BENDING ANALYSIS OF RC–UHPC PANELS IN SEMI-PRECAST SLAB USING FINITE ELEMENT METHOD

Tuan-Anh Cao[1], Hoang-Anh Pham[1,2]*, Van-Thuc Luu[1], Ngoc-Khoa Ho[1]

[1]*Hanoi University of Civil Engineering, Hanoi, Vietnam*

[2]*Frontier Research Group of Mechanics of Advanced Materials and Structures (MAMS), Hanoi University of Civil Engineering, Hanoi, Viet Nam*

*Corresponding author: anhph2@huce.edu.vn

(Received: March 24, 2026; Revised: May 06, 2026; Accepted: June 05, 2026)

DOI: 10.31130/ud-jst.2026.24(7B).574E

Abstract - This paper presents a finite element (FE) analysis of the flexural behavior of a semi-precast RC-UHPC panel during the construction stage. The panel consists of a prefabricated assembly comprising a bottom ultra-high-performance concrete (UHPC) layer, an expanded polystyrene (EPS) lightweight concrete core, and a steel truss mesh. The FE model employs flat triangular shell elements for the concrete layers and 3D bar elements for the steel truss, implemented using the CALFEM-Python framework. The model is validated against experimental data from the literature. A parametric study investigates the effects of the UHPC and EPS thicknesses as well as steel reinforcement diameters on panel deflection under self-weight and construction load. Results show that the EPS thickness is the dominant parameter governing flexural stiffness, while reinforcement diameter has a comparatively minor influence. Specific configurations satisfying practical deflection limits are identified, providing design guidance for this emerging slab typology in the construction phase.

Key words - RC-UHPC panels; semi-precast slab; finite element analysis

1. Introduction

Ultra-high-performance concrete (UHPC) is a cement-based composite material with a water-to-cement ratio below 0.25, resulting in minimal porosity and an exceptionally dense microstructure. UHPC achieves compressive strengths of 150–250 MPa (compared to approximately 30 MPa for normal concrete), post-cracking tensile strength exceeding 5 MPa, and elastic modulus of 45–55 GPa [1], [2]. In particular, the superior tensile strength of UHPC prevents cracking during transportation and construction, while also raising the cracking load under service conditions, thereby extending structural service life. Compared to normal concrete, UHPC enables: reduction in cross-section size, longer spans, fewer beams and supports, and consequently lower self-weight. UHPC is widely applied in flat components such as bridge deck overlays, composite slabs, connection joints, structural strengthening and rehabilitation, and thin-walled members [3].

One application of UHPC is as a precast base plate for reinforced concrete (RC) semi-precast slab systems. The typical configuration consists of a reinforced UHPC precast bottom layer combined with a cast-in-place normal concrete (NC) top layer, reinforced by a steel truss frame, forming a UHPC–NC composite slab [4]. This approach combines factory production with rapid site assembly, improving construction efficiency and quality control. Compared to conventional precast panels with a normal concrete base plate of relatively low tensile strength, the use of UHPC as the base plate significantly reduces the risk of cracking during transportation, installation, and service. However, a limitation of this slab type is that the UHPC layer is typically thin (due to high material cost), which may result in insufficient structural stiffness during construction.

An innovative solution proposed by Cao et al. [5] introduces an additional EPS lightweight concrete layer between the UHPC base plate and the upper concrete, prefabricated together with the UHPC layer. The EPS core increases the flexural stiffness of the precast panel while keeping the overall slab weight low. However, research on this RC-UHPC semi-precast slab system is still in its early stages. In [5], the authors conducted an experimental study on the flexural behavior of the complete panel under service conditions (i.e., with all three layers - UHPC, EPS, and NC).

While previous studies predominantly focus on the service-stage performance of the slabs, there is a lack of comprehensive research on their behavior during the critical construction stage. In this phase, the prefabricated panel (UHPC-EPS-Steel truss) must independently support the weight of fresh concrete and construction live loads before the NC top layer hardens. This study aims to fill the research gap by investigating the flexural behavior of the RC-UHPC precast panel under construction-stage conditions. To this end, a finite element (FE) model combining 3D bar elements and shell elements is developed in Python using the CALFEM-Python toolkit [6]. The FE model is validated against experimental results from [5]. The parametric study examines the effects of the upper and lower chord reinforcement cross-sectional areas, the web bar cross-sectional area, and the UHPC base plate and EPS core thicknesses on the flexural performance of this composite panel. It establishes for the first time that the EPS core thickness is the dominant governing parameter for flexural stiffness in the construction phase, rather than the reinforcement ratio.

2. Configuration of the RC-UHPC panel

The panel cross-section, from bottom to top, consists of: a UHPC base plate serving as permanent formwork during construction; an EPS lightweight concrete core that reduces self-weight; and a cast-in-place normal concrete

(NC) top layer that resists compression under service loading. The steel reinforcement comprises three components - a lower chord mesh, an upper chord mesh, and diagonal web bars - forming a three-dimensional truss. The diagonal bars create a robust cage that enhances composite action between concrete layers and prevents delamination. Figure 1 illustrates the configuration of the RC-UHPC panel.

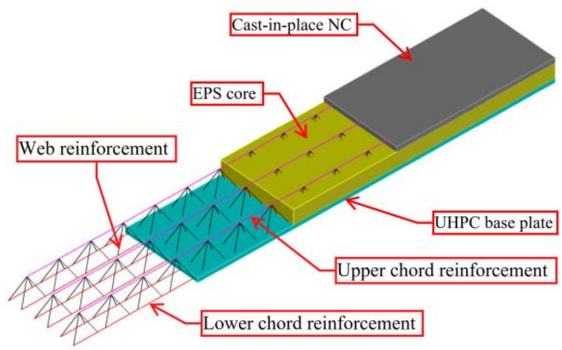

Figure 1. *Schematic diagram of the RC-UHPC slab panel*

3. FE modeling

In order to study the flexural behavior of the RC-UHPC panel, the finite element method (FEM) is employed. Perfect bond between concrete layers is assumed and the equivalent single-layer formulation is used to construct the shell element stiffness matrix. This assumption is based on experimental results in [5] indicating that no slip occurred between the EPS concrete surface and the UHPC surface. Moreover, all materials are assumed to behave in a linear elastic manner. This is a requirement for the panel in the construction state and shall be verified during the model analysis, ensuring that the tensile stress in the concrete is lower than the allowable tensile stress of the concrete, and that the tensile stress in the steel is lower than its yield strength.

A three-node flat triangular shell element, named CS-DSG3-A, is established for the analysis purpose. Section 3.1 gives the basis of this shell element. The finite element model is implemented based on an open-source Python package, CALFEM-Python [6]. The 3D bar element available in the CALFEM-Python element library is used for the steel mesh.

3.1. Shell element CS-DSG3-A

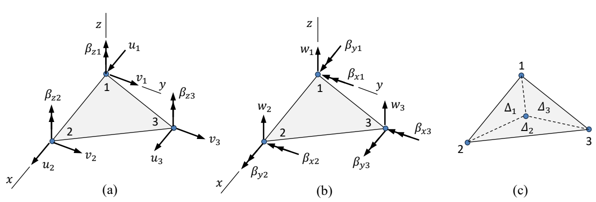

Figure 2. *CS-DSG3-A element: a) membrane DOFs; b) bending DOFs; c) sub-element*

The CS-DSG3-A shell element is constructed by combining the flat plate element CS-DSG3 [7] with the Allman membrane strain formulation [8]. The CS-DSG3-A element is designed to effectively handle both thin and thick plates by utilizing the Cell-based Smoothed Discrete Shear Gap method, which addresses the shear locking problem inherent in standard three-node triangular elements. This approach ensures numerical stability and accuracy by smoothing the strain fields over sub-elements [7].

Figure 2 illustrates a typical element in the x-y plane of the local coordinate system x-y-z. The shell consists of two material layers: UHPC (thickness t_{UHPC}) and EPS (thickness t_{EPS}). Each node has three translational and three rotational degrees of freedom (DOFs), giving 18 DOFs per element. For notational convenience, these DOFs are arranged in the vector $\{\mathbf{d}_e\}$ as follows:

$$\{\mathbf{d}_e\} = \lfloor \boldsymbol{u}_i \quad \boldsymbol{v}_i \quad \boldsymbol{\beta}_{zi} \quad \boldsymbol{w}_i \quad \boldsymbol{\beta}_{xi} \quad \boldsymbol{\beta}_{yi} \rfloor \tag{1}$$

where $\boldsymbol{u}_i = \lfloor u_1 \quad u_2 \quad u_3 \rfloor$, $\boldsymbol{v}_i = \lfloor v_1 \quad v_2 \quad v_3 \rfloor$, and the displacement vectors $\boldsymbol{\beta}_{zi}$, $\boldsymbol{w}_i$, $\boldsymbol{\beta}_{xi}$, $\boldsymbol{\beta}_{yi}$ are defined analogously.

The element stiffness matrix in the local coordinate system is given by:

$$[\mathbf{K}_e] = \begin{bmatrix} [\mathbf{K}_m] & [\mathbf{K}_{mb}] \\ [\mathbf{K}_{bm}] & [\mathbf{K}_b] \end{bmatrix} \tag{2}$$

In Eq. (2), $[\mathbf{K}_m]$ is the 9×9 membrane stiffness matrix, $[\mathbf{K}_b]$ is the 9×9 bending stiffness matrix, and $[\mathbf{K}_{mb}]$ is the 9×9 membrane-bending coupling stiffness matrix. These matrices are determined as follows:

- Bending stiffness matrix:

$$[\mathbf{K}_b] = \Delta([\bar{\mathbf{B}}_b]^T[\mathbf{D}_b][\bar{\mathbf{B}}_b] + [\bar{\mathbf{B}}_s]^T[\mathbf{D}_s][\bar{\mathbf{B}}_s]) \tag{3a}$$

$$[\bar{\mathbf{B}}_b] = \frac{1}{\Delta}\sum_{i=1}^{3}\Delta_i[\mathbf{B}_{bi}],\ [\bar{\mathbf{B}}_s] = \frac{1}{\Delta}\sum_{i=1}^{3}\Delta_i[\mathbf{B}_{si}] \tag{3b}$$

where, Δ is the element area; Δ_i $(i = 1, 2, 3)$ is the area of the i-th sub-triangle; and $[\mathbf{B}_{bi}]$ and $[\mathbf{B}_{si}]$ are respectively the bending and shear gradient matrices of the i-th sub-triangle, determined by the CS-DSG3 method [7]. The matrices $[\mathbf{D}_b]$ and $[\mathbf{D}_s]$ are the elasticity matrices for bending and shear deformation of the plate, defined as follows:

$$[\mathbf{D}_b] = \sum_{j=1}^{2}[\mathbf{Q}_m^j]\frac{z_j^3 - z_{j-1}^3}{3} \tag{4a}$$

$$[\mathbf{D}_s] = k\frac{h^2}{(h^2+\alpha l_e^2)}\sum_{j=1}^{2}[\mathbf{Q}_s^j]\,(\mathrm{z}_j - \mathrm{z}_{j-1}) \tag{4b}$$

where z_j and z_{j-1} represent the vertical coordinates of the top and bottom surfaces of the j-th material layer (UHPC or EPS) relative to the reference plane (i.e. for this two-layer system, z_0 is the bottom of the UHPC, z_1 is the interface, and z_2 is the top of the EPS layer); $k = 5/6$ is the shear correction factor; h is the element thickness $(h = t_{UHPC} + t_{EPS})$; l_e is the length of the element edge; and α is the stabilization parameter taken as 0.1 as in [7]. The term $\frac{h^2}{(h^2+\alpha l_e^2)}$ is introduced to improve shear accuracy and stability. The in-plane and transverse shear elasticity matrices for concrete layer j (UHPC or EPS) are respectively:

$$[\mathbf{Q}_m^j] = \frac{E^j}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{5a}$$

$$[\mathbf{Q}_s^j] = \frac{E^j}{2(1+\nu)}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (5b)$$

- Membrane stiffness matrix:

$$[\mathbf{K}_m] = [\mathbf{C}]^T\left(\int_\Delta [\mathbf{B}_m]^T[\mathbf{D}_m][\mathbf{B}_m]d\Delta\right)[\mathbf{C}] \quad (6a)$$

where $[\mathbf{C}]$ is provided in the Appendix of reference [8]; and the matrices $[\mathbf{B}_m]$ and $[\mathbf{D}_m]$ are respectively given by:

$$[\mathbf{B}_m] = \begin{bmatrix}1 & y & 0 & 0 & 0\\ 0 & 0 & 1 & x & 0\\ 0 & -x & 0 & -y & 1\end{bmatrix} \quad (6b)$$

$$[\mathbf{D}_m] = \sum_{j=1}^{n}[\mathbf{Q}_m^j]\,(z_j - z_{j-1}) \quad (6c)$$

- Membrane-bending coupling stiffness matrix:

$$[\mathbf{K}_{mb}] = [\mathbf{C}]^T\left(\int_\Delta [\mathbf{B}_m]^T[\mathbf{D}_{mb}][\overline{\mathbf{B}}_b]d\Delta\right) \quad (7a)$$

$$[\mathbf{K}_{bm}] = \left(\int_\Delta [\overline{\mathbf{B}}_b]^T[\mathbf{D}_{mb}][\mathbf{B}_m]d\Delta\right)[\mathbf{C}] \quad (7b)$$

where the matrix $[\mathbf{D}_{mb}]$ is defined as follows

$$[\mathbf{D}_{mb}] = \sum_{j=1}^{2}[\mathbf{Q}_m^j]\frac{z_j^2 - z_{j-1}^2}{2} \quad (7c)$$

The matrix $[\mathbf{D}_{mb}]$ in Eq. (7c) represents the membrane-bending coupling effect. In traditional single-layer symmetric plates, this matrix is zero. However, for the proposed RC-UHPC panel, the asymmetry of the material properties across the thickness (due to the distinct elastic moduli of UHPC and EPS) leads to a non-zero $[\mathbf{D}_{mb}]$, meaning that in-plane forces will induce bending moments and vice versa.

3.2. CALFEM-Python

CALFEM-Python is a software package developed in Python that provides tools for structural analysis using the finite element method. It integrates mesh generation capabilities from GMSH [9], which is very useful for establishing discrete models of plate/shell structures.

The basic workflow of a structural analysis program in CALFEM-Python follows the standard FEM procedure: define the discretized model; calculate element stiffness matrices and element nodal force vectors; assemble the global stiffness matrix and load vector; apply boundary conditions; solve the system of equations; compute element displacements, strains, and internal forces; and post-process results (see [6] for further details).

Automatic mesh generation is a notable feature of CALFEM-Python. The package provides two modules: the geometry module (*geometry.py*) and the mesh module (*mesh.py*).

- The *geometry.py* module provides functions to define the structural geometry through point, line, and surface objects.

- The *mesh.py* module generates 2D or 3D meshes from the geometry defined by the *geometry.py* module. The user may select either triangular or quadrilateral mesh types for shell element. In addition, *mesh.py* also supports meshing from *.geo files (native GMSH input format).

After meshing, the user obtains complete discretization data through the following arrays: *coords* (nodal coordinates), *edofs* (element connectivity), *dofs* (nodal DOFs), *bdofs* (boundary DOF groups by labeled points/edges), and *emarker* (element surface labels). These arrays are then used in subsequent analysis steps.

4. Result analysis

Consider an RC-UHPC panel with dimensions and reinforcement details as shown in Figure 3. A load case reflecting the construction condition is assumed, which includes the following loads:

- Self-weight of the UHPC layer, EPS core, and steel truss mesh;

- Weight of the fresh NC layer cast in-place on site, with an overload factor of 1.1;

- Construction live load: 2.5 kN/m^2 with an overload factor of 1.3.

Material properties of the panel are taken from Ref. [5] and listed in Table 1. The panel is simply supported at two ends.

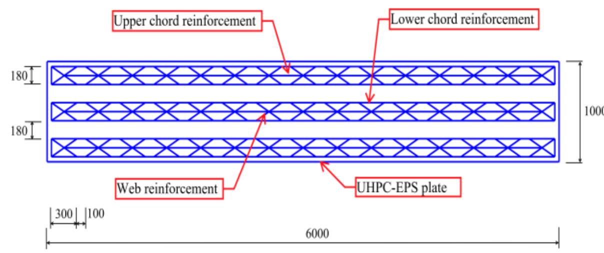

***Figure 3.** Dimension and reinforcement of RC-UHPC panel*

***Table 1.** Material properties of RC-UHPC panel*

Material	*E* (Gpa)	*ρ* (kg/m^3)	*ν*
UHPC	42.5	2645	0.3
EPS	3.35	1180	0.3
NC	33.20	2340	-
Steel	200	7850	-

All codes for the analysis are implemented in Python. Firstly, the geometry of the panel is defined by line and surface entities: The lines represent the steel mesh and panel edges; The surfaces represent the base plate. Then, the finite element mesh is automatically generated. The maximum element size is controlled by a size factor when defining geometry points (see [6] for further details). Connection between bar elements and shell elements is established at mesh nodes, which are automatically generated by GMSH.

4.1. FE model verification

Since there are no experimental data for the corresponding panel under construction conditions, the analysis results are compared with the experimental results for the finished panel in Ref. [5] (including the top NC layer). Deflection is computed for a panel with total thickness of 210 mm, configuration 30-120-60 (i.e., UHPC, EPS, and NC layer thicknesses are of 30 mm, 120 mm, and 60 mm, respectively), lower and upper chord reinforcement diameter of 10 mm, and web bar diameter of 6 mm. The applied loading consists of two symmetric point loads P spaced 1500 mm apart (see Ref. [5] for further details). The FE mesh of the panel with size factor of 200 mm is shown in Figure 4.

Table 2 lists the FE-predicted midspan deflection at P = 2 kN corresponding to different mesh refinements. The FE-predicted midspan deflection at P = 2 kN is approximately 3.1 mm. The experimental results from Ref. [5] for specimens M1 and M2 at the same load level are 4.4 mm and 3.2 mm, respectively. The relative error of the model results compared with the experiment results is 3.1% (specimen M2) and 29.5% (specimen M1).

From Table 2, it can be seen that the deflection error of the panel with a mesh corresponding to a size factor of 200 mm (1271 nodes) compared to a mesh with a size factor of 50 mm (3922 nodes) is very small. Therefore, a size factor of 200 mm is used for numerical investigation in the following sections.

Table 2. *Midspan deflection with different mesh refinements*

Size factor	600	200	100	50
No. nodes	1221	1271	2158	3922
Deflection (mm)	2.97	3.05	3.09	3.11

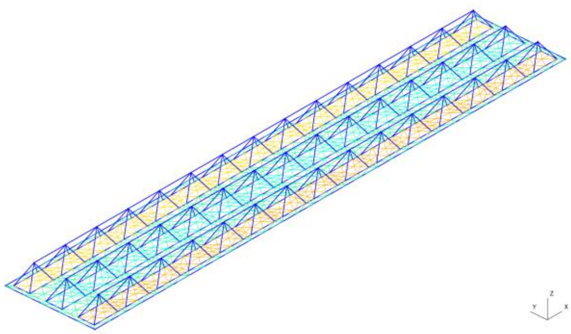

Figure 4. *FE mesh of RC-UHPC panel*

4.2. Bending in construction condition

The panel is analyzed in the construction stage, in which only the prefabricated assembly (UHPC base plate, EPS core, and steel truss mesh) is present - the NC layer is included only as fresh concrete load. Maximum deflection results for panels with varying UHPC and EPS layer thicknesses are given in Table 3, with the NC layer thickness held constant at 60 mm. Tables 4 and 5 present deflection results for the 50-180-60 configuration with varying lower chord, upper chord, and web bar diameters.

Table 3. *Maximum deflection with different thicknesses of UHPC and EPS core*

t_{EPS} (mm)	t_{UHPC} (mm)		
	30	40	50
120	46.18	40.02	34.80
150	29.98	26.50	23.59
180	20.74	18.55	16.77

Table 4. *Maximum deflection with different lower and upper reinforcement of 50-180-60 panel*

$\emptyset_{low}$ (mm)	$\emptyset_{up}$ (mm)		
	12	14	16
10	15.61	14.54	13.57
12	15.55	14.48	13.51
14	15.47	14.41	13.44

Table 5. *Maximum deflection with different web reinforcement of 50-180-60 panel*

$\emptyset_{low,up}$ (mm)	$\emptyset_{web}$ (mm)		
	6	8	10
10	16.77	16.64	16.58
12	15.55	15.33	15.23
14	14.41	14.09	13.93

Table 3 shows that both parameters t_{UHPC} and t_{EPS} have a pronounced effect on flexural stiffness, though with different magnitudes and mechanisms. Increasing t_{EPS} from 120 mm to 180 mm (50% increase) substantially reduces deflection - at t_{UHPC} = 30 mm, deflection decreases from 46.18 mm to 20.74 mm, a reduction of 55.1%. This trend is consistent across all three UHPC thickness levels examined. Figure 5 illustrates the variation of maximum panel deflection as a function of UHPC and EPS layer thicknesses. The effect arises from the role of the core in increasing the separation between the UHPC layer and the upper chord reinforcement, thereby amplifying the second-order contribution to the cross-sectional moment of inertia. The thickness t_{UHPC} shows a similar but smaller effect. When t_{EPS} = 120 mm, increasing t_{UHPC} from 30 mm to 50 mm (67% increase), deflection decreases by only about 24.7% (from 46.18 mm to 34.80 mm). This reflects the characteristic behavior of sandwich sections: the UHPC layer contributes to stiffness primarily through its position, while its own thickness has a relatively smaller contribution when $t_{UHPC} \ll t_{EPS}$. Notably, the combination t_{EPS}= 120 mm and t_{UHPC}= 30 mm yields a deflection of 46.18 mm, far exceeding the common limit of L/300 for typical spans (refer to TCVN 5574:2018 [10]), indicating that this configuration fails under normal construction conditions. Conversely, when t_{EPS}= 180 mm combined with t_{UHPC} ≥40 mm, deflection drops below 19 mm, which is more compatible with practical construction requirements. These results suggest that, in design optimization, prioritizing an increase in EPS core thickness yields a greater improvement in flexural stiffness than increasing UHPC thickness, while also benefiting self-weight reduction and cost.

Table 4 presents maximum deflection of the 50-180-60 panel for varying lower chord diameter ($\emptyset_{low}$) and upper chord diameter ($\emptyset_{up}$). The most notable observation is that the deflection range is far smaller than in Table 3 - all values fall between 13.44 mm and 15.61 mm, a maximum spread of 2.17 mm (14%) despite substantial changes in bar diameter. The influence of $\emptyset_{up}$ is more pronounced than that of $\emptyset_{low}$. Increasing $\emptyset_{up}$ from 12 mm to 16 mm reduces deflection by approximately 1.9–2.0 mm (about 13%) at all levels of $\emptyset_{low}$. By contrast, increasing $\emptyset_{low}$ from 10 mm to 14 mm reduces deflection by only 0.13–0.14 mm - essentially negligible. This contrast is explained by the position of the two reinforcement groups: the upper chord lies outside the concrete and contributes primarily to compressive zone stiffness; whereas the lower chord is embedded within the 50 mm UHPC layer and contributes comparatively little to flexural stiffness. Unlike the findings of Wang et al. [3] for UHPC-NC panel, which indicated that the upper chord reinforcement has a negligible influence during the service stage, these results show that in the construction stage, the upper chord area is significantly more influential than the lower chord in controlling deflection.

Table 5 similarly shows that increasing the web bar diameter has a negligible effect on panel stiffness.

Comparing Tables 4 and 5 with Table 3, it is evident that for the 50-180-60 panel, deflection is controlled primarily by the ‘section geometry’ (EPS core thickness = 180 mm) rather than by the amount of steel reinforcement. Increasing bar diameter within the investigated range yields only limited improvement in flexural stiffness. This is due to the area ratio of the steel bars is relatively small compared to that of EPS core. From a practical design standpoint, increasing reinforcement bar size is not an effective strategy for reducing deflection of this panel. If further deflection reduction is required, adjusting the core or UHPC layer thickness - as demonstrated in Table 3 - is far more effective than increasing the reinforcement ratio. If panel height is constrained, increasing the upper chord reinforcement area is preferable to increasing the lower chord.

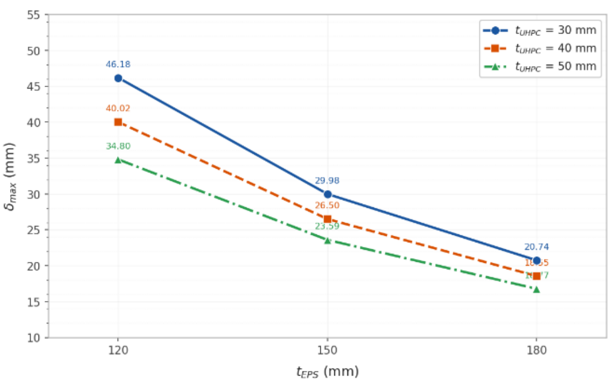

***Figure 5**. Maximum deflection of RC-UHPC panel with different thickness of UHPC plate and EPS core*

5. Conclusion

This paper presents an investigation on the flexural behavior of a semi-precast RC-UHPC panel with an EPS lightweight concrete core during the construction stage for the first time. A specialized numerical approach using the CS-DSG3-A shell element implemented in the open-source CALFEM-Python framework provides a transparent and efficient tool for parametric design of these specific sandwich-like panels. The main findings are as follows. First, the FEM model shows good agreement with experimental results from the literature, validating its use for parametric studies. Second, the EPS core thickness is the dominant parameter controlling panel deflection, with a 50% increase in core thickness (120 mm to 180 mm) reducing midspan deflection by up to 55%, consistent with sandwich beam mechanics. Third, the UHPC base plate thickness has a secondary but still meaningful effect, contributing through both direct stiffness and face-layer positioning. Fourth, reinforcement bar diameter - both chord and web - has only a minor influence on flexural stiffness in the construction stage; this is in contrast to the service stage, where tensile reinforcement governs flexural capacity. Fifth, for practical design, the 50-180-60 configuration achieves deflections in the 13–16 mm range, which is compatible with common construction-stage serviceability requirements. Sixth, design optimization should prioritize increasing core thickness over adding reinforcement, as this is more structurally efficient and reduces self-weight. The results provide a rational basis for the preliminary design of this emerging semi-precast slab typology.

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