtain the "raw material" to "link" together and create possible ideas and solutions for the given problem. This activity is manifested in Step 1, Step 2, and Step 3 (mainly in Step 1 and Step 2). The sources of data exploited include: 1/ from the assumptions of the problem; 2/ from the object of the problem - in this source, the mathematical agent needs to discover the properties, characteristics, formulas, theorems, and implications of the object, and also from objects related to this object (in certain problems, usually difficult ones); 3/ from the conditions of the problem - these data require the agent to narrow the range of the answer; 4/ from the requirement of the problem, or what needs to be found/proved - the agent needs to discover the properties and implications derived from what needs to be found/proved as a basis for orienting thought toward a solution; 5/ from experience in solving similar previous problems; 6/ from knowledge related to the problem. Given these sources, if thought has not carefully examined all of them, the agent runs the risk of lacking data - lacking grounds for reasoning to draw a logically valid conclusion satisfying the requirement of the problem statement. Therefore, from a general perspective, the agent's thinking needs to exploit data within the scope of the six sources above.

Third, the activity of logical reasoning by thought. This activity is clearly manifested in Step 2, Step 3, Step 4, and Step 5 of the general thinking process. It may be said that the characteristic of the thinking activity in solving mathematical problems is the mode of classical formal logical thinking. That is, logical reasoning is the principal activity in analyzing the object, exploiting data, and forming the solution. Each mathematical proposition has only one of two truth values: true or false. When solving a problem, the agent does not merely manipulate numbers, symbols, or terms, but in essence develops a chain of reasoning in which the conclusion is drawn necessarily and logically from determinate data. Vu Van Vien affirmed: "The mathematical method is the method of classical formal logic on abstract mathematical structures (on relations described by mathematical language)" [8, p. 63].

The following illustrates several important aspects of logic manifested in mathematical reasoning (Table 1).

Table 1. *Law of the excluded middle*

Logical content	Law of the excluded middle: $a \vee \neg a = 1$
Manifestation/ illustration in mathematics	A mathematical proposition can have only one of two values: true or false.
Mathematical significance	Applied in the method of proof by contradiction.
Note	The symbol ∨ denotes disjunction, and ¬ denotes negation.

Table 2. *Pure conditional inference*

Logical content	We have: $a \to b \to c \ldots \to k \to a \to k$
Manifestation/ illustration in mathematics	For example, prove that if $x > 3$ then $x^2 + 1 > 10$. We have $x > 3 \to x^2 > 9 \to x^2 + 1 > 10$. Hence, from $x > 3 \to x^2 + 1 > 10$, the statement to be proved follows.
Mathematical significance	In mathematical reasoning, many solutions are constructed by chaining intermediate conditions: from the assumption a one infers property b, from b one infers c, and so on, until the final conclusion k is reached. When the chain of implications is logically valid, the overall inference $a \to k$ is necessarily true.
Note	The symbol → denotes implication.

Table 3. *Determinate conditional inference*

Logical content	There are two forms: Form 1 – Modus Ponens: $\begin{cases} a \to b \\ a \end{cases} \rightarrow$ b Form 2 – Modus Tollens: $\begin{cases} a \to b \\ \neg b \end{cases} \rightarrow$ ¬a
Manifestation/ illustration in mathematics:	In Form 1: If a line is parallel to the base of a triangle, then it creates pairs of proportional corresponding sides. $a \to b$ holds according to Thales' theorem. According to the assumption, the line $x$intersects triangle ABCat $MN$and is parallel to BC; therefore, it follows that $AM/AB = AN/AC = MN/BC$. In Form 2: Given that $n^2$is odd, prove that $n$is not even. We have: if $n$is even, then $n^2$is even, so $a \to b$is true. By the assumption that $n^2$is odd, the statement that $n^2$is even is false, so we have $\neg b$. Therefore, it follows that $n$is not even, i.e., it is odd.
Mathematical significance	In Form 1, this is very important because it represents the application of theorems, formulas, or mathematical knowledge in general to problem solving. In Form 2, this is one of the methods of proof in mathematics. Step 1: state a necessary implication $a \to b$. Step 2: affirm that $\neg b$holds. Step 3: infer $\neg a$, where this conclusion is what must be proved.
Note	

***Table 4.** Disjunctive inference*

Logical content	There are two forms. Form 1: affirmation to negate: $\begin{cases} a \vee b \\ a \end{cases} \rightarrow \neg b$ Form 2: negation to affirm: $\begin{cases} a \vee b \\ \neg a \end{cases} \rightarrow b$ Finite extension: $\begin{cases} a_1 \vee a_2 \vee \ldots \vee a_n \\ \neg a_2, \ldots, \neg a_n \end{cases} \rightarrow a_1$
Manifestation/ illustration in mathematics:	In Form 1: Determine the position of point M relative to a circle. There are three possible positions of point M with respect to the circle: 1/ inside the circle; 2/ on the circle; 3/ outside the circle. According to the assumption, point M is not inside the circle and not on the circle. Therefore, the conclusion is that point M lies outside the circle.
Mathematical significance	This type of inference is applied in eliminative reasoning; it is an indirect method of finding/proving commonly used in problem solving. In the special case of proof by contradiction, when we have $a \vee \neg a$ and have $\neg a$, then infer a. In addition, this form of inference is also used in the technique of considering cases one by one.
Note	

***Table 5.** Syllogism*

Logical content	Syllogism $\begin{cases} M \rightarrow P \\ S \rightarrow M \end{cases} \rightarrow S \rightarrow P$
Manifestation/ illustration in mathematics	For example, the number 5 is a positive integer. A positive integer is a natural number. Therefore, 5 is a natural number. Another example: Every number divisible by 4 is divisible by 2. The number 28 is divisible by 4. Therefore, 28 is divisible by 2.
Mathematical significance	Syllogism is commonly used in mathematics and in problem solving. Its model is as follows: Major premise (general proposition): every object having property A has property B. Minor premise (specific case): the object under consideration has property A (data of the problem). Conclusion: the object under consideration has property B (what must be proved).
Note	

Fourth, the activity of equivalent transformation. In problem-solving thinking, mental operations that transform the object and the components of the problem are very important, especially for difficult and intractable problems. Only when the agent can transform the object and the components of the problem does the problem reveal new data, structures, and states that provide the basis, foundation, and material for the agent to form a solution direction and ideas for resolving the problem. In particular, there are cases in which the result of the transformation simultaneously leads to satisfying what needs to be found - for example, transformation in the direction of simplification. Transformation in mathematics is equivalent transformation, meaning that the transformation changes only the form and structure of the object and the other components of the problem (conditions, requirements) without changing their nature.

The methods of equivalent transformation include five basic types as follows: 1/ the method of simplification; 2/ the method of detailing; 3/ the method of division into parts; 4/ the method of reduction to a general form; and 5/ the representative method. In addition, there are quite many transformation methods based on combinations of the basic methods above. The thorough exploitation of all possible equivalent transformation methods is a prominent strength of the general methodology.

Fifth, creativity in problem-solving activity. Besides the mode of classical formal logical thinking, which is the characteristic and basic mode, the general methodology cannot fail to take into account the creative factor in the problem-solving process. Difficult and intractable problems, in particular, require the agent to bring into play his or her creativity. However, this creativity remains within the framework of logical reasoning. The objective basis for creativity lies in the fact that each answer may have many different solutions; therefore, creativity is directed not only toward finding a correct solution but also toward finding the shortest, the "most elegant," and the most surprising solution, one that goes beyond traditional approaches. From the perspective of the general methodology, creative techniques must always be taken into account.

b. Inheritance

The construction of a general methodology for problem solving is based on inheriting the positive and sound ideas of earlier theoretical views on this subject. The author has adopted many views from previous researchers. For example, Pólya's view emphasizes the importance of analyzing a problem into easier parts so that it can be solved systematically: To solve a problem, you must first understand it. To understand a problem, you must break it into smaller parts, analyze the relationships between them, and think creatively about how to solve each one [1].

Likewise, Terence Tao's view emphasizes that solving a problem is not merely a matter of applying available methods, but also requires the ability to adjust and test conjectures: The process of solving a problem often involves making educated guesses, testing them, and refining them as needed [6].

Or, Arthur Engel's view emphasizes that solving a problem requires not only the application of formulas but also a clear understanding of the principles and theory behind that problem: A key to solving problems is not merely memorizing techniques, but understanding the underlying principles that govern the problem [4].

In addition to the above inheritance, the author also applies the method "C.P.A: Comprehensive Method of Problem Determination, Transformation and Analysis" from a previous work [9].

3.3. Content of the general method for solving problems

Object: The general method is directed toward the group of finding/proof problems based on known knowledge.

Purpose: to help the agent solve, or improve efficiency in solving, finding/proof problems based on known knowledge, especially difficult, very difficult, and intractable problems.

Structure: The method consists of two phases: the preparation phase and the implementation phase.

Requirement: users must firmly grasp the theoretical basis, carry out the preparation phase well, and correctly apply the basic content of the method, while striving to promote the positive and creative role of the agent on the basis of the stated principles and orientations.

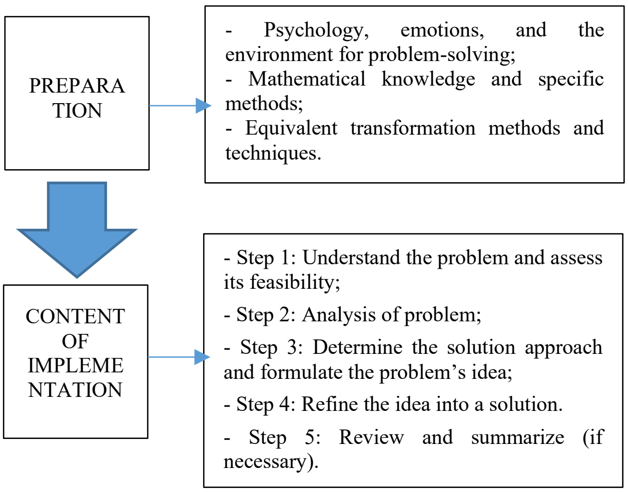

Figure 3. *Diagram of the general method for solving problems*

3.3.1. The first phase: Preparation

Users of the general method need to grasp the following principles and requirements before directly applying it:

*** Principles from the perspective of psychology and environmental influence**

First, the psychological state of the agent when solving a problem directly affects the efficiency of thinking and the ability to mobilize knowledge. The mathematical agent needs to maintain a confident, positive, and optimistic attitude, while also retaining the necessary caution to avoid subjectivity, omission of data, or misunderstanding of the problem requirement. Studies in positive psychology show that positive emotions play a role in enhancing the ability to connect information and broaden the scope of attention, thereby supporting the process of reasoning and idea formation in problem solving. According to B. L. Fredrickson, positive emotions not only improve the immediate mental state but also contribute to building long-term cognitive resources for the individual [10].

Second, in the problem-solving process, mental impasse is a common phenomenon, especially for difficult or exploratory problems. In such cases, instead of falling into a negative psychological state, the solver needs to show persistence and determination, and actively return to review the data, constraints, and possible transformations. According to M. W. Eysenck et al., when subjects experience anxiety and psychological stress at a high level, the efficiency of memory-based information processing declines, thereby limiting the ability to mobilize existing knowledge and control attention [11]. This explains why maintaining a stable psychological state is a rather important condition for avoiding "thought blockage" in problem solving.

Third, the problem-solving environment - including the learning environment, the way the classroom is organized, and the social context - has a significant influence on the agent's cognitive psychology and thinking performance. A supportive environment that encourages and acknowledges effort contributes to strengthening intrinsic motivation, self-confidence, and the solver's sense of autonomy. Self-Determination Theory, proposed by E. L. Deci and R. M. Ryan, affirms that the learning environment plays a key role in promoting sustainable learning motivation [12]. However, findings in psychology also indicate that moderate pressure, especially reasonable time limits, can create positive cognitive pressure, helping to improve the agent's concentration and thinking performance. In addition, the Yerkes–Dodson law shows that performance reaches an optimal level when stress is neither too low nor too high [13].

From the above analyses, it can be seen that preparation in terms of psychology, emotions, and environment is both supportive and a prerequisite condition for the general methodology for solving problems to be effective in teaching and learning practice. Underestimating this preparatory phase may significantly reduce the effectiveness of problem-solving methods and techniques, no matter how rigorously they are designed.

*** Requirements regarding mathematical knowledge**

Mathematical knowledge: problem solving especially requires and makes thorough use of mathematical knowledge. How mathematical knowledge is received and used in order to achieve effectiveness must satisfy the following requirements:

First, the teaching/learning of mathematical knowledge must be conducted systematically, so that the relationships among concepts, theorems, and formulas can be seen, while clearly indicating their scope and conditions of application. From a systemic perspective, learners not only see the internal relationships but also understand to which objects and under what circumstances they apply.

Second, it is necessary to see where these types of mathematical problems are applied in practice: within mathematics itself, in science and technology, or in life. Such connections create inspiration and motivation for learners to remember. There may be content that human beings have not yet been able to apply, but in principle, it should still be clearly oriented as playing a foundational role for higher-level knowledge and models.

Third, it is necessary to see the transformations of formulas and methods: from where they are derived, into what equivalent forms they can be transformed, and how new problems can be reduced to known forms. In this way, learners use mathematical tools flexibly and know how to extend and generalize, rather than merely applying them rigidly and mechanically.

*** Specific methods**

The general method not only does not deny the specific methods studied and synthesized by generations of mathematicians and teachers, but also emphasizes the necessity of using specific methods for specific types of problems. Such use makes problem solving faster and more effective. The author requires users to accumulate specific methods whenever possible.

*** Methods of equivalent transformation and techniques**

Methods of equivalent transformation, together with the associated techniques, are among the most important contents of the general method. The problem-solving subject needs to grasp these methods and techniques, including:

First, the method of detailing transformation: making the object, or a part of the object, possess an internal structure (detailing internal relations), or placing the object (or a part of the object) into other relations (detailing external relations).

This includes the following techniques:

1.1. Establishing relations in geometry: In geometry, drawing segments, extending segments, and constructing additional figures (the agent needs to consider all possible cases in constructing additional figures).

1.2. Structuring: Representing the object in a new and more detailed structural form through equivalent transformation - applying properties, theorems, and formulas.

For example, solve the equation: $sin\,3\,x + sin\,x = 0$.

Apply: $sin\,a + sin\,b = 2\,sin\left(\frac{a+b}{2}\right).cos\left(\frac{a-b}{2}\right)$

sina + sinb = 2sin$\left(\frac{a+b}{2}\right)$. $cos\left(\frac{a-b}{2}\right)$

Hence, sin 3x +sin x = 2sin 2x cos x

We have:

$$sin\,3\,x + sin\,x = 0 \Leftrightarrow 2\,sin\,2\,x.cos\,x = 0$$

$$\Leftrightarrow\; sin\,2x\; = \;0\;or\;cos\,x\; = \;0$$

1.3. Representing the requirement of the problem in a more specific form. For example, the problem statement is: “find all integers divisible by 6”. Applying this method, we may represent the requirement in the form: find all x satisfying: $x = 6k, k \in \mathbb{Z}$.

1.4. Representing the condition of the problem in a more specific form. For example: Find integers n such that n is odd and $n^2 + n$ is divisible by 4. Here, the condition that $n$is odd may be represented as follows: $n = 2k + 1, k \in \mathbb{Z}$.

Second, the method of simplifying transformation: this has two directions: first, simplifying the object (by equivalent transformations); second, de-structuring (introducing a substitution), including the following techniques:

1.5. Substitution: setting one part of the object in the form of a symbol, for example A, and then handling the relation among the parts of the object through A. This is a relatively common technique at present.

1.6. Simplifying the object: based on the characteristics of the object in the problem statement, applying properties to simplify it. This is also a commonly used measure.

Third, the method of division transformation: dividing the object into smaller parts, into separate cases, and solving each part or each case one by one.

This includes the following techniques:

1.7. Separation: separating one part of the object for treatment, then treating each remaining part.

For example, consider the problem of computing the sum:

$$S = \frac{1}{1.3} + \frac{1}{3.5} + \ldots + \frac{1}{k(k+2)}\,(k > 1)$$

Separate: $\frac{1}{k(k+2)} = \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right)$

Thus, we have:

$$S = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots + \frac{1}{k} - \frac{1}{k+2}\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{k+2}\right) = \frac{k+1}{2(k+2)}$$

1.8. Considering cases one by one: the object falls into a number of cases, and each case is treated separately. Or, for one part of the object, that part is considered as falling into a number of cases, and each of those cases is treated separately. This technique often begins with the phrase “Suppose” or “Assume.”

For example, consider the problem of finding min f(x) = |x| + |x-2|

Assume x<=0: f = 2-2x (decreasing in x) →min at x = 0, f = 2;

Assume 0<=x<=2: f = 2 (constant).

Assume: x>=2: f = 2x – 2 (increasing) →min at x = 2, f =2.

Conclusion: min f = 2 (for all x ∈ $[0,2]$

1.9. Dividing the conditions of the problem.

For example, solve the inequality: $\frac{1}{x-1} \geq 2$

Split the condition, x $\neq 1$, and consider the sign of x-1

- If $x > 1$: multiplying does not reverse the inequality, so $1 \geq 2(x-1) \Leftrightarrow x \leq \frac{3}{2}$,

Combining with the condition $x > 1$ we get $x \in 1; \frac{3}{2}$

- If $x < 1$ multiplying reverses the inequality, so $1 \leq 2(x-1) \Leftrightarrow x \geq \frac{3}{2}$ (which contradicts the condition $x < 1$) so this case is excluded.

Therefore, $1 < x \leq \frac{3}{2}$

Fourth, the method of reduction to a general form: reducing the object so that it becomes a case, a part belonging to, or a form of a more general type, or a problem of larger scope.

1.10. Finding the general form: the problem being solved is only a particular case.

Fifth, the method of establishing a representative: introducing an unknown X to represent what needs to be found. Then establishing the relationship between X and other objects.

In addition, flexible combination of the above transformation methods is required.

*** Basic and common logical inferences:** The content is shown in Table 3. Table 3: illustration of several important aspects of logic manifested in mathematical reasoning (problem solving).

3.3.2. The second phase: implementation content

The general method for problem solving includes the following steps:

STEP 1: Understand the problem / identify the correct problem to be solved and consider feasibility

In terms of content: clearly understand or identify the four components of the problem: the object, assumptions, requirement, and conditions (there may be no conditions arising from the problem itself, but it is always necessary to determine the maximum time for solving that problem). The content must be clear. In special cases, it is also necessary to clarify the terms used in describing the object, requirement, assumption, or conditions of the problem.

In terms of form: the above four components should be represented in a simplified, summarized, schematic, diagrammatic, or modeled form if possible. Such representation is necessary to help the agent see more clearly the logic of the problem as externally manifested.

After reading, understanding, and presenting the problem, is the problem of a familiar type? Has a specific method for that type already been established? If so, one should begin writing the idea and complete it into a solution.

After reading, understanding, and presenting the problem, the agent needs to determine whether the problem is feasible or not (usually applied to difficult and intractable problems), that is, whether there are sufficient conditions to proceed with solving it. The agent needs to answer the following questions:

Is the problem beyond one's own ability?

Is the maximum amount of time allocated for solving sufficient?

Does the problem fall into the category of an unsolvable problem? (that is, a problem with insufficient data or internal contradiction)

If the answer is "no" to the above three questions, the agent moves to Step 2.

STEP 2: Analyze the problem

The main objective of this step is to collect and exploit the data in the problem in order to prepare the premises for the following step. Without sufficient necessary data, there can be no correct direction of solution. The more difficult the problem, the more thoroughly it must be analyzed. The degree of analysis and data exploitation depends on the type of problem and its level of difficulty. Note: during the analysis process, if a correct solution direction is obtained, or a familiar problem type is recognized, or a specific method is identified, then that direction should be immediately carried out.

Problem analysis focuses on the following components: the object, assumptions, requirement (what needs to be found/proved), and conditions. Information is exploited on the basis of answering the following questions:

1/ What can be inferred from this? (used for all four components); 2/ What characteristics and properties does it have? (used for all four components); 3/ What are all the possible cases leading to it? (often used for "what needs to be found/proved"); 4/ To what theorems, formulas, or equations is it related? (often used for the "object"); 5/ From where does it originate? (often used for "what needs to be found/proved").

If, during the analysis process, a solution direction is found, one should immediately move to writing the solution.

STEP 3: Determine the solution direction and form the idea of the problem

This is the most important and decisive step in problem solving. The formation of the idea has the following characteristics and requirements:

First, before providing a complete and clear solution, the mathematical agent needs to determine the solution direction and thereby build the idea.

Second, determining the solution direction and the idea is conjectural in nature, and therefore trial-and-error in character, but the conjecture must be based on definite grounds and foundations, not on random groping.

Third, the basis and foundation for determining the solution direction and idea arise from the characteristics of what needs to be found/proved, the object, and the collected data (in the analysis step).

Fourth, in proposing a solution direction and idea, the mathematical agent should first give priority to experience, intuition, familiar problem types (already solved), and the use of specific methods (for certain types of problems) in order to conduct well-grounded trials. Only if these are ineffective should the general method be used to form the idea.

Fifth, because a mathematical solution is the result of a process of logical reasoning, the formation of the idea and the solution requires the agent to move toward flexible use of forms of logical inference, methods of equivalent transformation, and thorough exploitation of data - clear and determinate data.

Sixth, t the mathematical agent may need to return to Step 2 (analysis) to exploit additional data, or even to Step 1 to determine whether the problem falls into the category of an unsolvable problem.

Seventh, the mathematical agent needs to maintain a confident and optimistic attitude, but must also be cautious to avoid subjectivity; there may be moments of impasse, so greater persistence and determination are needed.

Eighth, the mathematical agent needs to promote creative ability, according to the following principles: 1/ find at least two approaches to the solution; 2/ make full use of methods and techniques of equivalent transformation and logical inferences to solve the problem in an original, concise, and "elegant" way; 3/ view the problem from many angles; 4/ think associatively of problems already solved that bear some similarity to the current one; 5/ exploit and combine all possible possibilities.

Ninth, the more difficult the problem, the more necessary it is to: 1/ fully exploit all possible data; 2/ consider all possible solution directions; 3/ make full use of all possible methods of reasoning; 4/ effectively use all possible methods of equivalent transformation; 5/ maximize the creative ability of each subject. Therefore, listing fully the above possible contents is a necessary operation when confronting difficult and intractable problems.

Tenth, the formation of a direction and idea for the solution may proceed through the following concrete measure (applied to difficult and intractable problems, when experience, intuition, and familiar problem types are ineffective):

The determination of the solution direction should be directed toward: (1): the characteristics of what needs to be found/proved;

(2): the object, conditions, and assumptions of the problem. After directing attention to these, the agent needs to determine the following:

First, whether the problem is to be solved by a direct or an indirect method (whether it is a finding problem or a proof problem).

In proof problems, the indirect proof method is a method of constructing grounds to reason indirectly toward affirming what needs to be proved. This method includes two forms: proof by contradiction and eliminative proof. The direct proof method is a method of constructing grounds to reason directly toward what must be proved. This method includes many different forms.

In finding problems, the indirect finding method is a method in which the search requirement falls only into a certain number of cases (the agent needs to reason this out). Then, by means of reasoning that excludes the other cases, only one case remains (and this remaining case often has no direct data for reasoning). The direct finding method is a method in which what needs to be found is satisfied by the very data mainly provided by the object and assumptions. This method also includes many different forms.

Second, after determining whether the problem uses a direct or indirect method, the agent continues to determine a more specific method of solution, but before that, in many cases (especially difficult problems), methods of equivalent transformation need to be used.

In proof problems, if the indirect proof method can be used, it is necessary to determine whether to use proof by contradiction or eliminative proof.

- **Proof by contradiction:** the agent needs to perform: Step 1: formulate the negation of the proposition to be proved. Step 2: prove that this negation is false, or contrary to the assumption. Therefore, conclude that the given proposition is true - which is what had to be proved.

- **Eliminative proof:** when proving that A is true, the agent needs to perform the following steps: Step 1: affirm that the object in the problem can only fall into the possible (finite) cases (list the cases, among which there is the case containing what must be proved, namely the one containing A). Step 2: successively prove that the cases that are not the case of what must be proved are false. Step 3: finally infer that the only remaining case must be true - that is A.

Or, if there are no data for carrying out an indirect proof method, the agent should move toward a direct proof method, and then examine whether the following possible methods may be used:

(1): the transitive method (this method is quite common);

(2): the method of mathematical induction (used for certain problems);

(3): the general proof method (placing the problem in a general model and proving it in that general form);

(4): the method of proof by a chain of reasoning - with deduction as the core (on the basis of applying formulas, theorems, etc.). This method is common and widely used. For example, with A as what needs to be proved, one may have the chain of reasoning: E →D→C→B→A (which proves the desired statement).

(5): the method of proof by backward reasoning: instead of thinking in the forward direction as in method (4), thought may proceed in the reverse direction, that is, beginning from A, the agent determines B, then determines C, then D, and finally E;

(6): the method of proof by division: dividing what needs to be proved into constituent cases, and then proving each case in turn.

Note: it is necessary to combine the use of methods of equivalent transformation in order to use the above methods.

In finding problems, if the indirect finding method can be used, then the eliminative finding method should be implemented immediately. This method is carried out through the following steps: Step 1: argue that the answer to the problem can only fall into the cases: $A, B, C, D, \ldots$ Step 2: argue that the cases $B, C, D, \ldots$are false. Step 3: conclude that A is the answer, satisfying the search requirement of the problem.

In finding problems, if the indirect finding method cannot be used, then one moves toward direct finding methods, including the following possible methods:

(1): the finding method based on methods of equivalent transformation (including the five methods of equivalent transformation and methods combining them).

(2): the backward finding method, namely assuming that $A$is what needs to be found, then the agent's thought takes Aas the starting point and then searches for what leads to A, call this B, then searches for what leads to B, and so on, until reaching the final items, which are the assumptions and related knowledge mobilized.

(3): the remainder method: in the two sides of an equation, if the parts on both sides are the same, then the remaining part (containing the unknown) must be equal. For example, in the equation: $5^x + 7 = 5^9 + 7$. the solution is x=9.

(4): the reverse transformation method: let Bbe the object to be transformed, and A the result of the transformation; the agent's thought starts from A and performs a chain of reverse reasoning back to B.

Note: if the agent cannot find a solution direction, it is necessary to return to Step 2, analysis, in order to exploit more data; if this is still unsuccessful, then continue returning to Step 1 to examine whether the problem falls into the category of an unsolvable problem.

After obtaining the correct solution direction, the agent exploits the data, uses methods of logical reasoning, and techniques of equivalent transformation to arrive at what needs to be found/proved.

STEP 4: Complete the idea into a solution

After obtaining the idea, the problem-solving subject needs to begin writing out the detailed solution. Requirement: the solution must be clear, logically rigorous, and satisfy the stated requirement.

STEP 5: Recheck and draw conclusions (if necessary)

After finishing the written solution, the agent must review and check the steps again to see whether there are any errors or whether any numerical data have been written incorrectly. This is a necessary phase to ensure that the solution is presented coherently and avoids confusion.

*** The relationship between the method and AI**: In the context of the development of AI, this problem-solving methodology is meaningful not only for the agent solving problems and for mathematics teaching, but also provides programmers with a reference thinking framework for building and refining problem-solving algorithms. Modeling the steps of thinking, reasoning activities, and equivalent transformations helps programmers design algorithms with tighter structure and logic. Conversely, AI tools can support the agent in applying the method more quickly and effectively by retrieving relevant mathematical knowledge, suggesting approaches, and checking reasoning. However, AI is only a supporting tool; orientation, evaluation, and decision regarding the solution still belong to the thinking of the human solver. This two-way interaction helps clarify the boundary and complementarity between human problem-solving methods and the computational power of AI.

3.4. Illustrative examples of the general method

Example 1: Find the minimum value of P $= \frac{x^2+4}{x+2}, x > -2$

Step 1: Identify the problem – consider feasibility

Object: P $= \frac{x^2+4}{x+2}$

Condition: $x > -2$

Time allocation: 15 – 20 minutes.

Feasibility: feasible.

Requirement: Find P_{min}

Step 2: Analysis

X> -2 ↔ X+2>0.

P_{min} → means finding x such that P is minimal under the condition $x + 2 > 0$. Finding the minimum value should be reduced to the form $P \geq$, where the equality sign represents the minimum value of P.

Observation: for P, the method of simplifying transformation is needed. Moreover, since the denominator contains $x + 2$, let

In terms of knowledge, it is necessary to exploit knowledge related to inequalities. There is the arithmetic mean–geometric mean inequality as follows:

∀a, b >0: $\frac{a+b}{2} \geq \sqrt{ab}$, with equality if and only if a = b

Step 3: Determine the solution direction and form the idea

Solution direction: for the above problem, the direct method should be used, carrying out the method of simplifying transformation together with the substitution technique and the application of an inequality.

Idea formation:

Let t = x + 2>0: By simplifying transformation, we have:

$$P = \frac{x^2+4}{x+2} = \frac{t^2-4t+8}{t} = t + \frac{8}{t} - 4$$

Apply the above inequality with a = t, b $= \frac{8}{t} \rightarrow t + \frac{8}{t} \geq 4\sqrt{2} \leftrightarrow t + \frac{8}{t} - 4 \geq 4\sqrt{2} - 4 \; or \; P \geq 4\sqrt{2} - 4$.

Equality holds if and only if a = b that is, t $= \frac{8}{t} \rightarrow t = 2\sqrt{2} \; since \; t = x + 2 \; we \; get \; x = 2\sqrt{2} - 2$.

Thus, when $x = 2\sqrt{2} - 2$ we have $P_{min} = 4\sqrt{2} - 4$.

Steps 4 and 5: Complete the detailed solution according to the above idea and then recheck it.

Example 2: Prove that for every integer n, $n^3 - n$ is divisible by 3.

Step 1: Identify the problem – consider feasibility

Object: $n^3 - n$

Condition: n $\in Z$

Time allocation: 20 – 25 minutes

Requirement: Prove that the object is divisible by 3.

Feasibility: feasible.

Step 2: Analysis

- Transform the object: $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$

- Mobilize mathematical knowledge: among any three consecutive integers, there is always exactly one number divisible by 3.

Step 3: Determine the solution direction and form the idea

Solution direction: direct proof, using the method of detailing transformation - factorization - and the property of divisibility by 3 of consecutive integers.

Idea: $n, n-1$, and $n+1$ are three consecutive integers, so there is always exactly one number divisible by 3. Hence, the product of the three consecutive integers $n(n-1)(n+1)$ must be divisible by 3.

Step 4: Complete the solution

We have: $n^3 - n = n(n^2 - 1) = n(n-1)(n+1)$

By the assumption: so $n, n-1$, and $n+1$ are three consecutive integers. Therefore, one of them is divisible by 3, and hence the product of the three consecutive integers $n(n-1)(n+1)$ must also be divisible by 3. We conclude that $n^3 - n$ is divisible by 3 for all $n \in \mathbb{Z}$. Proved.

Step 5: Recheck and draw lessons (if any)

Test with $n = 0$,the result is 0, which is divisible by 3. With $n = 2$,the result is 6, which is divisible by 3. With $n = 5$, the result is 120, which is divisible by 3.

4. Conclusion

The article has proposed a general methodological framework for problem solving, with a structure consisting of a preparation phase and an implementation phase within a systematic five-step process. The novelty of this work does not lie in introducing isolated problem-solving techniques, but in modeling problem-solving activity as an integrated whole, in which the problem, the solver, the thinking process, the activities of logical reasoning, equivalent transformation, creativity, and psychological-environmental factors are placed within a unified structure. This method allows the solver to proactively control the thinking process, flexibly return to previous steps and make adjustments when confronting difficult and intractable problems, rather than merely applying isolated strategies or heuristics as in many previous works. In particular, placing methods of equivalent transformation at the center helps clarify the mechanism of the formation and development of problem-solving ideas. In this way, the work not only provides an effective tool to support problem solving, but also contributes to clarifying the structure of human problem-solving thinking in the context of modern mathematics education. Further studies may focus on testing, refining, and concretizing the method for each content area (such as algebra, geometry, calculus, etc.) and for different educational levels.

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