ASPECT RATIO EFFECTS OF SUPERQUADRIC GRANULAR COLUMN COLLAPSE ON FLOW DYNAMICS AND KINETIC ENERGY FLUX

TỶ LỆ CỘT VẬT LIỆU ĐA DIỆN ẢNH HƯỞNG ĐẾN ĐỘNG LỰC HỌC DÒNG CHẢY VÀ THÔNG LƯỢNG ĐỘNG NĂNG TRONG HIỆN TƯỢNG SẠT TRƯỢT

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Abstract - This study investigates the dynamics and potential destructive energy of granular column collapse on a horizontal plane using the Discrete Element Method (DEM). The granular column is modeled using superquadric particles with initial geometric aspect ratios $a = h_0/l_0$ ranging from 0.3 to 2.0. The results show that, increasing the aspect ratio leads to a significant increase in the runout distance, whereas the final deposit height decreases. The conversion of potential energy to kinetic energy becomes more pronounced as the aspect ratio increases, resulting in a higher peak kinetic energy and greater mobility of the granular mass. The kinetic energy flux across virtual sections exhibits three distinct stages: rapid growth, peak, and gradual decay over time, and it decreases progressively with increasing distance from the initial collapse region. These findings provide valuable insights for assessing the destructive potential of granular flows and offer guidance for designing protective structures against landslide hazards.

Key words - Discrete Element Method; superquadric particle; granular column collapse; kinetic energy flux

Tóm tắt - Nghiên cứu này khảo sát động lực học và khả năng phá hoại tiềm năng trong bài toán sụp đổ khối vật liệu trên mặt phẳng nằm ngang bằng phương pháp phần tử rời rạc. Khối vật liệu được mô hình hóa bằng các phần tử đa diện superquadric có tỉ lệ hình học ban đầu $a = h_0/l_0 = 0{,}3 - 2{,}0$. Kết quả cho thấy, khi a tăng, phạm vi lan truyền tăng đáng kể, trong khi chiều cao khối sạt cuối cùng giảm. Sự chuyển hóa thế năng thành động năng trở nên mạnh mẽ hơn đối với các cột có tỉ lệ a lớn, dẫn đến giá trị động năng cực đại cao hơn và tác động lan xa hơn. Tốc độ truyền năng lượng qua các mặt cắt thể hiện ba giai đoạn đặc trưng gồm tăng nhanh, đạt cực đại và suy giảm theo thời gian, đồng thời giảm dần theo khoảng cách từ vùng sụp đổ ban đầu. Những kết quả này là tài liệu tham khảo để đánh giá khả năng phá hoại của dòng vật liệu hạt và hỗ trợ thiết kế các công trình phòng chống sạt trượt.

Từ khóa - Phương pháp phần tử rời rạc; phần tử đa diện; khối vật liệu sạt trượt; thông lượng động năng

1. Introduction

Granular mass flows such as landslides, debris flows, and rockfalls are natural hazards that cause severe damage to infrastructure and human settlements, particularly in mountainous regions [1], [2], [3]. The destructive potential of these phenomena is governed not only by the total mobilized mass but also by the flow dynamics, including velocity, runout distance, and tremendous impact forces on protective downstream structures [4], [5]. Therefore, understanding the mechanics of energy transfer within granular flows is of critical importance for hazard assessment and the design of effective structural countermeasures.

Experimental studies by Lajeunesse et al. [6] and Lube et al. [7] revealed fundamental scaling laws linking the runout distance and final deposit morphology to the initial aspect ratio of the granular column. The collapse process of a granular column on a horizontal plane has been widely adopted as a benchmark problem to investigate gravity-driven granular flow behavior [8] - [11]. Subsequently, numerical investigations using the Discrete Element Method (DEM) have further improved the understanding of the flow dynamics, energy dissipation during column collapse.

In experimental models, the particle shapes used in granular column studies range from disk to various non-spherical geometries [6], [7], [12], [13]. In numerical simulations, granular materials are often represented by disk-shaped particles in two dimensions [14] - [17], or spherical particles in three dimensions [10], [18] - [20] due to their computational simplicity. However, natural granular materials consist of irregular, angular grains or complex geometries, so the idealized materials used in the above studies tend to spread excessively, causing a larger displacement, especially for circular (2D) or spherical (3D) particles whose high mobility results in extended runout distance. Even for polygonal (2D) or cubic (3D) particles, the simplified geometry may still inadequately capture interlocking effects, leading to discrepancies in predicted deposition morphology and energy evolution during collapse. Previous studies on granular column collapse have primarily focused on macroscopic quantities such as the maximum runout distance, final deposit morphology, and scaling laws relating these quantities to the initial aspect ratio of the column [21], [22]. While these metrics provide valuable information on the overall flow behavior, they offer limited insight into how granular flows interact with downstream structures or how the destructive potential evolves at specific locations where protective structures may be installed.

To address these limitations, energy-based approaches have been proposed as an alternative framework for characterizing the intensity of granular flows impacting structures. In this context, the concept of kinetic energy flux across a virtual section, introduced by Utili et al. [10], provides a physically meaningful macroscopic measure of the destructive potential of granular flows. This quantity represents the rate at which kinetic energy is transported across a prescribed cross-section, simultaneously accounting for particle velocity magnitude and flow direction relative to the protected structure. However, the study by Utili et al. employed spherical particles, which may lead to an overestimation of particle mobility, velocity, and kinetic energy flux due to reduced interlocking effects. Therefore, the kinetic energy flux across a virtual section can be regarded as a representative indicator of the potentially destructive impact of granular flows at a given location.

Motivated by these considerations, the present study investigates the dynamics of granular column collapse on a horizontal plane using three-dimensional DEM simulations with superquadric particles. The study examines the final deposit morphology, temporal evolution of potential and kinetic energy, and particularly the time-dependent behavior of kinetic energy flux across multiple virtual sections positioned along the flow direction. Special attention is given to the influence of the initial aspect ratio of the granular column on the intensity, timing, and spatial attenuation of energy transmission. By analyzing the peak values of kinetic energy flux, the occurrence time of peak energy, and their decay with distance, this study aims to provide additional scientific insight for hazard assessment and the design of downstream protective structures.

2. Numerical method and model validation

2.1. Numerical method

This study employs the Discrete Element Method (DEM) to investigate the collapse behavior of a granular mass on a horizontal plane using a three-dimensional (3D) numerical model. In DEM, originally introduced by Cundall and Strack [23], particles are treated as rigid bodies during contact interactions. When two particles come into contact, the interaction forces are decomposed into normal and tangential components. The normal force governs the repulsive interaction between particles and prevents excessive overlaps, whereas the tangential force accounts for frictional resistance during sliding. These contact forces determine the translational and rotational motion of each particle according to Newton's laws of motion. The granular mass is composed of superquadric particles, and all simulations are performed using the open-source software LIGGGHTS (version 3.8.0), as illustrated in Figure 1. In this paper, granular columns with different initial geometries are investigated by varying the initial aspect ratio $a = h_0/l_0$ = 0.3 – 2.0, where l_0 = 0.1 (m) is the initial length of the superquadric granular column in the X-direction, which is kept constant in all simulations, and h_0 is the initial height of the column in the Z-direction. The selected range of aspect ratios ensures that collapse regimes are captured: for a < 1, the spreading process is a friction-dominated regime, while for a > 1, the collapse is governed by an inertia-dominated regime. This choice allows the study to systematically compare the transition between these two regimes. Furthermore, Utili (2015) has already investigated the kinetic energy flux across virtual sections for larger columns, so focusing on the interval a = 0.3–2.0 provides a deeper understanding of the flow dynamics and allows the investigation of the kinetic energy flux in this current work. The granular column width is set to W_0 = 0.02m along the Y-direction, which is sufficiently large to minimize boundary effects from the lateral walls during the collapse process. Each superquadric particle has a major dimension of 2.1mm and a minor dimension of 1.4mm. Previous studies have shown that when the column width exceeds ten times the mean particle diameter, the influence of lateral boundary walls on the flow dynamics in the Y-direction becomes negligible [24], [25]. Therefore, the selected column width in this study ensures that wall effects do not significantly affect the collapse behavior.

The granular column is initially generated inside a confining box. The collapse is triggered by removing the right retaining wall along the X-axis. Under the action of gravity, the superquadric particles first collapse downward in the Z-direction and subsequently spread over the horizontal plane until a final stable configuration is reached. In the final deposit, the maximum runout distance of the granular column is denoted as l_f and the final deposit height is defined as h_f, as illustrated in Figure 1. The total number of superquadric particles involved in all simulations ranges from approximately 8,700 to 60,000 particles, depending on the initial aspect ratio a.

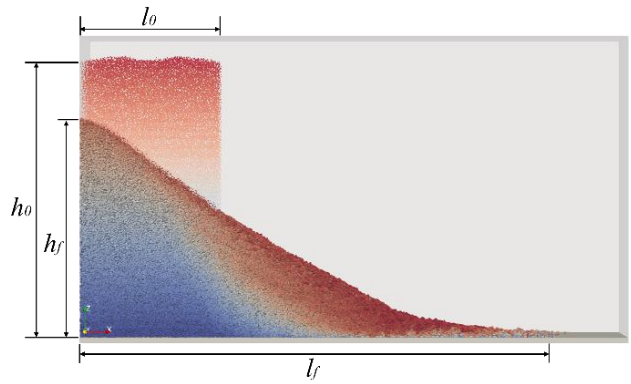

Figure 1. *Numerical model of a granular column composed of superquadric particles at the initial stage and final stage*

2.2. Model validation

To validate the numerical model, the simulation results obtained in this study are compared with available experimental data [12] and numerical results reported in the literature [21], [26], [27], as shown in Figure 2, for granular columns with initial aspect ratios a ranging from 0.3 to 2.0. It can be observed that the present results are in good agreement with previously published studies, including the experimental investigations conducted by Man et al. [12] using cubic particles, numerical simulations of superquadric particles performed by Hoang & Nguyen

[26] as well as Jiang et al. [27], and the two-dimensional numerical simulations with pentagonal particles reported by Nguyen & Vo [21].

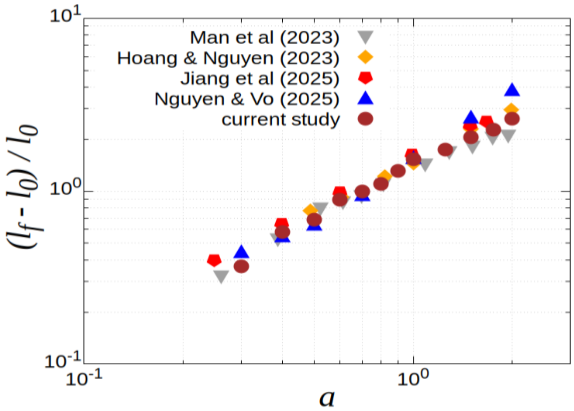

Figure 2. *Model validation with previous results about runout distance*

Table 1. *Principal parameters used in all simulations*

Parameter	Symbol	Value	Unit
Initial column height	h_0	0.03 – 0.2	m
Initial column length	l_0	0.1	m
Particle density	ρ	2500	kg m^{-3}
Young's modulus	E	5.84	MPa
Poisson's ratio		0.3	
Gravity	g	9.81	$m\ s^{-2}$
Interparticle friction coefficient	μ_p	0.3	-
Particle – wall friction coefficient	μ_w	0.3	-
Coefficient of rolling friction	μ_r	0.3	
Time step	Δt	5 x 10^{-6}	s

The main parameters of the superquadric granular column, as well as the interaction between particle and boundary walls, are detailed in Table 1. In this table, the coefficient of interparticle friction and particle-wall friction were set to 0.3. The particle density was maintained constant at 2500 kg/m^3, the Young's modulus at 5.84 MPa and the time step at 5 x 10^{-6} s, ensuring numerical stability throughout the collapse process across all investigated aspect ratios.

3. Results

The present study investigates the collapse dynamics of a granular column on a horizontal plane over a range of initial aspect ratios $a = h_0/l_0$ = 0.3 – 2.0, with the initial column length fixed at l_0 = 0.1m. To characterize the flow dynamics, the analysis focuses on the resulting deposit morphology, the temporal evolution of energy during the collapse, including the normalized potential energy E_p, total kinetic energy E_k, and its vertical and horizontal components E_{kz}, E_{kx}. In addition, the study examines the rate of energy transmission across virtual sections placed downstream of the collapsing column, as described in the following sections.

3.1. Final deposit morphology

After triggering, under the action of gravity, the granular column initially collapses in the vertical direction and subsequently spreads in the horizontal direction until a stable state is reached. Figure 3 illustrates the final deposit morphology corresponding to different initial column sizes. Figure 3(a) depicts the stable deposit shapes, whereas Figure 3(b) reports the normalized final height and runout length relative to the initial height h_0 and length l_0.

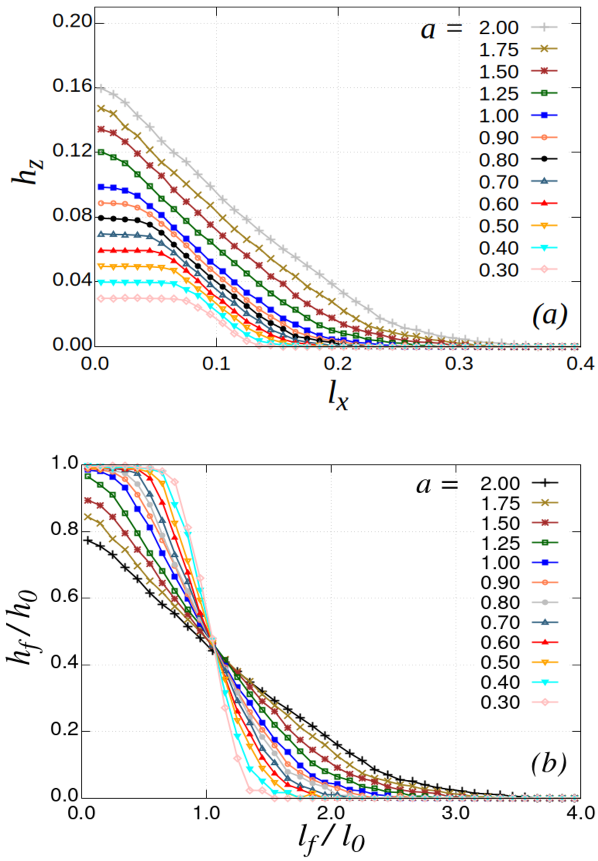

Figure 3. *Final deposit morphology of granular columns with different aspect ratios*

The results shown in Figure 3(a) highlight distinct deposit morphologies associated with different initial column heights. For columns with small aspect ratios (a <1.0), the final deposit exhibits an approximately trapezoidal shape, whereas for larger aspect ratios (a > 1.0), the final morphology tends to be triangular. This behavior is governed by the amount of initial potential energy stored in the column. For small aspect ratios, the collapse process is mainly governed by interparticle interactions, particularly frictional effects. In contrast, for higher aspect ratios, the collapse dynamics are primarily governed by the gravitational potential energy of the column.

Figure 3(b) presents the normalized final deposit morphology with respect to the initial column height h_0 and length l_0. For columns with a < 1.0, only a limited portion of the material participates in the collapse, primarily on the downstream side (positive X-direction), and the final column height remains close to its initial value. Conversely, for higher aspect ratios (a > 1.0), the enhanced influence of gravitational potential energy leads to a significantly larger horizontal runout distance, accompanied by a pronounced reduction in the final deposit height as a increases.

3.2. Temporal evolution of energy

As discussed in the analysis of the deposit morphology, for columns with small aspect ratios $a < 1.0$, only a portion of the granular mass undergoes collapse, whereas for larger aspect ratios $a > 1.0$ the column collapses in the vertical direction before spreading extensively in the horizontal direction, resulting in a reduction of the column height and an increase in the runout distance. These contrasting collapse mechanisms are clearly reflected in the energy evolution throughout the collapse process, as illustrated in Figure 4.

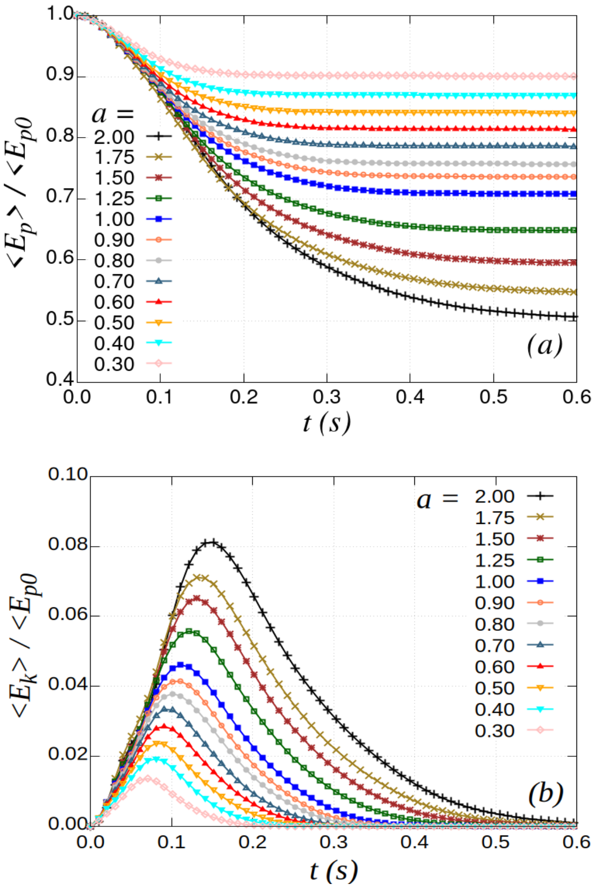

Figure 4. *Evolution of the normalized potential energy (a) and kinetic energy (b) as a function of the collapse process for all aspect ratios*

Figure 4(a) shows the temporal evolution of the normalized potential energy $\langle E_p \rangle / E_{p0}$ with respect to the initial potential energy for the different granular columns throughout the collapse process. The potential energy decreases slightly as the column aspect ratio a increases. The potential energy is defined as $E_p = m_i g h_i$, where m_i and g denote the mass of superquadric particle i and the gravitational acceleration, respectively, and h_i is the elevation of particle i at different stages of the simulation. The initial potential energy E_{p0} corresponds to the initial energy of the granular column. For columns with small aspect ratios, the potential energy decreases rapidly during the early stage of the collapse ($t = 0.0 – 0.25$ (s)) and then stabilizes, indicating that only a portion of the granular mass collapses vertically before quickly reaching a stable configuration. In contrast, for columns with large aspect ratios, the granular mass undergoes an almost complete collapse in the vertical direction, followed by extensive spreading in the horizontal direction. As a result, the potential energy decreases markedly over a longer duration $t = 0.0 – 0.5$ (s), before the system reaches a stable state. The transformation of energy from potential to kinetic energy is demonstrated throughout the collapse process, the initial phase mainly involves collapse along the vertical direction, followed by a transformation into kinetic energy, at which point the superquadric particles mainly move along the horizontal direction.

Figure 4(b) presents the temporal evolution of the normalized kinetic energy $\langle E_k \rangle / E_{p0}$ as a function of the flow time t for granular columns with different aspect ratios a. Here, the total kinetic energy is defined as $E_k = 0.5 m_i v_i^2$, v_i denotes the velocity of superquadric particle i evaluated at the corresponding time instants. The results show that the kinetic energy increases rapidly from zero, reaches a peak value, and then gradually decreases towards zero. Both the temporal evolution and the peak value of the kinetic energy increase with increasing aspect ratio a, due to the influence of the large column height, which provides greater potential energy available for conversion to kinetic energy. The time at which the maximum normalized kinetic energy $\langle E_k \rangle / E_{p0}$ increases from approximately 0.05(s) for the smallest column to about 0.15(s) for the column with $a = 2.0$. After the kinetic energy peak, particle velocities progressively decrease due to interactions among particles and between particles and walls, leading to the system towards a stable state. Consequently, taller columns exhibit longer collapse and spreading durations compared to shorter columns, as illustrated in Figure 4(b).

To further elucidate the mechanisms governing collapse dynamics, the vertical and horizontal components of kinetic energy, E_{kz} and E_{kx}, are presented in Figure 5(a) and (b). These kinetic energies are defined as $E_{kz} = 0.5 m_i v_{iz}^2$, and $E_{kx} = 0.5 m_i v_{ix}^2$, where v_{iz}, v_{ix} denote the vertical and horizontal velocity components of particle i, respectively. Figure 5(a) shows that the normalized vertical kinetic energy E_{kz}/E_{p0} rises rapidly during the early stage of collapse, reaching a maximum at $t \approx 0.07 - 0.13$ s depending on the aspect ratio, with higher columns (larger a) exhibiting substantially larger peaks. After the peak, E_{kz} decreases quickly as vertical motions are dissipated through inter-particle collisions and basal friction. The normalized horizontal kinetic energy E_{kx}/E_{p0} shown in Figure 5(b) reaches its maximum later, at $t \approx 0.09 - 0.22$ s. Although E_{kx} also increases with a, its magnitude remains significantly lower than E_{kz}.The shape of the E_{kx} curves highlights the sustained horizontal flow that governs the runout and final deposit morphology.

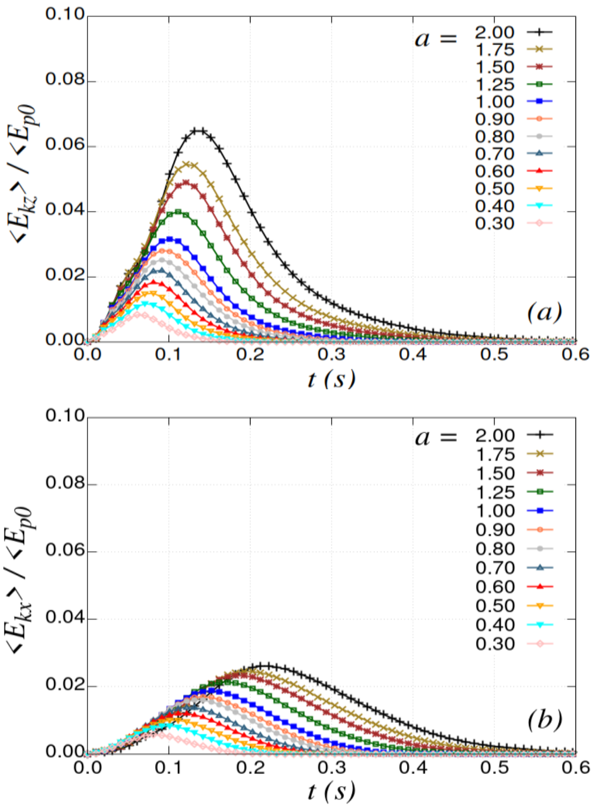

Figure 5. Evolution of the normalized kinetic energy in the vertical (a) and horizontal (b) as a function of the collapse process

3.3. Kinetic energy flux

The analysis of energy evolution plays a crucial role in granular flow and landslide problems, as it provides a fundamental basis for estimating the runout distance and the peak energy points, which are essential for the design of mitigation measures and the reduction of downstream hazards. To investigate the rate of energy transmission at different downstream locations, the current study evaluates the kinetic energy flux across five sections, denoted as sections A, B, C, D, and E. The virtual sections are located at X_section = 0.105, 0.15, 0.20, 0.25 and 0.30m, respectively. The selected locations start immediately after the initial column to capture the energy transmission associated with smaller columns and progressively extend further downstream to account for larger columns with longer runout distances. The location of the virtual sections is illustrated in Figure 6.

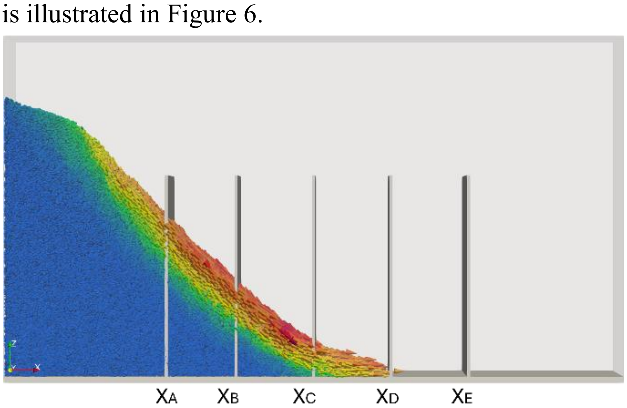

Figure 6. Snapshot of the kinetic energy flux across virtual sections A, B, C, D and E for the granular column with aspect ratio a = 2.0

The energy transmitted across virtual sections, ΔE as sections A-E over a given time interval Δt, is evaluated by accounting for all superquadric particles that cross each section. This analysis enables the quantification of the kinetic energy flux across virtual sections located downstream of the initial granular column, providing a rational basis for assessing the impact potential and for the design of protective structural countermeasures downstream [10].

Figure 7 illustrates the kinetic energy transmission across virtual sections A and B, shown in Figure 7(a), and 7(b), respectively, for simulations with different aspect ratios a = 0.3 – 2.0 over the entire collapse process. The kinetic energy flux P is defined as $P = \dfrac{\Delta E / E_{p0}}{\Delta t / t_0}$, where $t_0 = \sqrt{h_0 / g}$. For all investigated cases, the kinetic energy flux P exhibits a characteristic three-stage pattern: (i) an initial rapid increase phase, (ii) reaching the maximum value $P_{\max}$, and (iii) the prolonged decrease phase over time. This behavior reflects the conversion of the initial gravitational potential energy into kinetic energy during the collapse stage, followed by progressive energy dissipation due to interparticle collisions, friction effects.

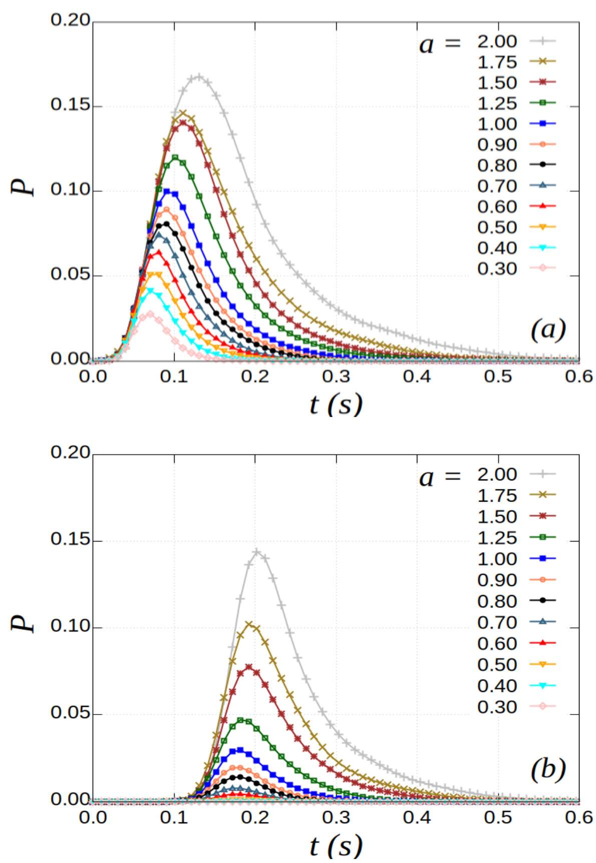

Figure 7. Kinetic energy flux across section A (a) and B (b) for all simulated granular column configurations

Consequently, the energy transmission section A begins only after approximately t > 0.03s, while for section B it occurs later, at about t > 0.12s. A comparison between the two sections shows that, at the farther downstream location, the magnitude of P is significantly reduced, both in terms of peak amplitude and slope, and the occurrence of the energy peak is delayed. This phenomenon indicates that the kinetic energy flux transmitted through virtual sections decreases with distance from the initial collapse zone.

Figure 8 shows the temporal evolution of the kinetic energy flux across virtual sections A, B, C, D, and E for two granular columns with aspect ratios $a = 1.0$ (Figure 8a) and $a = 2.0$ (Figure 8b). For the column with $a = 1.0$, the energy flux transmitted across section A ($P \approx 0.1$) is significantly higher than that through section B ($P \approx 0.03$) and it becomes nearly zero at the downstream sections. This behavior arises because only a portion of the granular mass participates in the collapse, resulting in a limited runout distance before section C. Consequently, the kinetic energy flux across sections C, D, and E is negligible.

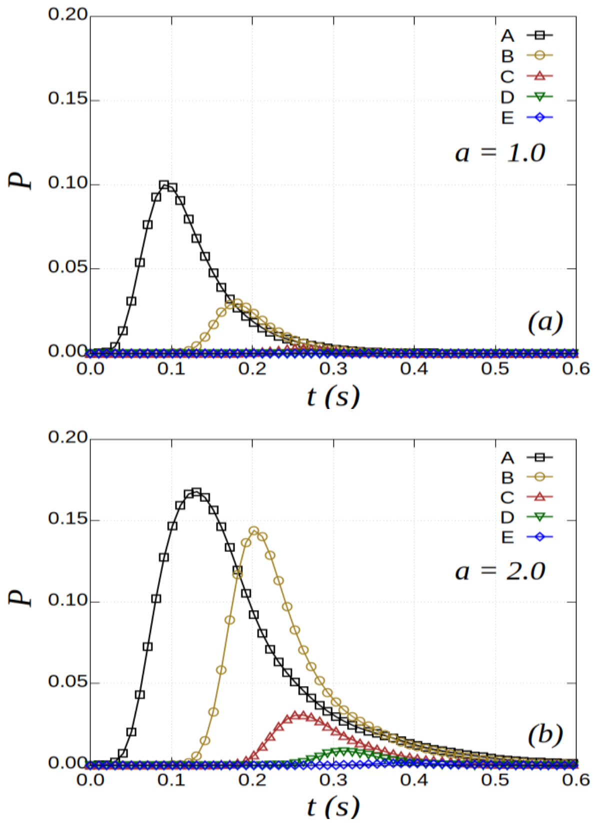

Figure 8. *Kinetic energy flux across virtual sections A, B, C, D and E for granular columns with aspect ratio a = 1.0 (a) and a = 2.0 (b)*

For the taller column with $a = 2.0$ (Figure 8(b)), the energy transmitted is substantially greater than that observed for $a = 1.0$, due to the influence of a larger initial column height. Although the investigated sections are approximately equally spaced, the kinetic energy flux varies markedly between sections, particularly between sections B and C. From section A to section B, the energy peak P at a time interval of 0.08s, but from section B to C, this time interval is much shorter, approximately 0.03s, and the energy decreases from $P \approx 0.14$ at section B to $P \approx 0.03$ at section C. This happens because the kinetic energy of the granular mass decreases rapidly during the period from 0.2 to 0.3s, causing the granular mass to spread along the horizontal plane and gradually reach a stable configuration. As discussed above, columns with larger aspect ratios are governed by higher initial potential energy. As shown in Figure 8b, the kinetic energy flux transmitted through section B remains relatively high compared to that observed for smaller columns.

The results have important implications for the design and assessment of downstream mitigation measures. In scenarios with large aspect ratios a, protective structures located at considerable distances from the initial granular column collapse may still be subject to significant impacts from the granular flow. In contrast, for a smaller aspect ratio, the energy transmitted decreases more rapidly, thereby reducing the risk of damage at farther downstream locations. Therefore, a combined evaluation of the initial column size and the maximum runout distance is sufficient for accurately assessing hazard levels and for proposing effective solutions of protective structures.

4. Conclusions

This paper investigates the collapse of a granular column on a horizontal plane using three-dimensional Discrete Element Method (DEM) simulations. The granular column composed of superquadric particles is characterized by initial aspect ratios ranging from $a = 0.3 – 2.0$. The study focuses on examining the influence of the initial column geometry on flow dynamics, temporal evolution of energy, and the kinetic energy flux across virtual sections located downstream of the collapsing column.

The results indicate that the initial geometric aspect ratio has a pronounced influence on the final deposit morphology. For columns with small aspect ratios $a < 1$, only a portion of the granular mass participates in the collapse process, leading to a final deposit with an approximately trapezoidal shape, and a limited horizontal runout distance. In contrast, for columns with larger aspect ratios $a > 1$, the granular mass undergoes an almost complete vertical collapse, then spreads on a horizontal plane, resulting in a nearly triangular final deposit and a significantly larger runout distance. As the aspect ratio a increases, the potential energy decreases more substantially and over a longer duration during collapse. The conversion from potential energy to kinetic energy becomes more pronounced, leading to higher peak kinetic energy values. This behavior highlights the dominant role of the initial potential energy in governing the flow dynamics of the granular mass. This study also investigates the kinetic energy flux across virtual sections, which exhibits a characteristic three-stage evolution consisting of a rapid increase, peak, and decline over time. The peak energy flux decreases with increasing distance from the initial granular column, while the occurrence of the energy peak is progressively delayed at farther downstream sections. For columns with small aspect ratios, significant energy transmission is confined to sections located close to the initial granular column, whereas for larger aspect ratios, the energy remains significant at locations further downstream. The kinetic energy flux can serve as an important parameter for evaluating the destructive potential of granular flows. Larger kinetic energy fluxes indicate more intense momentum transfer within the flow, which may lead to stronger impacts on protective structures such as dams, barriers, and retaining systems. Consequently, understanding the variation of kinetic energy flux during the collapse process can contribute to hazard assessment and the design of mitigation measures for landslides and debris flows.

Although the present study is limited to investigating initial granular column aspect ratios in the range a = 0.3 – 2.0, the results provide deeper insight into the dynamics of granular column collapse involving superquadric particles. Further studies may extend the proposed numerical framework to examine the effects of basal surface roughness, as well as the interactions between granular flows and downstream protective structures.

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