MODEL PREDICTIVE CONTROL FOR TWIN ROTOR MIMO SYSTEM (TRMS)

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Abstract - A Twin Rotor MIMO System (TRMS) is an aerodynamic experimental system with high nonlinearity which includes two inputs, two outputs, and six states. In the world, this system has been studied and applied in reality in order to evaluate and implement the advanced control algorithms [1], [2], [3], [8], [9]. In Vietnam, although the TRMSs have been installed in some university laboratories, it is still difficult to use them for testing modern control algorithms because there is no exact mathematical model of the system. The documents and software provided on a laboratory equipment provider in the algorithm are confined to the classical PID controller. In this paper we will present the results from the application of Model Predictive Control (MPC) for TRMS based on its mathematical model we have built recently [12].

Key words - Model Predictive Control (MPC); State parametters; Twin rotor MIMO system (TRMS); cross-coupling channels; yaw angle (horizontal angle); pitch angle (vertical angle

1. Introduction

MPC is one of the advanced control techniques suitable for the problems of controlling industrial processes. The construction of the predictive model built on complex domain as GPC (General Predictive Control), or an equivalent as DMC (Dynamic Matrix Control) is the most suitable for SISO objects [10], [11]. The TRMS is a MIMO and a nonlinear system, therefore constructing predictive models is performed in the time domain because it is easy to linearize and calculate.

2. Construction of Methodology for MPC algorithms

Consider a nonlinear system with n_u inputs, n_x outputs and n_y states are described as the state space equations below:

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k) = h(x(k)) \end{cases} \quad (1)$$

Where *x(k)* is the state vector, *u(k)* is the input vector, and *y(k)* is the output vector, all at instant *k*. It can be linearised adaptively at each real sample time *k* (In model predictive control, two sample instants are considered and should be clarified to prevent from misunderstanding. One is the real sample time, and the other is the internal sample time. In term $u(k+i|k)$, *k* is the real sample time and *k + i* is the internal sample time) as the state equations of the discrete space below:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) \end{cases} \quad (2)$$

or can be represented by a combination of state- dependent state-space equations as:

$$\begin{cases} x(k+1) = A(x(k))x(k) + B(x(k))u(k) \\ y(k) = C(x(k))x(k) \end{cases} \quad (3)$$

The state variables and the inputs related to previous instant are used as initial conditions to linearise the non-linear system at each time. Making linearized nonlinear system *Np* times at each sampling instance adaptively according to *Np* operating points from earlier periods of the optimum result:

$$\begin{cases} \hat{x}(k+i+1|k) = A(x(k+i|k))\hat{x}(k+i|k) + B(x(k+i|k))\hat{u}(k+i|k) \\ \hat{y}(k+i|k) = C(x(k+i|k))\hat{x}(k+i|k) \end{cases} \quad (4)$$
$$i = 0, 1, ..., N_p - 1$$

In order to simplify the representation of the equations, the state dependent matrix $A(x(k+i|k))$ is shown as $A(k+i|k)$ and similar are the other state-dependent matrices.

To find the linear models, one can use the known values of $x(k+i|k-1)$ instead of the unknown $x(k+i|k)$, where $i = 0, 1, \ldots, N_p - 1$. In order to solve the optimization problem of the MPC, and obtain the relationship between the internal model outputs during the prediction horizon interval, $1 \le i \le N_P$, and the internal model inputs during the control horizon interval, $1 \le i \le N_C$, where *Np* and *Nc* are the prediction and control horizons. If the relationship is linear and the constraints are also linear, there is an optimization problem in quadratic form.

In the prediction horizon, the state vector can be expressed in terms of the state available vector *x(k)* and the future input vectors:

$$\hat{x}(k+i+1|k) = \left[\prod_{j=0}^{i} A(k+i-j|k)\right] x(k) + \sum_{n=0}^{i} \left\{ \left[\prod_{j=0}^{i-n-1} A(k+i-j|k)\right] B(k+n|k)\hat{u}(k+n|k) \right\} \quad (5)$$

It is common to use the input difference between two consecutive instants, $\Delta\hat{u}(k+i|k)$, instead of the input itself, $\hat{u}(k+i|k)$, using $\Delta\hat{u}(k+i|k) = \hat{u}(k+i|k) - \hat{u}(k+i-1|k)$ [5]. The only input changes during rest-of-control and did not change after, namely $\hat{u}(k+i|k) = \hat{u}(k+N_C-1|k)$ this means that $\Delta\hat{u}(k+i|k) = 0$for $N_c \le i \le N_p-1$. The input vectors related to the reference input vector:

$$\hat{u}(k+j|k) = u(k-1) + \sum_{i=0}^{j} \Delta\hat{u}(k+i|k) \quad (6)$$
$$j = 0, 1, ..., N_C - 1$$

Subsituting equation (6) into equation (5) we obtain:

$$\begin{aligned} &\hat{x}(k+i+1|k) \\ &= \left[\prod_{j=0}^{i} A(k+i-j|k)\right] x(k) \\ &+ \sum_{n=0}^{i} \left\{ \left[\prod_{j=0}^{i-n-1} A(k+i-j|k)\right] B(k+n|k) \right\} u(k-1) \\ &+ \sum_{m=0}^{\min(i,N_C-1)} \left(\sum_{n=m}^{i} \left\{ \left[\prod_{j=0}^{i-n-1} A(k+i-j|k)\right] B(k+n|k) \right\} \Delta\hat{u}(k+m|k) \right) \\ &i = 0, ..., N_p - 1 \end{aligned} \quad (7)$$

The predicted outputs are represented as:

$$\hat{y}(k+i|k) = C(k+i|k)\hat{x}(k+i|k) + \hat{d}(k+i|k), \quad (8)$$
$$i = 1, ..., N_p$$

where $\hat{d} \in \Re^{n_y x1}$ is the disturbance. Subsituting equation (7) into equation (8) we obtain:

$$\begin{aligned} Y(k) &= M_C(k)M_A(k)x(k) + M_C(k)M_B(k)u(k-1) \\ &+ M_C(k)M_U(k)\Delta U(k) + M_d(k) \end{aligned} \quad (9)$$

In which the matrix /vector:

$$\begin{aligned} &Y(k) \in \Re^{n_y N_p x1}, M_C(k) \in \Re^{n_y N_p x n_x N_p}, \\ &M_A(k) \in \Re^{n_x N_p x n_x}, M_B(k) \in \Re^{n_x N_p x n_u}, \\ &M_U(k) \in \Re^{n_x N_p x n_u N_C}, \Delta U(k) \in \Re^{n_u N_C x1}, \\ &M_d(k) \in \Re^{n_y N_p x1} \end{aligned}$$

3. Objective function

Suppose that the following objective function minimization as the constraint conditions (11) to (13):

$$\begin{aligned} J(k) &= \sum_{i=1}^{N_P} \left[r(k+i) - \hat{y}(k+i|k) \right]^T \delta(i) \left[r(k+i) - \hat{y}(k+i|k) \right] \\ &+ \sum_{i=1}^{N_C} \left[\Delta\hat{u}(k+i-1|k) \right]^T \lambda(i) \left[\Delta\hat{u}(k+i-1|k) \right] \end{aligned} \quad (10)$$

$$y_{\min} \le \hat{y}(k+i|k) \le y_{\max}, i = 1,2,...,N_p \quad (11)$$

$$u_{\min} \le \hat{u}(k+i-1|k) \le u_{\max}, i = 1,2,...,N_C \quad (12)$$

$$\Delta u_{\min} \le \Delta\hat{u}(k+i-1|k) \le \Delta u_{\max}, i = 1,2,...,N_C \quad (13)$$

Where

r: Reference trajectory with dimension (n_y x 1);

δ: The weight matrix of tracking errors with dimension (n_y x n_y);

λ: The weight matrix of control efforts with dimension (n_u x n_u).

The objective function can be written as:

$$\begin{aligned} J(k) &= \left[M_r(k) - Y(k) \right]^T Q \left[M_r(k) - Y(k) \right] \\ &+ \Delta U^T(k) R \Delta U(k) \end{aligned} \quad (14)$$

Subsituting equation (9) into equation (14) the objective function is a quadratic form:

$$\begin{aligned} J(k) &= \frac{1}{2}\Delta U^T(k)H(k)\Delta U(k) \\ &+ \Delta U^T(k)G(k) + c(k) \end{aligned} \quad (15)$$

where

$$\begin{aligned} &H(k) = 2(M_U^T(k)M_C^T(k)QM_C(k)M_U(k) + R) \\ &G(k) = -2M_U^T(k)M_C^T(k)QE(k) \\ &c(k) = E^T(k)QE(k) \\ &E(k) = M_r(k) - M_C(k)M_A(k)x(k) \\ &-M_C(k)M_B(k)u(k-1) - M_d(k) \end{aligned}$$

4. TRMS Objects

The proposed multistep Newton-type MPC based on the state - dependent is implemented on the TRMS, Figure 1. The control objective is to control the yaw and the pitch angles (α_h, α_v) as accurate as possible.

The state variables, the input and output vectors of TRMS are as follows:

$$\begin{aligned} x(k) = \big[&i_{ah}(k) \quad \omega_h(k) \quad S_h(k) \quad \alpha_h(k) \\ &i_{av}(k) \quad \omega_v(k) \quad S_v(k) \quad \alpha_v(k) \big]^T \end{aligned} \quad (16)$$

$$u(k) = \left[U_h(k) \quad U_v(k) \right]^T \quad (17)$$

$$y(k) = \left[\alpha_h(k) \quad \alpha_v(k) \right]^T \quad (18)$$

Where:

i_{ah}: Armature current of the tail motor (A);

ω_h: Rotational velocity of the tail rotor (rad/s);

S_h: Angular velocity of TRMS beam in the horizontal plane without affect of the main rotor (rad/s);

i_{av}: Armature current of the main motor (A);

ω_v: Rotational velocity of main rotor(rad/s);

S_v: Angular velocity of TRMS beam in the vertical plane without affect of the tail rotor (rad/s).

α_v: Vertical position (pitch angle) of the TRMS beam (rad)

U_h: Input voltage signal of the tailmotor (V)

U_v: Input voltage signal of the main motor (V)

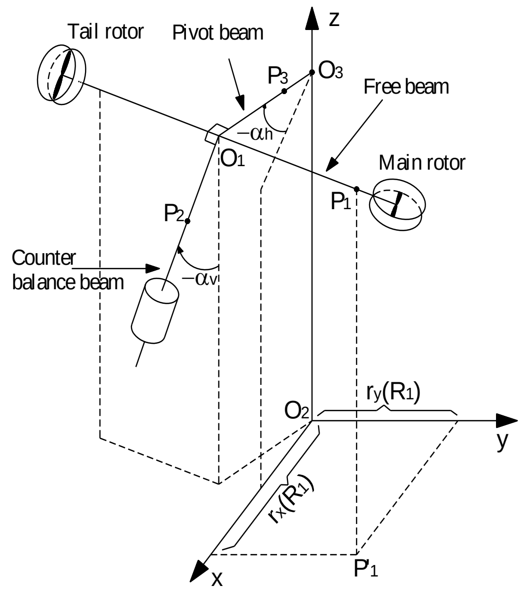

Figure 1. *TRMS Model*

The nonlinear continuous state space equations of the TRMS are expressed in [8]:

$$\frac{d}{dt}\begin{bmatrix} i_{ah} \\ \omega_h \\ S_h \\ \alpha_h \\ i_{av} \\ \omega_v \\ S_v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -\dfrac{R_{ah}}{L_{ah}} i_{ah} - \dfrac{k_{ah}\varphi_h}{L_{ah}}\omega_h + \dfrac{1}{L_{ah}} f_6(U_h) \\ \dfrac{k_{ah}\varphi_h}{J_{tr}} i_{ah} - \dfrac{B_{tr}}{J_{tr}}\omega_h - \dfrac{f_1(\omega_h)}{J_{tr}} \\ \dfrac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \dfrac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\dfrac{R_{av}}{L_{av}} i_{av} - \dfrac{k_{av}\varphi_v}{L_{av}}\omega_v + \dfrac{1}{L_{av}} f_8(U_v) \\ \dfrac{k_{av}\varphi_v}{J_{mr}} i_{av} - \dfrac{B_{mr}}{J_{mr}}\omega_v - \dfrac{f_4(\omega_v)}{J_{mr}} \\ \dfrac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} \\ \dfrac{+g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \dfrac{k_t}{J_v}\omega_h \end{bmatrix} \quad (19)$$

where

$$\begin{aligned} &R_{ah}, L_{ah}, k_{ah}\varphi_h, J_{tr}, B_{tr}, l_t, D, E, F, k_m, R_{av}, L_{av}, \\ &k_{av}\varphi_v, J_{mr}, B_{mr}, l_m, k_g, g, A, B, C, H, J_v, k_t \end{aligned}$$

is the positive constant, Ω_h and Ω_v is defined as

$$\Omega_h = S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \qquad (20)$$

$$\Omega_v = S_v + \frac{k_t\omega_h}{J_v} \qquad (21)$$

f_1 to f_9 is the nonlinear functions.

When $L_{ah}<<R_{ah}$ và $L_{av}<<R_{av}$ without loss of accuracy, the number of levels of the system can be reduced to grade of 6$x(k)=\left[\omega_h(k)\quad S_h(k)\quad \alpha_h(k)\quad \omega_v(k)\quad S_v(k)\quad \alpha_v(k)\right]^T$ as follows:

$$\frac{d}{dt}\begin{bmatrix}\omega_h\\ S_h\\ \alpha_h\\ \omega_v\\ S_v\\ \alpha_v\end{bmatrix} = \begin{bmatrix} -\frac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \frac{B_{tr}}{J_{tr}}\omega_h - \frac{f_1(\omega_h)}{J_{tr}} + \frac{k_{ah}\varphi_h}{J_{tr}R_{ah}} f_6(U_h) \\ \frac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\frac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \frac{B_{mr}}{J_{mr}}\omega_v - \frac{f_4(\omega_v)}{J_{mr}} + \frac{k_{av}\varphi_v}{J_{mr}R_{av}} f_8(U_v) \\ \frac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} + \frac{g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \frac{k_t}{J_v}\omega_h \end{bmatrix} \qquad (22)$$

Although this reduced-order model 6 does not affect the accuracy of the model, it can significantly affect the boot capacity calculations that reduce processor load and the speed of the optimization problem. The nonlinear state-space equation above can be approximated and represented as a state space equation follows: $\dot{x} = A(x)x + Bu$

5. Simulation results

Figure 2 shows the block diagram of the MPC approach for TRMS

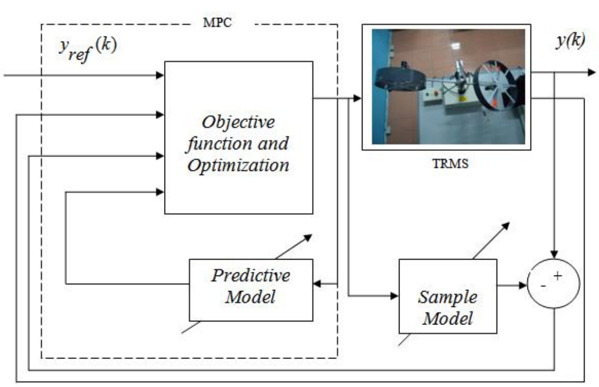

Figure 2. *Block diagram of the MPC approach*

The simulation results with square and substep wares are represented in the following figures. Figure 3 is the response of the pitch angle in which the reference is a square ware. Figure 4 is the response of the Yaw angle in which the reference is a square ware. Figure 5 is the response of the pitch angle in which the reference is a substep. Figure 6 is the response of the Yaw angle in which the reference is a substep.

Based on the simulation results in 200 seconds when applying Model Predictive Control for TRMS, the output responses of Yaw angle and pitch angle track the reference in predictive window. Especially, the cross-coupling channels between Yaw angle and pitch angle is best known. As soon as α_h varies, α_v changes and vice versa. Then the outputs track the inputs.

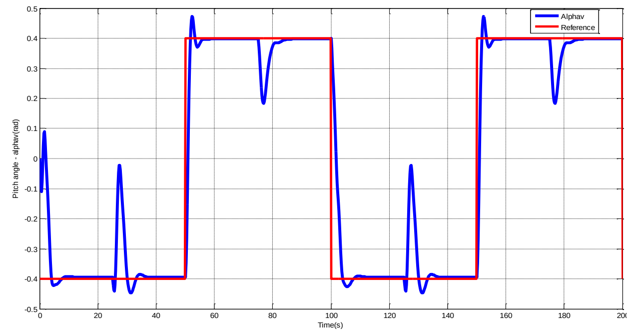

Figure 3. *The response of the pitch angle control loop with respect to a square - ware*

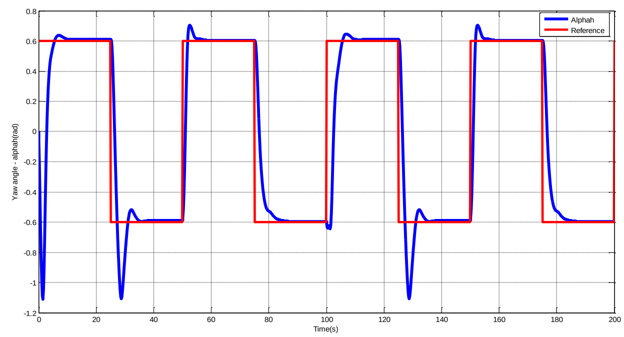

Figure 4. *The response of the Yaw angle control loop with respect to a square - ware*

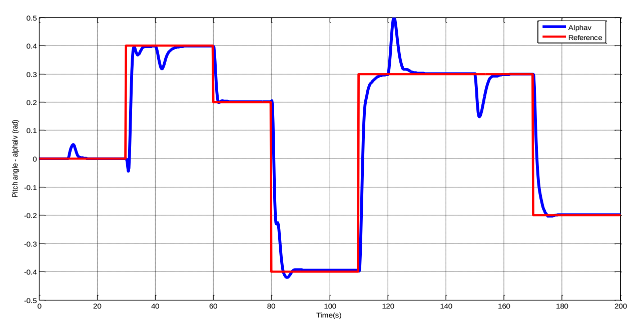

Figure 5. *The response of the pitch angle control loop with respect to a substep*

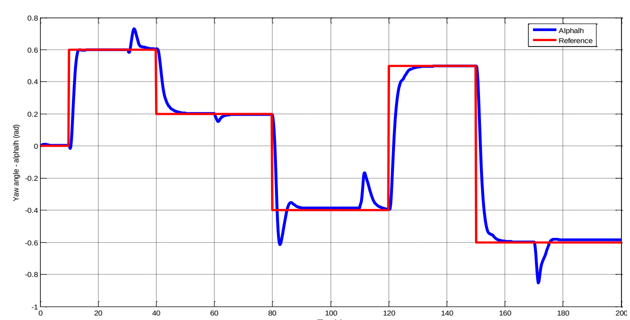

Figure 6. *The response of the Yaw angle control loop with respect to a substep*

6. Conclusion

In this paper, the TRMS system is modelized following the Np linear models during the predicting horizon at each sample time *k*. Then the author applies the MPC for TRMS and sees that output responses of the yaw and pitch angle track the followed trajectory, especially cross-coupling channels in vertical and horizontal directions. However, in this paper the author has not conducted a test to know when the disturbances take place, hence this is for further research in the next study.

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