LYAPUNOV-BASED NONLINEAR MODEL PREDICTIVE CONTROL FOR SUSTAINABLE OPERATION OF SHIP-MOUNTED CONTAINER CRANES

ĐIỀU KHIỂN DỰ ĐOÁN MÔ HÌNH PHI TUYẾN DỰA TRÊN LYAPUNOV ĐỂ VẬN HÀNH BỀN VỮNG CẦN TRỤC CONTAINER GẮN TRÊN TÀU

Duy Lam Mac[1], Viet Thang Dang[1], Van Manh Nguyen[2], Van Binh Cao[3], Anh Duc Pham[3], Trong Hien To[3], Van Trieu Pham[3]*

[1]*Hanoi University of Science and Technology, Vietnam*
[2]*Unit 871, General Department of Politics, Ministry of National Defense, Vietnam*
[3]*Vietnam Maritime University, Vietnam*

*Corresponding author: phamvantrieu@vimaru.edu.vn

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Abstract - The rapid development of the marine industry has driven the emergence of ultra-large container ships, creating challenges in accessing small ports and ensuring efficient cargo handling. This paper investigates an advanced control framework for a ship-mounted container crane to enable direct transshipment at sea. Specifically, a Lyapunov-based nonlinear model predictive control (NMPC) approach is proposed to address system nonlinearities and environmental disturbances. The proposed method combines the rigorous stability guarantees of Lyapunov-based control with the optimization and constraint-handling capabilities of NMPC, thereby providing stronger theoretical stability guarantees while offering optimization and constraint-enforcement capabilities. Numerical simulations demonstrate that, compared with robust nonlinear control approaches, the proposed method effectively stabilizes the crane states and constrains the control inputs within prescribed domains, thereby ensuring reliable and sustainable operation.

Key words - Load sway suspension; Lyapunov-based nonlinear model predictive control; Ship-mounted container cranes; Sustainable transport technology; Sliding mode control

Tóm tắt – Sự phát triển nhanh chóng của ngành công nghiệp hàng hải đã thúc đẩy sự xuất hiện của các tàu container siêu lớn, tạo ra thách thức trong việc tiếp cận cảng nhỏ và đảm bảo xử lý hàng hóa hiệu quả. Bài báo này nghiên cứu một khung điều khiển tiên tiến cho cần cẩu container gắn trên tàu để chuyển tải trực tiếp trên biển. Cụ thể, bộ điều khiển dự báo mô hình phi tuyến dựa trên Lyapunov (NMPC) được đề xuất để giải quyết phi tuyến của hệ thống và nhiễu môi trường. Phương pháp đề xuất kết hợp đảm bảo ổn định của điều khiển dựa trên Lyapunov với khả năng tối ưu hóa và xử lý ràng buộc của NMPC, do đó cung cấp các phân tích ổn định lý thuyết chặt chẽ cũng như khả năng tối ưu hóa và thực thi ràng buộc. Mô phỏng cho thấy rằng, so với các bộ điều khiển phi tuyến, phương pháp đề xuất ổn định hiệu quả trạng thái của cần cẩu và ràng buộc điều khiển trong các miền được quy định, do đó đảm bảo hoạt động tin cậy và bền vững.

Từ khóa – Giảm lắc tải; Điều khiển dự đoán mô hình phi tuyến dựa trên Lyapunov; Cần cẩu container trên tàu; Công nghệ vận tải bền vững; Điều khiển trượt

1. Introduction

The maritime industry has advanced significantly, with ultra-large container ships enhancing transportation efficiency. However, these vessels pose challenges in port accessibility and cargo handling efficiency. Ship-mounted container cranes offer a solution by enabling offshore transshipment, reducing reliance on port infrastructure [1]. Unlike land-based cranes, these systems operate in dynamic ocean environments and are affected by wave-induced ship motions, flexible hoisting cables, and environmental disturbances, leading to oscillations that complicate stability control. Hence, robust control strategies are essential to ensure safe and precise cargo handling in unpredictable maritime conditions.

Recent years have witnessed growing interest in the development of advanced control strategies for ship-mounted container cranes, aiming to enhance stability and positioning accuracy under dynamic offshore conditions. The study presented in [2] introduced a control strategy where a sliding surface was designed using the linear quadratic regulator method, allowing the offshore crane to operate under bounded disturbances and system uncertainties. Following this, a nonlinear feedback control technique was introduced in [3], taking into account the combined effects of wave excitations, the viscoelasticity of seawater, and the elasticity of the hoisting cable. In a different approach, study [4] proposed a feedback and feed-forward control scheme that leverages predicted target position information, enhancing the container's trajectory tracking performance. Additionally, research [5] developed a control structure that combines nonlinear feedback proportional-derivative control with sliding mode proportional-derivative control, demonstrating improved robustness under external disturbances. According to [6], the sliding mode control (SMC) method is applied to address robustness requirements in offshore crane systems. To improve adaptability under time-varying disturbances, an adaptive-gain sliding mode controller was introduced in [7], while [8] integrated SMC with a neural network-based compensator to enhance disturbance rejection. Despite their effectiveness in enhancing robustness against uncertainties and disturbances, the aforementioned methods do not explicitly incorporate system constraints into the control design nor consider an optimization-based perspective. To overcome these limitations, nonlinear model predictive control (NMPC) has emerged as a promising control paradigm due to its capability to optimize system performance while systematically enforcing state and input constraints. For example, studies [9] and [10] applied NMPC to solve trajectory tracking problems and overload protection tasks for offshore cranes, showing the effectiveness of NMPC in managing state and input limits during operation. To enhance adaptability under varying conditions, an NMPC-based control with an auto-tuning method was introduced in study [11], enabling the controller to dynamically adjust its parameters. Furthermore, study [12] proposed a robust tube-based NMPC approach, which improves robustness by enclosing predicted trajectories within predefined tubes, thereby enhancing resilience against external disturbances. These studies have significantly advanced control strategies for ship-mounted container cranes, contributing valuable techniques for trajectory tracking, load sway suppression, and performance optimization. Nevertheless, rigorous stability guarantees have not been addressed in many of the existing approaches. As the complexity of offshore operations continues to increase, there remains a need for control frameworks that can explicitly account for operational constraints while simultaneously providing rigorous stability assurances throughout the control process.

Based on the analysis of previous studies, a robust control framework is developed to simultaneously address trajectory tracking, closed-loop stability, and the enforcement of input and state constraints in ship-mounted crane operations. First, a nonlinear robust controller is designed to drive the system states toward a predefined sliding surface, where a Lyapunov function is constructed to evaluate the stability of the closed-loop system under external disturbances. Then, a Lyapunov-based NMPC (L-NMPC) strategy is proposed, in which the Lyapunov function derived from the robust controller is incorporated into the predictive control optimization process. This integration ensures that the predicted states are driven toward a stable equilibrium throughout the prediction horizon through the incorporation of Lyapunov-based stability conditions, extending the formulation adopted in previous NMPC approaches in [9] – [12]. In addition, the proposed framework explicitly accounts for operational feasibility and safety-related constraints on both system states and control inputs, providing a more comprehensive treatment of stability and constraint satisfaction, an aspect not considered in [2] – [8]. Hence, the predictive controller is able to optimize control actions over a finite horizon while guaranteeing Lyapunov-based stability. The numerical simulation results based on the MATLAB platform, in comparison with robust nonlinear control methods, demonstrate that the proposed approach stabilizes the states and control inputs of the ship-mounted container crane within the prescribed domains while achieving more than 1.5% improvement in tracking accuracy and an approximately 11-fold reduction in control effort, thereby ensuring reliable and sustainable operation.

2. Ship-mounted crane model

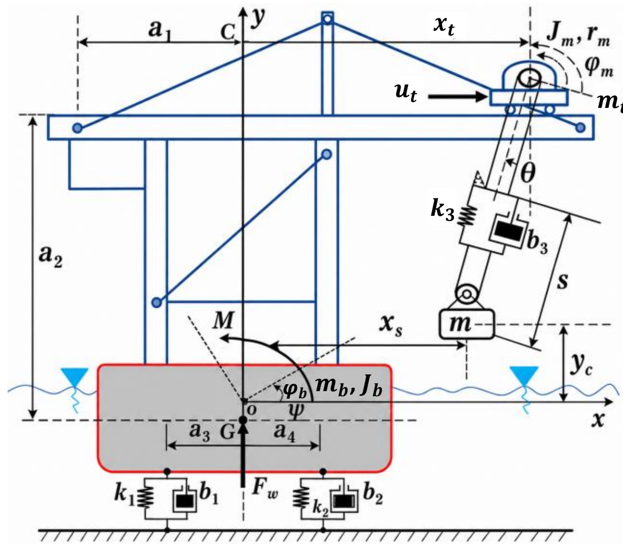

Figure 1. *Visual model of a ship-mounted container crane*

The visual model of a ship-mounted container crane is considered as illustrated in Figure 1, where the crane is installed on a floating platform. The ship has mass and moment of inertia $(m_b; J_b)$ is supported by the elastic foundation of seawater, characterized by stiffness coefficients k_1 and k_2. An inertial reference frame Oxy is selected as the absolute coordinate system. The center of mass of the ship is y, and the vessel is inclined by an angle φ_b. A crane is mounted on the ship, where a trolley of mass m_t is connected to a hoisting drum with a radius r_m and moment of inertia J_m. The hoisting cable, with a length l, is attached to the drum, and its length varies as the drum rotates. The cable is assumed to be elastic with a stiffness coefficient k_3, and the container has mass m_c. During operation, the hoisting cable exhibits swing motion characterized by the swing angle θ. In this context, the mathematical model of the ship-mounted container crane is developed by considering its motion through six generalized coordinates, including the displacement of the trolley x_t, the rotational angle of the hoisting drum φ_m, the swing angle of the container θ, the axial stretch of the hoisting cable s, the ship heave motion y, and the swing angle of the ship φ_b. According to [1], the dynamic model is expressed with matrix form as follows:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{G}(\mathbf{q})=\mathbf{U}+\mathbf{d} \tag{1}$$

where $\mathbf{q}=[q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6]^{\mathrm{T}} = [x_t \quad \varphi_m \quad \theta \quad s \quad y \quad \varphi_b]^{\mathrm{T}}$ is the vector of generalized coordination; $\mathbf{M}(\mathbf{q}), \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbb{R}^{6\times 6}$ are the symmetric mass matrix and the centrifugal damping matrix; $\mathbf{G}(\mathbf{q})$, $\mathbf{d}=\left[\mathbf{d}_1^{\mathrm{T}}\in\mathbb{R}^{1\times 2}\quad \mathbf{d}_2^{\mathrm{T}}\in\mathbb{R}^{1\times 4}\right]^{\mathrm{T}}\in\mathbb{R}^{6\times 1}$ are the gravitational force vector and uncertainties/ unmeasured disturbances, respectively. The control inputs and measured external forces are defined as:

$$\begin{aligned}\mathbf{U}&=\left[\mathbf{U}_1^{\mathrm{T}}\in\mathbb{R}^{1\times 2}\quad \mathbf{U}_2^{\mathrm{T}}\in\mathbb{R}^{1\times 4}\right]^{\mathrm{T}}\\&=\left[u_1=u_t\quad u_2=M_m\quad 0\quad 0\quad F\quad M\right]^{\mathrm{T}}\end{aligned}\tag{2}$$

where, u_t is the trolley driving force, M_m is the hoisting drum torque, and the ship body is periodically affected by the force and torque of regular waves described by:

$$\begin{cases}F=F_0\cos(\omega_1 t+\alpha_1)\\M=M_0\cos(\omega_2 t+\alpha_2)\end{cases}\tag{3}$$

Remark 1: The study [1] deemed that information on disturbances can be measured by equipped sensors. However, in the ocean environment, disturbances measurement is unlikely to achieve a high level of accuracy. Therefore, we considered not only measured disturbances as in (3) but also uncertainties and unmeasured disturbances.

3. Controller design

3.1. Robust control structure

In this part, a robust control method based on SMC theory is constructed, and the system stability is analyzed, which will be adopted as an optimization condition for the following NMPC. First, based on the study [1], the system in (1) is decomposed into actuated states $\mathbf{q}_a=[q_1\quad q_2]^{\mathrm{T}}$ and under-actuated states $\mathbf{q}_u=[q_3\quad q_4\quad q_5\quad q_6]^{\mathrm{T}}$ subsystem as follows:

$$\mathbf{M}_{11}\ddot{\mathbf{q}}_a+\mathbf{M}_{12}\ddot{\mathbf{q}}_u+\mathbf{C}_{11}\dot{\mathbf{q}}_a+\mathbf{C}_{12}\dot{\mathbf{q}}_u+\mathbf{G}_1=\mathbf{U}_1+\mathbf{d}_1\tag{4}$$

$$\mathbf{M}_{21}\ddot{\mathbf{q}}_a+\mathbf{M}_{22}\ddot{\mathbf{q}}_u+\mathbf{C}_{21}\dot{\mathbf{q}}_a+\mathbf{C}_{22}\dot{\mathbf{q}}_u+\mathbf{G}_2=\mathbf{U}_2+\mathbf{d}_2\tag{5}$$

By substituting (5) into (4), the actuator's dynamic can be expressed as:

$$\begin{aligned}\ddot{\mathbf{q}}_a&=\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)^{-1}\\&\times\begin{pmatrix}\mathbf{U}_1+\mathbf{d}_1-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\begin{pmatrix}\mathbf{U}_2+\mathbf{d}_2-\mathbf{C}_{21}\dot{\mathbf{q}}_a\\-\mathbf{C}_{22}\dot{\mathbf{q}}_u-\mathbf{G}_2\end{pmatrix}\\-\mathbf{C}_{11}\dot{\mathbf{q}}_a-\mathbf{C}_{12}\dot{\mathbf{q}}_u-\mathbf{G}_1\end{pmatrix}\end{aligned}\tag{6}$$

The sliding surface is chosen as:

$$\mathbf{s}=\dot{\mathbf{e}}_a+\boldsymbol{\alpha}\mathbf{e}_a+\boldsymbol{\gamma}\mathbf{e}_u\tag{7}$$

where, $\mathbf{e}_a=\mathbf{q}_a-\mathbf{q}_{ad}$ and $\mathbf{e}_u=\mathbf{q}_u-\mathbf{q}_{ud}$ with $\mathbf{q}_{ad}$ and $\mathbf{q}_{ud}$ are the desired values of actuated and under-actuated components, respectively, the matrices $\boldsymbol{\alpha}=\begin{bmatrix}\alpha_1&0\\0&\alpha_2\end{bmatrix}$ and $\boldsymbol{\gamma}=\begin{bmatrix}\gamma_1&0&0&\gamma_2\\0&\gamma_3&\gamma_4&0\end{bmatrix}$ are control parameters. Differentiating the sliding surface in (7), we have:

$$\dot{\mathbf{s}}=\ddot{\mathbf{q}}_a+\boldsymbol{\alpha}\dot{\mathbf{q}}_a+\boldsymbol{\gamma}\dot{\mathbf{q}}_u\tag{8}$$

Based on the SMC theory, the control vector is designed including two components: the equivalent control signal $\mathbf{U}_{1\_eq}$ and the switching control signal $\mathbf{U}_{1\_sw}$. Using (6) and (8), the equivalent control signal $\mathbf{U}_{1\_eq}$ is computed by solving $\dot{\mathbf{s}}=\mathbf{0}$:

$$\begin{aligned}\mathbf{U}_{1\_eq}&=\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\left(\mathbf{U}_2-\mathbf{C}_{21}\dot{\mathbf{q}}_a-\mathbf{C}_{22}\dot{\mathbf{q}}_u-\mathbf{G}_2\right)\\&+\mathbf{C}_{11}\dot{\mathbf{q}}_a+\mathbf{C}_{12}\dot{\mathbf{q}}_u+\mathbf{G}_1\\&-\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)\left(\boldsymbol{\alpha}\dot{\mathbf{q}}_a+\boldsymbol{\gamma}\dot{\mathbf{q}}_u\right)\end{aligned}\tag{9}$$

The switching control signal $\mathbf{U}_{1\_sw}$ is designed as follows:

$$\mathbf{U}_{1\_sw}=-\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)\mathbf{K}\,\mathrm{sgn}(\mathbf{s})\tag{10}$$

where, $\mathbf{K}=\mathrm{diag}(K_1,K_2)$ is a positive-definite diagonal matrix, the term $\mathrm{sgn}(\mathbf{s})$ represents the sign functions associated with the sliding surface. By combining (9) and (10), the control input is calculated as follows:

$$\begin{aligned}\mathbf{U}_1&=\mathbf{U}_{1\_eq}+\mathbf{U}_{1\_sw}\\&=\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\left(\mathbf{U}_2-\mathbf{C}_{21}\dot{\mathbf{q}}_a-\mathbf{C}_{22}\dot{\mathbf{q}}_u-\mathbf{G}_2\right)\\&+\mathbf{C}_{11}\dot{\mathbf{q}}_a+\mathbf{C}_{12}\dot{\mathbf{q}}_u+\mathbf{G}_1\\&-\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)\left(\boldsymbol{\alpha}\dot{\mathbf{q}}_a+\boldsymbol{\gamma}\dot{\mathbf{q}}_u\right)\\&-\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)\mathbf{K}\,\mathrm{sgn}(\mathbf{s})\end{aligned}\tag{11}$$

To prove stability, the following Lyapunov function is chosen as:

$$V=\frac{1}{2}\mathbf{s}^{\mathrm{T}}\mathbf{s}\tag{12}$$

Taking the derivative of the function V and using (6), we have:

$$\begin{aligned}\dot{V}&=\mathbf{s}^{\mathrm{T}}\dot{\mathbf{s}}=\mathbf{s}^{\mathrm{T}}\left(\ddot{\mathbf{q}}_a+\boldsymbol{\alpha}\dot{\mathbf{q}}_a+\boldsymbol{\gamma}\dot{\mathbf{q}}_u\right)\\&=\mathbf{s}^{\mathrm{T}}\begin{pmatrix}\begin{pmatrix}\mathbf{M}_{11}\\-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\end{pmatrix}^{-1}\\\times\begin{pmatrix}\mathbf{U}_1+\mathbf{d}_1-\mathbf{C}_{11}\dot{\mathbf{q}}_a-\mathbf{C}_{12}\dot{\mathbf{q}}_u-\mathbf{G}_1\\-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\begin{pmatrix}\mathbf{U}_2+\mathbf{d}_2-\mathbf{C}_{21}\dot{\mathbf{q}}_a\\-\mathbf{C}_{22}\dot{\mathbf{q}}_u-\mathbf{G}_2\end{pmatrix}\end{pmatrix}\\+\boldsymbol{\alpha}\dot{\mathbf{q}}_a+\boldsymbol{\gamma}\dot{\mathbf{q}}_u\end{pmatrix}\end{aligned}\tag{13}$$

By substituting (11) into (13), we can obtain:

$$\begin{aligned}\dot{V}&=-\mathbf{s}^{\mathrm{T}}\mathbf{K}\,\mathrm{sgn}(\mathbf{s})\\&+\mathbf{s}^{\mathrm{T}}\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)^{-1}\times\left(\mathbf{d}_1-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{d}_2\right)\end{aligned}\tag{14}$$

In practical applications, the model matrices or parameters and uncertainties/ unmeasured disturbances are typically bound due to physical mechanical constraints and finite energy. From this point of view, the term $\left(\mathbf{M}_{11}-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}\right)^{-1}\left(\mathbf{d}_1-\mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{d}_2\right)$ in (14) can be bounded by ψ. In this manner, equation (14) is rewritten as:

$$\dot{V}\le-(K_1-\psi)|s_1|-(K_2-\psi)|s_2|\tag{15}$$

Equation (15) implies that $V(t) \le V(0)$ for control parameters such that $K_1, K_2 > \psi$; therefore, the sliding surface is asymptotically stabilized. Thus, the SMC law ensures that the system states are driven to the sliding surface, where the Lyapunov function confirms the asymptotic stability of the closed-loop system. However, to guarantee trajectory tracking performance, the control parameters must be sufficiently large, which may cause actuator capabilities to be exceeded. Moreover, ensuring compliance with operational constraints in ship-mounted crane operations under external disturbances remains a challenge. To address these issues, the next section presents the L-NMPC approach, which integrates stability constraints within the predictive control process to systematically manage operational constraints while preserving the stability properties established in this section.

3.2. Lyapunov-based Model Predictive Control

According to the aforementioned stability condition, the proposed L-NMPC is executed through a sequential process to ensure stability, constraints enforcement, and trajectory tracking in ship-mounted crane operations. Based on equation (1), the mathematical model is reformulated as:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{bmatrix} = \mathbf{f}(\mathbf{x}, \mathbf{U}) = \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{U} - \mathbf{M}\ddot{\mathbf{q}} - \mathbf{C}\dot{\mathbf{q}} - \mathbf{G} \end{bmatrix} \tag{16}$$

The cost function to be minimized over a prediction horizon N_p is given by:

$$J^* = \min \int_0^{t_s} \left(\mathbf{e}^{\mathrm{T}}(\tau)\mathbf{Q}\mathbf{e}(\tau) + \mathbf{U}^{\mathrm{T}}(\tau)\mathbf{R}\mathbf{U}(\tau) \right) d\tau \tag{17}$$

where, $\mathbf{e} = \hat{\mathbf{q}} - \mathbf{q}_d$ denotes the tracking error between the predicted trajectory and the reference trajectory, $\mathbf{Q}$ and $\mathbf{R}$ are positive weighting matrices that penalize state deviations and control effort, respectively, $t_s = T_s N_p$ is defined as the predictive time with T_s is the sampling time. The optimal solution problem would be considered with the following conditions:

$$\begin{gathered} \mathbf{x}_{\min} \le \hat{\mathbf{x}} \le \mathbf{x}_{\max} \\ \mathbf{U}_{1\min} \le \hat{\mathbf{U}}_1 \le \mathbf{U}_{1\max} \\ \frac{\partial V_{\text{L-NMPC}}}{\partial \mathbf{e}} \mathbf{f}\left(\hat{\mathbf{x}}(0), \hat{\mathbf{U}}_1(0)\right) \le \frac{\partial V_{\text{Lyapunov}}}{\partial \mathbf{e}} \mathbf{f}\left(\hat{\mathbf{x}}(0), h(\hat{\mathbf{x}}(0))\right) \end{gathered} \tag{18}$$

Where, $\hat{\mathbf{x}}$ and $\hat{\mathbf{U}}_1$ denote the predicted states and control inputs, respectively. V_{Lyapunov} is the Lyapunov function derived from the sliding surface in section 3.1 and $h(\hat{\mathbf{x}}(0)) = \mathbf{U}_1(0)$ is the control law in (11). The details of the condition which satisfies stability is expressed as:

$$\begin{aligned} &\frac{\partial V_{\text{L-NMPC}}}{\partial \mathbf{x}} \mathbf{f}\left(\hat{\mathbf{x}}(0), \hat{\mathbf{U}}_1(0)\right) \\ &\le \mathbf{s}^T(0)\left(\ddot{\mathbf{q}}_a(0) + \boldsymbol{\alpha}\dot{\mathbf{q}}_a(0) + \boldsymbol{\gamma}\dot{\mathbf{q}}_u(0)\right) \\ &\le -(K_1 - \psi)\left|s_1(0)\right| - (K_2 - \psi)\left|s_2(0)\right| \end{aligned} \tag{19}$$

According to the designed L-NMPC, the control algorithm for the ship-mounted crane will be implemented as summarized in Algorithm 1. First, the cost function in (17) is declared. The current states $\mathbf{x}(t)$ of the ship-mounted crane are measured in step 2. Based on these states, the optimization problem in (17)-(19) is solved to find the optimal solution $\xi(\vartheta)$ with $\vartheta \in [0, T_s]$ under $\hat{\mathbf{x}}(0) = \mathbf{x}(t)$. During a sampling time $\mathbf{U}_1(t) = \xi(\vartheta)$, to determine the control inputs for the next sampling time, $t = t + T_s$ is installed to repeat from step 2. Thus, by incorporating the Lyapunov-based stability condition into the predictive control, the L-NMPC explicitly enforces stability within the optimization horizon, while ensuring that predicted states and control inputs remain within predefined operational and safety limits.

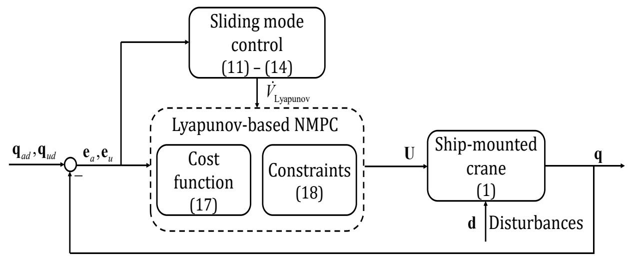

Figure 2. *Structure of the proposed method*

Algorithm 1: Lyapunov-based NMPC method	
Step 1	Input the cost function (17)
Step 2	Measure the current state $\mathbf{x}(t)$
Step 3	Solve the Lyapunov-based NMPC problem (17)-(19) with $\hat{\mathbf{x}}(0) = \mathbf{x}(t)$ to obtain the optimal solution, denoted by $\xi(\vartheta)$.
Step 4	Utilizing $\xi(\vartheta)$ for only one sampling period: $\mathbf{U}_1(t) = \xi(\vartheta)$ for $\vartheta \in [0, T_s]$.
Step 5	Repeat from Step 2 at the next sampling time $t = t + T_s$.

Figure 2 illustrates the structure of the proposed control framework. The tracking errors $\mathbf{e}_a, \mathbf{e}_u$ are first used to construct a sliding-mode-based Lyapunov stability condition (14). This stability condition is then incorporated into the Lyapunov-based NMPC formulation as a constraint (18), together with the optimization cost function (17). By solving the resulting constrained optimization problem, the control input $\mathbf{U}$ is generated and applied to the ship-mounted crane system. The closed-loop system simultaneously achieves trajectory tracking, constraint satisfaction, and stability assurance.

Remark 2: The SMC-based formulation is used only to derive the Lyapunov stability condition, as depicted in (18), which is embedded as a constraint in the NMPC problem; hence, it does not introduce an additional online controller or significant computational burden.

Remark 3: Unlike the robust control methods in [2] – [8] that primarily focus on disturbance rejection and tracking performance without explicitly addressing system constraints, the proposed approach combines Lyapunov-based control with NMPC to systematically account for operational feasibility and safety-related constraints on both system states and control inputs. Furthermore, by incorporating the Lyapunov-based stability condition in (18) into the optimization problem defined by the cost function in (17), the proposed framework provides explicit stability guarantees for the closed-loop system compared to existing NMPC approaches in [9] – [12] while preserving the optimization and constraint-handling capabilities of NMPC.

4. Simulation Results

In this section, a numerical simulation is conducted considering the operation of a ship-mounted crane under both measured and unmeasured disturbances. Based on the study [1], the model parameters and measured disturbances in Section 2 are presented as follows:

$$a_2 = 32\text{m},\ a_3 = a_4 = 12.5\text{m},\ r_m = 0.325\text{m},\ l_0 = 15\text{m},$$

$$m_b = 4.5\times10^5\text{kg},\ m_t = 5900\text{kg},\ m_c = 650\text{kg},$$

$$J_b = 571875000\text{kgm}^2,\ J_m = 41700\text{kgm}^2,$$

$$k_1 = k_2 = 1.25\times10^6\text{N/m},\ k_3 = 1.2\times10^4\text{N/m},$$

$$b_1 = b_2 = 200\text{Ns/m},\ \ b_3 = 220\text{Ns/m},\ \ b_t = 50\text{Ns/m},$$

$$g = 9.81\text{m/s}^2,\ b_m = 70\text{Ns/m},\ F_0 = 3\times10^5\text{N},$$

$$M_0 = -6\times10^5\text{N},\ \omega_1 = \omega_2 = 0.35\text{rad/s},\ \alpha_1 = 0,\ \alpha_2 = \pi.$$

In addition sto the measured disturbances, we also consider unmeasured disturbances in the present simulation. To emulate unusual and unforeseen factors affecting the ship-mounted crane, random disturbance is modeled as band-limited white noise with a variance of 0.01, ensuring that the disturbance remains bounded and complies with the assumption adopted in (15).

The initial condition of the system is defined as $\mathbf{q}(0) = \mathbf{0}$. The desired position of the container is set as the trolley motion $x_{td} = 3\text{m}$ and the cable length with the initial cable length $l_d = 12\text{m}$. Note that the cable length is calculated as:

$$l = l_0 - r_m\varphi_m + s \tag{20}$$

To highlight the effectiveness of the proposed method, our approach is compared with the SMC designed in Section 3 and backstepping SMC (B-SMC) schemes in [1]. The prediction horizon and control horizon are 20 and 10, respectively, with the sampling time $T_s = 0.1\text{s}$. The control parameters for L-NMPC are selected as:

$$\alpha_1 = 0.2,\ \ \alpha_2 = 0.4,\ \ \gamma_1 = 13,\ \ \gamma_2 = 1,$$

$$\gamma_3 = 4,\ \ \gamma_4 = 0.1,\ \ K_1 = K_2 = 2,$$

$$\mathbf{Q} = \text{diag}(50,50,1,1,1,1),\ \ \mathbf{R} = \text{diag}(1,1),$$

$$\mathbf{x}_{\max} = -\mathbf{x}_{\min} = \begin{bmatrix}3.5 & 10 & 0.02 & 0.1 & 0.2 & 0.2\end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{U}_{1\max} = -\mathbf{U}_{1\min} = \begin{bmatrix}400 & 1.5\times10^4\end{bmatrix}^{\mathrm{T}}.$$

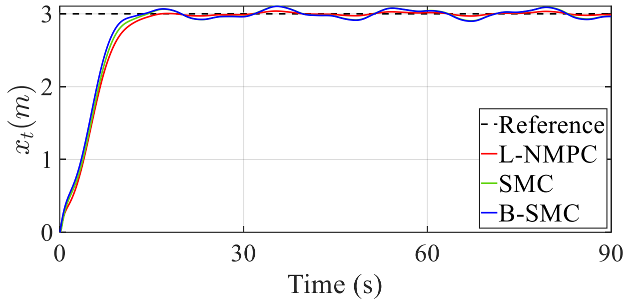

Figure 3. *Displacement of the trolley*

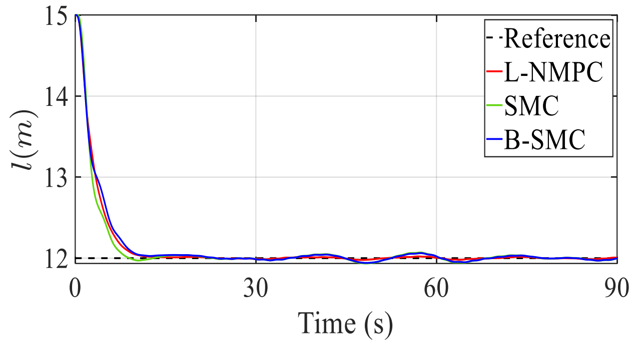

Figure 4. *Container – lifting motion*

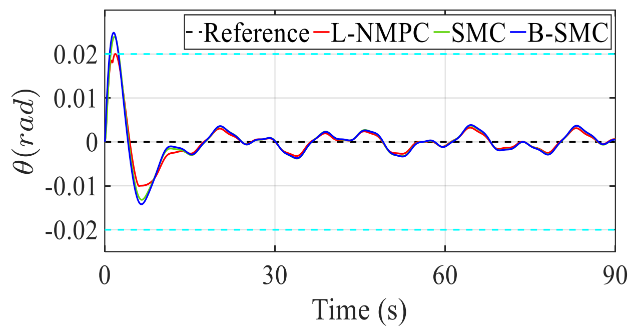

Figure 5. *Swing angle of the container*

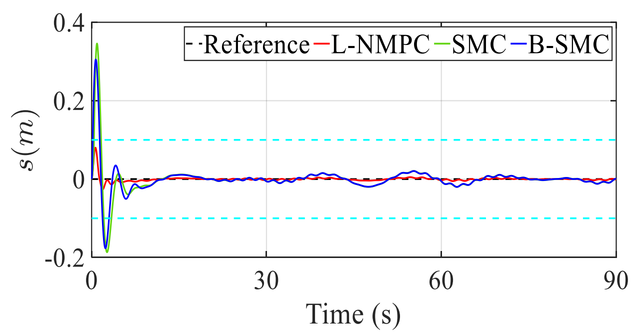

Figure 6. *Axial stretch of the hoisting cable*

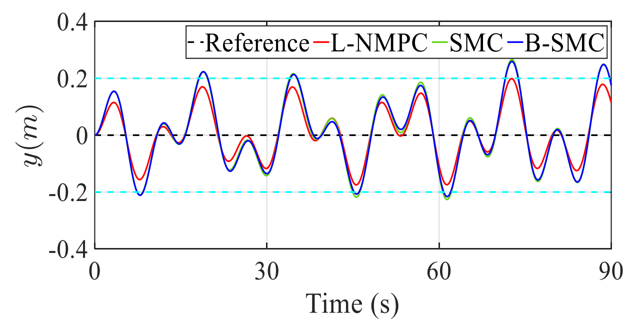

Figure 7. *Ship heave motion*

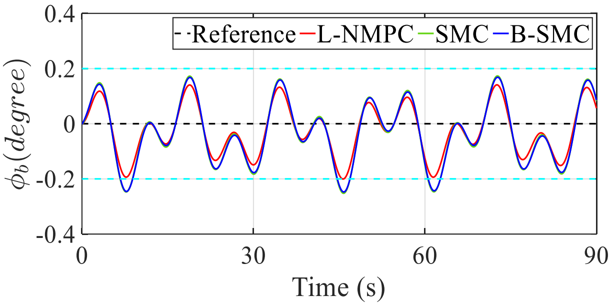

Figure 8. *Ship swing motion*

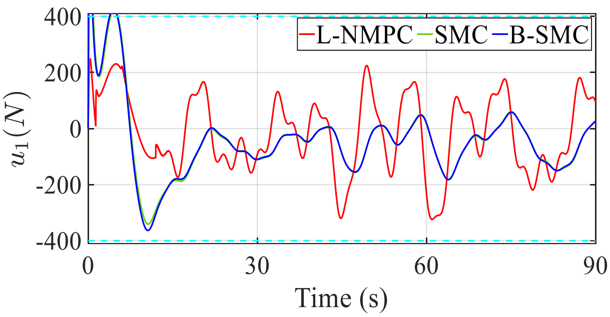

Figure 9. *Trolley driving force*

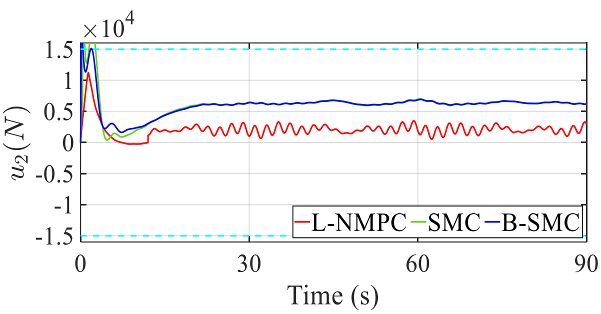

Figure 10. *Hoisting drum torque*

Table 1. *Comparison of performance index for tracking errors*

$\text{RMSE} = \sqrt{\frac{1}{T}\int_0^T e^2(t)\,dt}$	L-NMPC	SMC	B-SMC
x_t	0.6028	0.6124	0.6119
φ_m	1.2712	1.3392	1.3526
θ	0.2336	0.2578	0.2601
s	0.0071	0.0102	0.0086
y	0.0931	0.0968	0.0955
φ_b	0.0936	0.1021	0.1018

According to the aforementioned simulation setting, the compared results are presented in Figures 3 - 10. Specifically, the proposed L-NMPC ensures that the trolley and container converge to the desired positions after approximately 20s and 14s in Figures 3 and 4, correspondingly. Although the methods exhibit robustness under external disturbances, the proposed method shows small errors in comparison with SMC and B-SMC. Furthermore, the un-actuated states, including the container swing angle in Figure 5, the cable axial stretch in Figure 6, the ship heave motion in Figure 7, and the ship swing angle in Figure 8, also reveal the effectiveness of the proposed method in restricting and minimizing oscillations under predetermined domains (blue dashed line). Meanwhile, both SMC and B-SMC violate limited domains and generate higher oscillations due to uncertainties and unmeasured disturbances, which have not been considered in the study in [1]. Satisfying the working domains (blue dashed line) of the actuators is also guaranteed by applying the proposed L-NMPC in Figures 9 and 10. Tables 1 and 2 present the performance indices of tracking errors and control efforts for all controllers. It can be observed that the proposed framework achieves the smallest values among all methods, further demonstrating its high-precision tracking and optimization performance. By comparing the proposed method with SMC and B-SMC, we can affirm that the proposed method inherited robustness and stability from Lyapunov-based control and operational constraints via an optimization problem, which is necessary to achieve safety in practical applications.

Table 2. *Comparison of performance index for control efforts*

$\text{E} = \int_0^T u^2(t)\,dt$	L-NMPC	SMC	B-SMC
x_t	1.69×10^5	1.90×10^6	2.03×10^6
φ_m	5.45×10^8	3.76×10^9	3.59×10^9
θ	0	0	0
s	0	0	0
y	0	0	0
φ_b	0	0	0

Although the proposed L-NMPC, SMC, and B-SMC demonstrated robustness under both measured and unmeasured disturbances, or the proposed method works on operational domains, they also contain some errors and cannot eliminate uncertainties and disturbances completely as in Figures 3 - 8. This implies that the closed-loop system can be unstable in the case of high amplitude or frequency disturbances. Therefore, it is necessary to encourage research efforts in the development of disturbance competitions in future works to combine with the proposed L-NMPC in the present study.

5. Conclusion

In the present study, we proposed a Lyapunov-based model predictive control approach to simultaneously ensure trajectory accuracy and load sway suppression in ship-mounted crane operations. The proposed method integrated stability constraints to maintain system performance under real-world maritime conditions. The simulation results demonstrated the feasibility and efficiency of the proposed control algorithm compared with SMC and B-SMC methods. However, despite the implementation of the aforementioned robust control methods, tracking errors persist due to the presence of uncertainties and disturbances. In future works, further efforts will focus on developing adaptive robust control strategies to enhance the system's ability to handle time-varying disturbances and parameter uncertainties caused by changes in cargo weight, sea states, or operational configurations. Moreover, energy-based control methods could be explored to optimize both stability and energy consumption, which is critical for sustainable offshore operations.

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