

DIFFERENT MODELLING TECHNIQUES ON COMPRESSIVE BEHAVIOUR OF MASONRY COLUMNS

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Abstract - Many existing masonry columns have experienced significant material deterioration, requiring reliable methods for structural assessment and rehabilitation planning. This study evaluates the compressive behaviour of three-dimensional masonry columns under axial compression using two finite element modelling strategies: simplified micro-modelling and macro-modelling. The simplified micro-modelling approach integrates the Extended Finite Element Method (XFEM) with plasticity-based constitutive laws. Masonry units and mortar joints are represented as enlarged continuum elements, while surface-based cohesive interactions simulate interface behaviour. Material crushing under compression is described using the Drucker–Prager model, and XFEM enables crack initiation and propagation without predefined crack paths. In contrast, the macro-modelling approach represents the masonry column as a homogeneous continuum. Numerical simulations are performed using ABAQUS and validated against experimental data from previous studies. Results indicate that simplified micro-modelling provides more accurate predictions of nonlinear compressive response and failure mechanisms than macro-modelling, demonstrating superior capability for masonry column assessment.

Keywords – Simplified micro-modelling; Macro-modelling; Masonry columns; Extended Finite Element Method (XFEM); Axial compression load.

1. Introduction

Modern construction utilizes a broad spectrum of materials, among which masonry remains one of the most popular construction materials. Owing to its durability, thermal efficiency, fire resistance, economic advantages, and construction simplicity, masonry continues to play a significant role in civil engineering applications [1]. A masonry system is composed of masonry units and mortar joints, each exhibiting distinct mechanical characteristics that collectively determine the structural response of the assemblage. As a result, accurately modelling the structural behavior of masonry is challenging because of its heterogeneous composition and anisotropic nature.

Alongside experimental investigations, computational approaches (e.g., Finite Element Method (FEM)) have become an essential tool for estimating the structural behaviour of reinforced concrete [2], composite [3-5], and existing masonry [6–8] structures. Within the FEM framework, masonry systems can generally be modelled using either macro- or micro-modelling, depending on the accuracy level and time-consuming requirements, as well as computational resources.

In the macro-modelling technique, masonry is treated as

an equivalent homogeneous continuum whose material properties are derived from the averaged characteristics of its constituents. This strategy is particularly advantageous for geometrically complex structures where the general response of the structure is of primary concern. On the other hand, the micro-modelling approach models different units of masonry structures separately, thereby providing a more detailed description of local behaviour and failure mechanisms. However, the increased level of detail is accompanied by substantially higher computational costs, restricting its application to relatively small-scale problems. To overcome these limitations, the simplified micro-modelling technique was introduced. In this technique, masonry units are expanded to incorporate the mortar thickness, as mortar layers are not simulated as separate elements. Correspondingly, interface elements are employed to model the connection between adjacent units. This modelling technique offers a balance between time and resource efficiency and accuracy level, which has led to its adoption in numerous studies. One of the earliest applications was presented by Page [9], who modelled mortar joints as interface elements and masonry units as continuum elements. Subsequently, Lotfi and Shing [10] extended this approach by incorporating the fracture behaviour of mortar joints through interface modelling. Abdulla et al. [11] demonstrated the effectiveness of a simplified micro-model in reproducing the plastic behaviour of masonry walls subjected to different loading directions. More recently, a simplified micro-modeling technique, incorporating discrete elements and elastic bodies, was developed by Luo et al. [12] for the non-linear static analysis of masonry panels subjected to in-plane loading.

Although simplified micro-modelling has been widely adopted to model masonry structures, relatively limited research has been dedicated to the compressive behaviour of masonry columns, despite their prevalence in historic masonry construction. To address this research gap, the present study employs a simplified micro-modelling approach to investigate masonry columns subjected to axial compression. In this framework, masonry units and mortar joints are combined into enlarged continuum elements, while the behaviour of the interfaces is represented using surface-based cohesive interactions capable of simulating both elastic and inelastic responses. The Drucker–Prager plasticity model is adopted to capture crushing failure under compression. Furthermore, the Extended FEM (XFEM) is incorporated to simulate crack

initiation and propagation within masonry units without requiring predefined crack trajectories.

The primary objective of this study is to establish, validate, and extend a rigorous computational framework capable of capturing complex damage evolution and analyzing parametric sensitivities that are difficult to isolate experimentally. Rather than presenting new experimental data, this work leverages high-quality, peer-reviewed experimental benchmarks from the established literature to validate the developed numerical model.

By utilizing these established physical benchmarks, this simulation framework provides a deeper look into the internal stress distributions and localized failure mechanisms that cannot be directly observed or measured via traditional physical testing. Furthermore, the validated model is utilized to conduct a comprehensive parametric study, exploring material variations and structural configurations that would be financially and logistically prohibitive to execute via physical experiments. To evaluate the comparative efficiency of this framework, a macro-modelling strategy is also established. The resulting comparative analysis provides critical insights into the accuracy, limitations, and predictive capabilities of each modelling approach, demonstrating that this advanced numerical framework serves as a valuable, high-fidelity complement to existing experimental methodologies.

2. Test program description

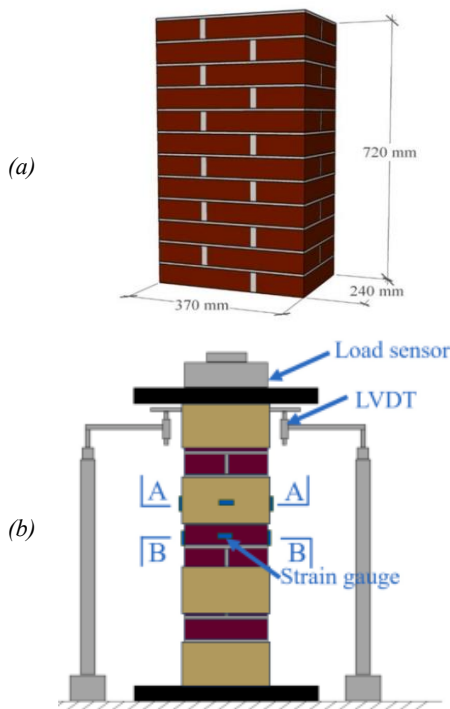


Figure 1. Test setup, dimensions, and configurations of masonry column [9]: a) Dimensions; b) Configurations

The experimental investigation reported by Zhang et al. [13] was adopted as the basis for the development and validation of the numerical models in this study. In their experimental program, fired clay bricks conforming to the requirements of GB/T 2542–2012 [14] were used. The bricks exhibited an average compressive strength of 16.11

MPa and a mean density of 1600 kg/m³. The tested masonry column measured 370 mm in length, 240 mm in width, and 720 mm in height, as illustrated in Figure 1. The specimen consisted of 12 brick courses, with each course containing three bricks. The bricks were bonded using mortar joints made from mortar with an average 28-day compressive strength of 8.3 MPa. Individual brick units had dimensions of 240 mm length $\times$ 115 mm width $\times$ 53 mm thickness. Axial compression testing was performed using a hydraulic jack with a 1000-ton load capacity. The load was applied under displacement-controlled conditions at a constant rate of 0.2 mm/min. Column configuration and test setup are presented in detail in Ref. [9] and shown in Figure 1.

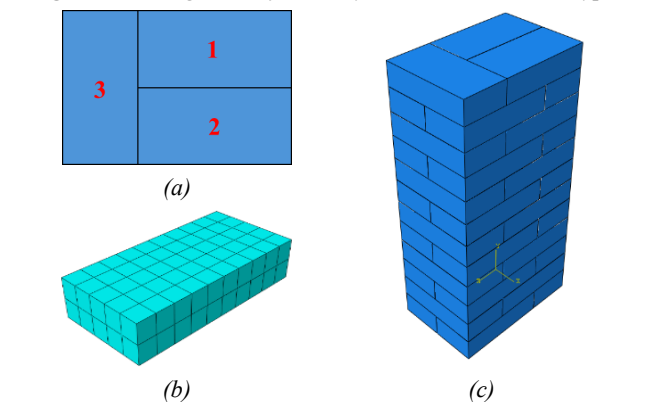
3. Finite element model approaches

3.1. Simplified micro-modelling

3.1.1. Expanded units, element type, and specimen modelling

As simplified micro-modelling is employed in this study, the mortar layers are excluded, and the brick is accordingly expanded in size, known as an expanded unit. The expanded units, named '1' and '2' in Figure 2(a), are 247.5 mm (length) $\times$ 120 mm (width) $\times$ 60 mm (thickness), calculated based on the brick and column dimensions. Similarly, the expanded unit named '3' in Figure 2(a) is 240 mm (length) $\times$ 122.5 mm (width) $\times$ 60 mm (thickness). The expanded unit is modelled using a solid element with reduced integration (C3D8R), as shown in Figure 2(b). The mesh size of 20 mm is used for the expanded unit in this study. The arrangement of masonry units in the FE model, which is shown in Figure 2(c), is similar to that shown in the experimental program of Ref. [13].

Figure 1. Arrangement of masonry units and its element type



and size: a) Arrangement of masonry units; b) Element type and size of masonry unit; c) 3D FE model of masonry column

3.1.2. Elastic behaviour of expanded masonry units

The compressive strength of mortar (f_j) and brick (f_b) are used to define masonry units' compressive strength f'_m , which is based on a proposed equation reported in [15] and shown in Eq. (1). The elastic modulus of expanded masonry units needs to be modified so that their elastic behavior aligns with that of the original masonry assemblage (including both units and mortar). The equivalent modulus of elasticity of masonry units E_m is calculated by considering the elastic properties of the original masonry brick and mortar, along with the geometry of the assemblage. According to Kaushik et al.

[15], E_m is defined in the range from 250 to 1100 times the compressive prism strength of masonry (f'_m).

In this study, the modulus of elasticity is approximately 500 times the compressive prism strength of masonry, as shown in Eq. (2). Based on the experimental data shown in Ref. [13], the compressive strength of masonry unit f'_m and its modulus of elasticity E_m are calculated as per Eqs. (1)-(2) and tabulated in Table 1.

$$f'_m = 0.63 f_b^{0.49} f_j^{0.32} \quad (1)$$

$$E_m \approx 500 f'_m \quad (2)$$

3.1.3. Drucker-Prager plasticity model

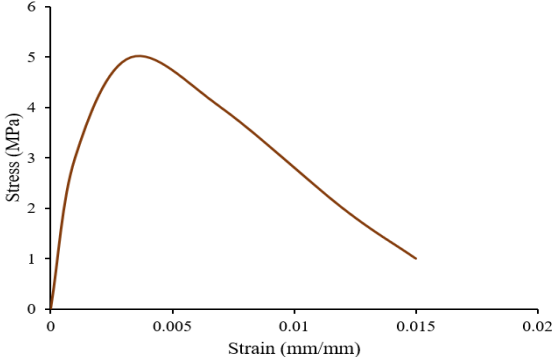


Figure 2. Compressive stress–strain curve of masonry unit

The Drucker–Prager (DP) plasticity model was established by Drucker and Prager [16] and is currently used as a constitutive model to describe the inelastic behavior of pressure-dependent materials, such as soils, rocks, and ceramics. It is often viewed as a smooth approximation of the Mohr–Coulomb criterion, making it more convenient for numerical simulations. The yield criterion under hydrostatic pressure was added by Lubarda et al. [17] to the model later on. To apply this criterion in the model, the following parameters must be determined: flow stress ratio R , dilation angle ψ , and friction angle β . The values of the parameters have been defined in Table 2. Compressive behaviour of the masonry unit is expressed by the compressive stress–strain curve shown in Figure 3 and applied to the model through the Drucker-Prager hardening suboption available in ABAQUS. Noted that the compressive stress–strain curve of masonry is represented by a parabolic curve expressed by Eq. (3):

$$\frac{f_m}{f'_m} = 2 \frac{\varepsilon_m}{\varepsilon'_m} - \left(\frac{\varepsilon_m}{\varepsilon'_m} \right)^2 \quad (1)$$

where f_m is compressive stress in masonry unit, and ε_m is compressive strain in masonry unit; f'_m is the corresponding value at which ε'_m is the peak strain. The value of peak strain ε'_m is determined based on Eq. (4):

$$\varepsilon'_m = C_j \frac{f'_m}{E_m^{0.7}} \quad (4)$$

where the peak strain is affected by the mortar's strength via the influence factor C_j shown in Eq. (5).

$$C_j = \frac{0.27}{f_j^{0.25}} \quad (5)$$

3.1.4. Extended finite element method (XFEM)

The XFEM, originally proposed by Belytschko and Black [18], enables the simulation of crack initiation and propagation within finite elements by enriching nodal displacements near the crack tip, thereby eliminating the need for remeshing. In the present study, XFEM is employed to define crack evolution in masonry units subjected to tensile and shear stresses, without the requirement of a predefined crack path or location. The crack propagation process is represented using the cohesive segment approach [19], which is governed by a traction–separation law describing the initiation and growth of cracks within the masonry material.

3.1.5. Surface-based cohesive behaviour model for expanded units' interface

A surface-based cohesive model in ABAQUS is used to represent crack initiation and propagation along a potential surface between expanded units, without explicitly inserting cohesive elements in the mesh. Instead of predefining a physical cohesive element between expanded units, a cohesive interaction property is assigned to a contact surface.

The model employs the traction–separation law to describe how stresses (tractions) relate to the relative displacements (separations) of the two surfaces. For the elastic response of the interface, a linear traction–separation relationship before damage is applied and expressed through the form of the elastic stiffness matrix K shown in Eq. (6), in which t is the term of the nominal traction vector and s is the corresponding separation vector.

Table 1. Mechanical properties of masonry unit employed in the FE model

f_b (MPa)	f_j (MPa)	f'_m (Eq. (1)) (MPa)	E_m (Eq. (2)) (MPa)	μ
16.11	8.3	4.84	2420	0.15

Table 1. Parameters in the Drucker-Prager (DP) plasticity model

Parameters	Dilation angle ψ	Friction angles β	Flow stress ratio R
Value	11.3	36	1

Table 2. Values of three components of elastic stiffness matrix K

K_{nn} (N/mm ³)	K_{ss} (N/mm ³)	K_{tt} (N/mm ³)
42	17	17

$$\begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix} = \begin{bmatrix} K_{nn} & 0 & 0 \\ 0 & K_{ss} & 0 \\ 0 & 0 & K_{tt} \end{bmatrix} \begin{bmatrix} \delta_n \\ \delta_s \\ \delta_t \end{bmatrix} \quad (2)$$

The values of three components of the elastic stiffness matrix K , including normal stiffness K_{nn} , shear stiffness in local direction K_{ss} , and shear stiffness in local direction K_{tt} , are shown in the Eq.(6) are computed using the mortar and unit's moduli of elasticity, as well as the mortar's thickness shown in Eqs.(7)-(8) and are provided in Table 3.

$$K_{nm} = \frac{E_u E_m}{h_m (E_u - E_u)} \quad (3)$$

$$K_{ss} \text{ and } K_{tt} = \frac{G_u G_m}{h_m (G_u - G_u)} \quad (4)$$

3.1.6. Boundary condition

According to the test of Zhang et al. [13], displacement control was applied to the masonry column at a rate of 0.2 mm/min. To mimic the test, in the numerical modelling of this study, the displacement was applied to the top surface of the column. On the other hand, the bottom surface was restrained in all three directions. The boundary condition of the numerical model is indicated in Figure 4.

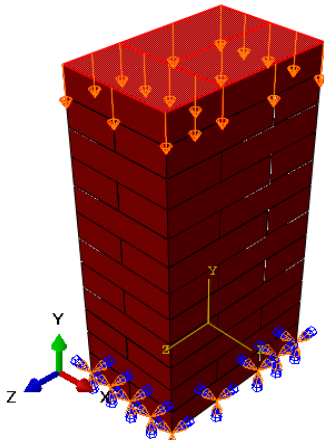


Figure 4. Boundary condition (Simplified Micro-Modelling) in ABAQUS software

3.2. Macro-modelling approach

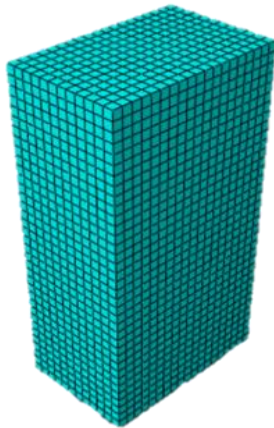


Figure 5. Masonry structure (Macro-Modelling) in ABAQUS software

In the macro-modelling approach, masonry is represented as a homogeneous continuum without explicitly distinguishing between the brick units and mortar joints. Accordingly, the masonry column modeled in ABAQUS possesses dimensions of 370 mm in length, 240 mm in width, and 720 mm in height, as illustrated in **Error! Reference source not found.** The mechanical properties assigned to the masonry column are consistent with those employed in the simplified micro-modelling approach. Drucker–Prager plasticity model for compressive behaviour and XFEM to define the cracks are employed in the macro

model, similar to the simplified micro model. It is noted that since the macro model in this study is treated as a single integrated block, surface-to-surface interactions between units and joints are not considered in the macro-modelling framework.

4. Results and discussions

The axial load-displacement curve achieved from the simplified micro-modelling and macro-modelling approaches compared to the experiment results of Ref. [13] is shown in Figure 3. It is indicated that in the experiment results, including the results of three columns “N-a”, “N-b”, and “N-c”, the load-displacement curves of these three columns showed a relatively consistent tendency, with the load increasing linearly up to the average load capacity of 420 kN at an axial displacement of 2.2 mm. Meanwhile, the load-displacement curve obtained by the simplified micro-modelling approach appeared very close to the average load-displacement curve of the experimental result, with an axial load capacity of 418 kN and a displacement of 2.16 mm recorded from the model. Comparing the two, the deviation is only 0.5% for load capacity and 1.8% for displacement values. Moreover, similar to the experimental results, the load-displacement curve achieved from the simplified micro-model showed a similar tendency as that in the experiment, with linear behaviour of load capacity before failure, and followed by a softening behaviour once passing the peak axial load. The comparison of the crack distribution and failure mode between the experiment and the simplified micro modelling is shown in Figure 4. The experimental crack development and ultimate failure mode of the unconfined masonry columns were heavily dictated by the low tensile strength of the masonry assembly, resulting in a highly brittle failure. Initially, micro-cracks emerged at the upper end of the specimens and progressively propagated through the vertical mortar joints and masonry units as the load increased. Upon approaching the ultimate load, a dominant, major vertical crack rapidly widened and propagated downward through the entire height of the specimen, leading to a sudden loss of stability and ultimate failure. The simplified micro-model captures this physical progression with high fidelity. In the numerical simulation, crack initiation is successfully predicted at the top of the column at a load level corresponding to the experimental damage onset. Furthermore, the model accurately traces the propagation path of the major vertical crack, reproduces the split through both the unit-to-unit interfaces and the bricks themselves before capturing the eventual global instability of the column at the ultimate limit state. This close agreement in both the initiation zone and the final split-failure path validates the capability of the numerical model to predict brittle localization in masonry elements.

On the other hand, the load-displacement curve obtained by the macro-modelling showed significantly higher stiffness than that of the experiment and the simplified micro-modelling. As there are no masonry units, which accordingly disregards the sliding and separation at the brick-mortar interfaces, subsequently the column is

more stiffened and thus appears higher in stiffness. In terms of load capacity, the macro modelling approach exhibits similar results to the two methods investigated before. However, the load reached the peak earlier than the experiment and the simplified micro-model. The reason behind this is attributed to the higher stiffness of the column in the macro-modelling.

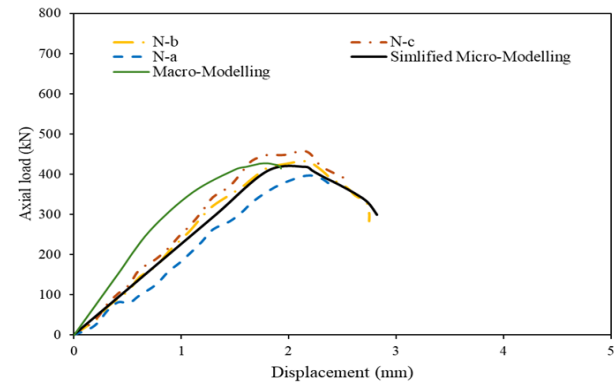


Figure 3. Axial load-displacement curves obtained by experiment and finite element modelling approaches

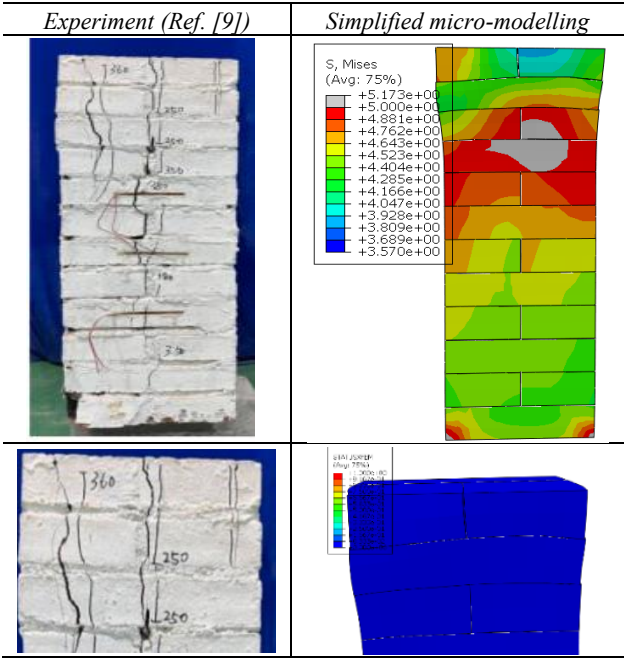


Figure 4. The result of the experiment and simplified micro-modelling approach

To guide practical implementation, the primary trade-off between the simplified micro-model and the macro-model lies in the modeling (pre-processing) setup time rather than the computational solver runtime. The simplified micro-model requires each individual masonry unit to be geometrically modeled and positioned, demanding substantial manual pre-processing effort and engineering time. Consequently, this approach is best suited for localized investigations where block-to-block interactions, joint slip, or discrete cracking patterns govern the structural response. On the other hand, the macro-model treats the masonry assembly as a homogeneous continuum, completely eliminating the need to layout individual units. This drastically reduces the initial

modeling time and complexity, making the macro-model the preferred, practical choice for large-scale structural simulations and rapid design optimization where global behavior is the primary focus.

5. Conclusions

This study investigated the compressive response of three-dimensional masonry columns under axial loading using two finite element modelling strategies. Based on the findings of this study, the following conclusions can be drawn:

(1) The simplified micro-modelling, by simulating relatively detailed components of the masonry column and interaction between them, appeared to be quite accurate in modelling the masonry column’s compressive behaviour. More specifically, the compressive load-displacement relationship obtained by the simplified micro-modelling approach is generally close to the average load-displacement curve of the experimental results, with the deviation being only 0.5% for maximum compressive load and 1.8% for displacement values. Moreover, the load-displacement relationship of the simplified micro model showed a similar tendency as that in the experiment, characterised by linear behaviour before failure, and followed by a softening behaviour once passing the peak axial load. Not only in terms of the compressive load-displacement relationship, but the crack distribution is also almost similar between the two. These results demonstrate that the simplified micro-modelling is a suitable approach to achieve effectiveness in both the degree of accuracy and computational resource consumption in simulating the compressive behaviour of masonry columns.

(2) The load-displacement curve achieved by the macro-modelling showed significantly higher stiffness than that of the experiment and the simplified micro-modelling. As there are no masonry units, which accordingly disregards the sliding and separation at the brick-mortar interfaces, subsequently the column becomes stiffer, and thus the load-displacement curve appears higher in stiffness. In terms of load capacity, the macro-modelling approach indicates similar results to the two methods investigated previously. However, the load reached the peak at the displacement point earlier than that in the experiment and the simplified micro-model.

(3) While the validated simplified micro-modeling and macro-modeling approaches successfully capture the fundamental compressive behavior of the masonry columns, the scope of the current parametric investigation remains limited to the tested specimen configurations. To establish the broader statistical reliability and universality of these numerical strategies, a comprehensive parametric analysis is designated as the immediate next phase of this research. This future work will systematically evaluate the sensitivity of both modeling approaches to variations in key structural parameters, such as masonry unit strength, mortar joint thickness, column aspect ratios, and boundary conditions, thereby transitioning these localized models into a generalized predictive framework for structural design.

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