MISALIGNMENT FADING EFFECTS ON PERFORMANCE OF AMPLIFY-AND-FORWARD RELAYING FSO SYSTEMS USING SC-QAM SIGNALS OVER LOG-NORMAL ATMOSPHERIC TURBULENCE CHANNELS

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Abstract - This paper presents the theoretical analysis of misalignment fading effects on performance of free-space optical (FSO) communication system based on Amplify-and-Forward (AF) relaying technology. This system uses subcarrier quadrature amplitude modulation (SC-QAM) over weak atmospheric turbulence modelled by Log-Normal distribution. The misalignment fading effect is studied by taking into account the influence of beamwidth, aperture size and jitter variance on the average symbol error rate (ASER). The influence of the number of relay stations, link distance on the system's ASER are also discussed in this paper. The numerical results show that the misalignment fading affect the performance of systems and how we use proper values of aperture size and beamwidth to improve the performance of such systems. The simulation results on ASER versus average signal-to-noise ratio (SNR) show a close agreement with analytical results.

Key words - AF; atmospheric turbulence; ASER; FSO; QAM; misalignment fading.

1. Introduction

Free-space optical (FSO) communications can provide high-speed links for a variety of applications. The most special characteristics are virtually unlimited bandwidth for achieving a very high aggregate capacity, no licensing requirements or tariffs for its utilization, excellent security, reduced interference, cost-effectiveness and simplicity of system design and setup [1]. FSO communication systems are made use of to solve the last mile problem when fiber-optic links are not practical, as well as a supplement to radio-frequency (RF) links. Among the most important disadvantages are the atmospheric propagation factors, such as haze, fog, rain and snow. However, there are several challenging issues in deployment of FSO systems, including the negative effects of scattering, absorption and turbulence. Among these impairment factors, the atmospheric turbulence has shown as the most serious problem on study of optical wireless communications. Atmospheric turbulence results in the fluctuationis of signal intensity, known as scintillation or fading, consequently degrades the system performance [2].

FSO systems using sub-carrier (SC) intensity modulation schemes, such as sub-carrier phase shift keying (SC-PSK) and sub-carrier quadrature amplitude modulation (SC-QAM), have been proposed. The use of the SC intensity modulation scheme also allows the combination of several radio frequency SC streams into an intensity modulated laser signal, which results in the higher system throughput and flexibility in signal multiplexing. The performance of FSO systems using SC-PSK has been extensively investigated [3]-[6]. Regarding the SC-QAM systems, the average SEP of the SISO/FSO systems using SC-QAM signals over atmospheric turbulence channel can be found in [7], [8]. However, to the best of our knowledge, the performance of relaying SISO/FSO systems using SC-QAM signals over atmospheric turbulence channels has not been clarified.

The rest of the paper is organized as follows: Section 2 introduce the system description. Section 3 discusses the atmospheric turbulence model of AF FSO/SC-QAM systems with misalignment fading. Section 4 is devoted to ASER derivation of AF FSO links. Section 5 presents the numerical results and discussion. The conclusion is reported in Section 6.

2. System description

A typical AF - FSO system employing SC-QAM is depicted in Figure 1. The source terminal S and destination terminal D can be connected using multiple wireless links arranged in an end-to-end conFigureuration so that the source terminal S can communicate with the destination terminal D through c relay terminals R_1, R_2, . . . , R_{c-1}, R_c.

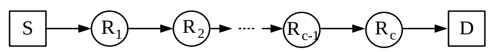

Figure 1. *A serial relaying SISO/FSO system*

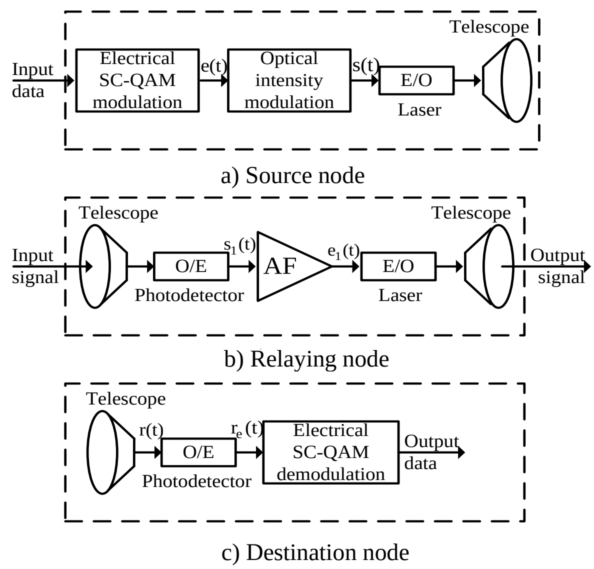

Figure 2. *The source node, relaying node and destination node of SISO/FSO systems*

The schemes of source node, relaying node, and destination node are illustrated in Figure 2. In Figure 2a, QAM symbol is first up-converted to an intermediate frequency f_c; this electrical subcar-rier QAM signal is then used to modulate the intensity of a laser. Generally, the electrical SC-QAM signal at the output of QAM modulator can be written as [9].

$$e(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad (1)$$

where $s_I(t) = \sum_{i=-\infty}^{i=+\infty} a_i(t) g(t - iT_s)$ and $s_Q(t) = \sum_{j=-\infty}^{j=+\infty} b_j(t) g(t - jT_s)$ are the in-phase and the quadrature signals, respectively. $a_i(t)$, $b_j(t)$ are the in-phase and the quadrature information amplitudes of the transmitted data symbol, respectively, $g(t)$ is the shaping pulse and T_s denotes the symbol interval. The QAM signal is used to modulate the intensity of a laser of the transmitter, the transmitted signal can be written as

$$s(t) = P_s\left\{1 + \kappa[s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)]\right\} \quad (2)$$

where P_s denotes the average transmitted optical power per symbol at each hop and κ is the modulation index. Due to the effects of both atmospheric loss, atmospheric turbulence and the misalignment fading, the received optical signal at the first relay node can be expressed as

$$s_1(t) = X\, P_s\left\{1 + \kappa[s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)]\right\} \quad (3)$$

where X presents the signal scintillation caused by atmospheric loss, atmospheric turbulence and the misalignment fading . At each relay node, AF module is used for signal amplification as shown in Figure 2b. Due to slow turbulence changes, the DC term $\{X P_s\}$ can be filtered out by a bandpass filter. The electrical signal output of AF module at the first relay node therefore can be expressed as

$$e_1(t) = \Re X P_1 P_s \kappa e(t) + \upsilon_1(t) \quad (4)$$

where $\Re$ is the PD's responsivity and P_1 is the amplification power of the first relaying node. The receiver noise $\upsilon_1(t)$ can be modeled as an additive white Gaussian noise (AWGN) process.

Repeating such manipulations above, the electrical signal output of the PD at the destination node can be derived as follows

$$r_e(t) = P_s e(t)\left[\prod_{i=0}^{c} \Re^{2i+1} X_{i+1} P_i\right] + \sum_{i=0}^{c} \upsilon_i(t) \quad (5)$$

where, *c* is the number of relay station and *n(t)* is the average gauss function. X denotes the stationary random process for the turbulence channel.

When the equal gain combining (EGC) detector is employed at the destination node to the estimate the transmitted signal, to analyze the ASER performance of the AF FSO/SC-QAM system, we define the instantaneous signal-to-noise ratio (SNR), denoted as γ, at the input of the electrical demodulator of an optical receiver. The γ is defined as the ratio of the time-averaged AC photocurrent power to the total noise variance, and it can be expressed as

$$\gamma = \frac{\left(\kappa \Re^{2i+1} P_s \prod_{i=1}^{c} X_{i+1} P_i\right)^2}{N_0} = \bar{\gamma}\left(\prod_{i=0}^{c} X_{i+1}\right)^2 \quad (6)$$

In this equation, $\bar{\gamma} = \overline{\text{SNR}} = \left(\kappa \Re^{2i+1} P_s \prod_{i=1}^{c} P_i\right)^2 / N_0$ is defined as the average electrical SNR and N_0 is the total noise variance.

3. Atmospheric turbulelce models with misalignment fading

As we derived above, the received electrical signal can be expressed in Eq (6) where X is the channel state. X models the optical intensity fluctuations resulting from atmospheric loss X_l, atmospheric turbulence fading X_a and misalignment fading X_p, which can be described as

$$X = X_l X_a X_p \quad (7)$$

3.1. Atmospheric lost

Atmospheric lost X_l is a deterministic component and no randomness, thus acting as a fixed scaling factor over a long time period. It is modeled in [10] as

$$X_l = e^{-\sigma L} \quad (8)$$

where σ denotes a wavelength and weather dependent attenuation coefficient, and L is the link distance.

3.2. Log-Normal atmospheric turbulence

For weak turbulence, the most widely accepted model is the Log-Normal distribution, which has been validated through studies [1]. The pdf of the irradiance intensity in the weak turbulent is given by

$$f_{X_a}(X_a) = \frac{1}{X_a \sigma_I \sqrt{2\pi}} \exp\left(-\frac{[\ln(X_a) + 0.5\sigma_I^2]^2}{2\sigma_I^2}\right) \quad (9)$$

where $\sigma_I^2 = \exp(\omega_1 + \omega_2) - 1$ the log intensity with ω_1 and ω_2 are respectively defined as

$$\omega_1 = \frac{0.49\sigma_2^2}{\left(1 + 0.18d^2 + 0.56\sigma_2^{12/5}\right)^{7/6}}, \quad (10)$$

$$\omega_2 = \frac{0.51\sigma_2^2\left(1 + 0.69\sigma_2^{12/5}\right)^{-5/6}}{1 + 0.9d^2 + 0.62\sigma_2^{12/5}} \quad (11)$$

In Eqs (10) and (11), $d = \sqrt{kD^2/4L}$ where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, L is the link distance, and D is the receiver aperture diameter, and σ_2 is the Rytov variance, defined as [1]

$$\sigma_2 = 0.492 C_n^2 k^{7/6} L^{11/6} \quad (12)$$

In Eq (12), C_n^2 is the refractive-index structure parameter, which is altitude dependent and varies from $10^{-17} m^{-2/3}$ to $10^{-13} m^{-2/3}$ to the turbulence conditions, accordingly. Through c relay terminals, we derive the probability distribution function of X_a^c for amply-and-forward SISO/FSO systems as follows [11].

$$f_{X_a}\left(X_a^{c+1}\right)=\frac{1}{(c+1)X_a^{c+1}\sigma_I\sqrt{2\pi}}\exp\left[-\frac{\ln(X_a+0.5\sigma_I^2)^2}{2\sigma_I^2}\right] \quad (13)$$

3.3. Misalignment fading model

A statistical misalignment fading model is developed in [12, 13], the pdf of X_p is given as [12]

$$f_{X_p}(X_p)=\frac{\xi^2}{A_0^{\xi^2}}X_p^{\xi^2-1}, \quad 0\le X\le A_0 \quad (14)$$

where $A_0=\left[\mathrm{erf}(v)\right]^2$ is the fraction of the collected power at radial distance 0, v is given by $v=\sqrt{\pi}r/(\sqrt{2}\omega_z)$ with r and ω_z respectively denote the aperture radius and the beam waist at the distance z and $\xi=\omega_{zeq}/2\sigma_S$, where the equivalent beam radius can be calculated by

$$\omega_{zeq}=\omega_z(\sqrt{\pi}\,\mathrm{erf}(v)/2v\times\exp(-v^2))^{1/2} \quad (15)$$

where $\omega_z=\omega_0\left[1+\varepsilon(\lambda L/\pi\omega_0^2)^2\right]^{1/2}$ with ω_0 is the transmitter beam waist radius at $z=0$, $\varepsilon=(1+2\omega_0^2)/\rho_0^2$ and $\rho_0=(0.55C_n^2k^2L)^{-3/5}$ is the coherence length.

3.4. Combined channel model

The complete statistical model of the channel considering the combined effect of atmospheric turbulence, atmospheric lost and misalignment fading. The unconditional pdf of the channel state is obtained [13].

$$f_X(X)=\int f_{X|X_a}(X|X_a)f_{X_a}(X_a)dX_a \quad (16)$$

where $f_{X|X_a}(X|X_a)$ denotes the conditional probability given a turbulence state, and it can be expressed by [2].

$$f_{X|X_a}(X|X_a)=\frac{1}{X_a.X_l}f_{X_p}\left(\frac{X}{X_a.X_l}\right)$$
(17)

As a result, we can derive the unconditional pdf for weak atmospheric turbulence conditions. For weak turbulence, the unconditional pdf through c relay terminals can be expressed by

$$f_X(X)=\frac{\xi^2}{(\mathrm{c}+1)(A_0X_l)^{\xi^2}}X^{\xi^2-1}\int_{(X/X_lA_0)}^{\infty}\frac{1}{X_a^{\xi^2+c+1}\sigma_I\sqrt{2\pi}}\times\exp\left\{\frac{[\ln(\mathrm{X}_a)+0.5\sigma_I^2]^2}{2\sigma_I^2}\right\}dX_a \quad (18)$$

Letting $t=\dfrac{\ln(X_a)+a}{\sqrt{2}\sigma_I}$ the Eq. (18) can be obtained in a closed-form expression as

$$f_X(X)=\frac{\xi^2}{(\mathrm{c}+1)(A_0X_l)^{\xi^2}}X^{\xi^2-1}\frac{1}{2}e^b\times\mathrm{erf}c\left(\frac{\ln(\mathrm{X}/\mathrm{X}_l\mathrm{A}_0)+a}{\sqrt{2}\sigma_I}\right) \quad (19)$$

where $a=0.5\sigma_I^2+\sigma_I^2(\xi^2+c)$ and $b=\sigma_I^2(\xi^2+c)\{1+(\xi^2+c)\}/2.$

4. Aser calculation

The average symbol error rate of AF relaying MIMO/FSO/SC-QAM, can be generally expressed as

$$P_{se}=\int_0^\infty P_e(\gamma)f_\gamma(\gamma)d\gamma \quad (20)$$

where $P_e(\gamma)$ is the conditional error probability (CEP). For using SC-QAM modulation, the conditional error probability presented as

$$P_e(\gamma)=1-\left[1-2q(M_I)Q(A_I)\sqrt{\gamma}\right]\left[1-2q(M_Q)Q(A_Q\sqrt{\gamma})\right] \quad (21)$$

where $q(x)=1-x^{-1}$, $Q(x)$ is the Gaussian Q-function, $Q(x)=0.5\,\mathrm{erfc}(x/\sqrt{2})$, $A_I=\left(6/\left[(M_I^2-1)+r^2(M_Q^2-1)\right]\right)^{1/2}$, $A_Q=\left(6r^2/\left[(M_I^2-1)+r^2(M_Q^2-1)\right]\right)^{1/2}$, in which $r=d_Q/d_I$ as the quadrature to in-phase decision distance ratio, M_I and M_Q are in-phase and quadrature signal amplitudes, respectively. Eq. (21) can further be written as follows

$$P_e(\gamma)=2q(M_I)Q(A_I\sqrt{\gamma})+2q(M_Q)Q(A_Q\sqrt{\gamma}) -4q(M_I)q(M_Q)Q(A_I\sqrt{\gamma})Q(A_Q\sqrt{\gamma}) \quad (22)$$

Assuming that SISO sub-channels' turbulence processes are uncorrelated, independent and identically distributed, the joint pdf $f(\gamma)$ can be reduced to a product of the first-order pdf of each element. Eqs (6), (19) and formula contact between probability density function, the pdf for AF - SISO/FSO systems in the case of weak turbulence channels can be, respectively, given as

$$f_\gamma\left(\gamma^{\frac{c+1}{2}}\right)=\frac{\xi^2}{2(\mathrm{c}+1)(A_0X_l)^{\xi^2}}\frac{\gamma^{0.5\xi^2-1}}{\gamma^{-0.5\xi^2}}\frac{1}{\sqrt{\pi}}e^b\times \mathrm{erf}c\left(\frac{0.5\ln(\gamma/\mathrm{X}_l^2A_0^2\bar{\gamma})+a}{\sqrt{2}\sigma_I}\right) \quad (23)$$

Substituting Eq (22) and Eq (23) into Eq. (20), the ASER of the systems can be obtained as

$$P_{se}(\gamma)=2q(M_I)\int_0^\infty Q(A_I\sqrt{\gamma})f(\gamma)d\gamma+2q(M_Q)\int_0^\infty Q(A_Q\sqrt{\gamma})f(\gamma)d\gamma -4q(M_I)q(M_Q)\int_0^\infty Q(A_I\sqrt{\gamma})Q(A_Q\sqrt{\gamma})f(\gamma)d\gamma \quad (24)$$

5. Numerical results and discussion

Using previous derived expressions, Eq (23) and Eq (24), we present numerical results for ASER analysis of the AF relaying FSO systems. The systems's ASER can be estimated via multi-dimensional numerical integration with the help of the Matlab™ software. Relevant parameters considered in our analysis are provided in Table 1.

Table 1. *Sysem parametters and constants*

Parameter	Symbol	Value
Laser Wavelength	λ	1550 nm
Photodetector responsivity	$\Re$	1 A/W
Modulation Index	κ	1
Link distance	L	1km
Total noise variance	N_0	10^{-7}A/Hz
In-phase/Quadrature signal amplitudes	M_I/M_Q	8/4
Index of refraction structure	C_n^2	$10^{-15}m^{-2/3}$

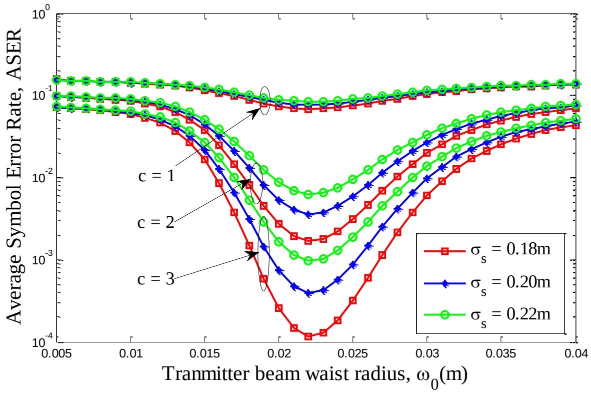

Figure 3. *ASER performance versus transmitter beam waist radius ω_0 for various values of σ_s, the aperture radius $r = 0.07m$, the average $SNR = 22$dB.*

Figure 3 depicts the ASER against transmitter beam waist radius for various values of the misalignment fading displacement standard deviation $\sigma_s = 0.18$m, 0.02 m and 0.22 m. It is clearly depicted that for a given condition including specific values of number relay stations, aperture radius and average SNR, the minimum of ASER can be reached to a specific value of ω_0. This value is called the optimal transmitter beam waist radius. Apparently, the more the value of transmitter beam waist radius comes close to the optimal one, the lower the value of system's ASER. The system's performance is therefore improved. The optimal value of transmitter beam waist radius $(\omega_0)_{op} \approx 0.022$m.

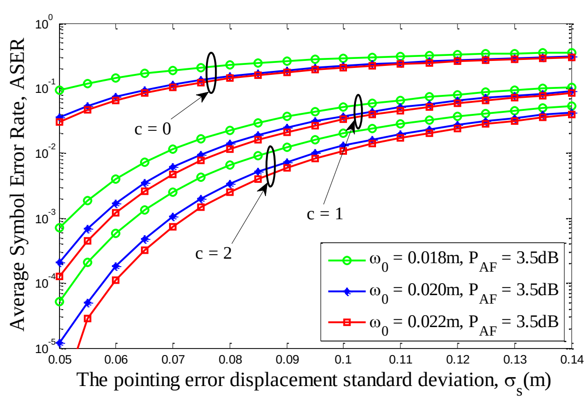

Figure 4. *ASER performance against the misalignment fading displacement standard deviation σ_s for various values of ω_0 with $r = 0.055$m, the average $SNR = 27$dB.*

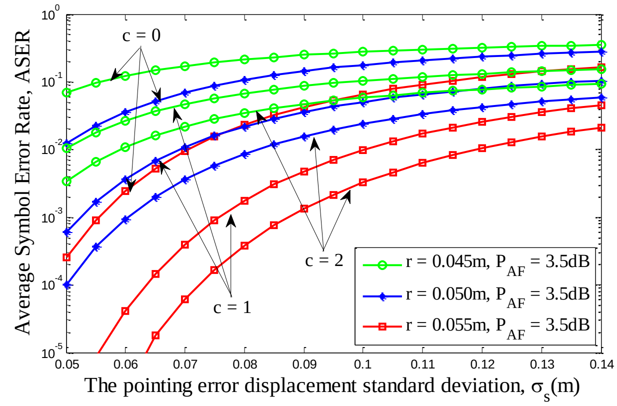

Figure 5. *ASER performance against the misalignment fading displacement standard deviation σ_s for various values of r with $\omega_0 = 0.022$m, the average $SNR = 27$dB.*

Figures 4 and 5, illustrate the ASER performance against the misalignment fading displacement standard deviation under various relay stations. The system's ASER significantly decreases when the misalignment fading displacement standard deviation decreases. Therefore, the system's performance is greatly improved when σ_s decreases. In addition, Figureures also show that, the misalignment fading effects impact more severely on the system's performance since higher values of ASER are gained. The impact of the aperture radius and the transmitter beam waist radius on the system's performance is more significant in low σ_s regions than in high σ_s regions.

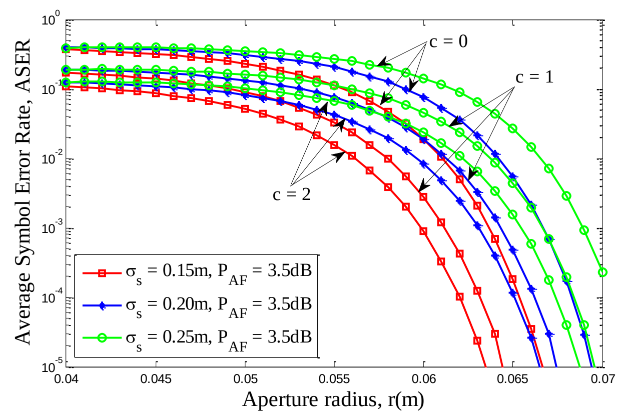

Figure 6. *ASER performance against the aperture radius r for various values of σ_s with $\omega_0 = 0.022$m, the average $SNR = 27$dB.*

Figure 6, illustrate the ASER performance agains the aperture radius under various misalignment fading displacement standard deviation. As a result, the system's ASER significantly decreases when the values of aperture radius and number relay stations increase. It is found that, in low-value region when aperture radius increases, system's ASER does not change much. However, when aperture radius exceeds the threshold value, ASER plummets when aperture radius increases.

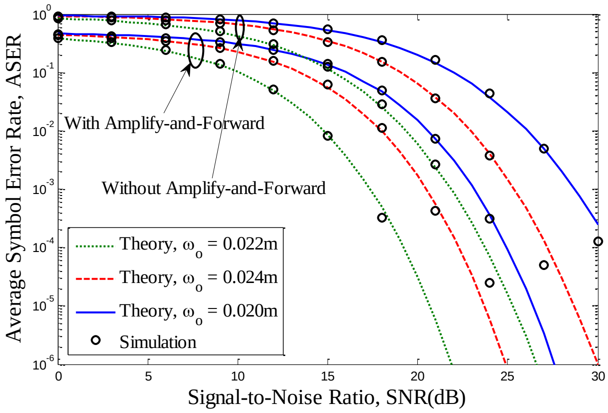

Figure 7. *ASER performance versus average SNR for various values of transmitter beam waist radius,* $\sigma_s = 0.16\,\text{m}$, *the number of relay stations c= 0, 1 and* $r = 0.055\,\text{m}$.

In Figure 7, the system's average symbol error rate, ASER, is presented as a function of average signal to noise ratio, SNR, under various values of the transmitter beam waist radius, misalignment fading displacement standard deviation $\sigma_s = 0.16\,\text{m}$. Besides, ASER performance with misalignment fading increases compared to that without AF. Again, the theoretical results are in accordance with the simulation results. It can be seen from Figure 7 that the ASER decreases with the increase of the SNR. ASER performance will be better when the wider beam waist radius of 0.024m is used instead of 0.02m. The best ASER performance is carried out when the optimal beam waist radius of 0.022m is applied. It can be found that simulation results show a close agreement with analytical results.

6. Conclusion

This paper has theoretically analyzed the performance of AF - FSO systems employing SC-QAM over weak atmospheric turbulence channels in the presence of misalignment fading. ASER of the system is theoretically derived taking into account various system's parameters, link atmospheric conditions, AF relaying and the misalignment fading effect. The numerical results have shown the impact of misalignment fading on the system's ASER. By analyzing ASER performance, we can conclude that using proper values of aperture radius transmitter beam waist radius regardless of partially surmounted misalignment fading and number relay stations can greatly benefit the performance of the systems. In addition, simulations results are also performed to validate the theoretical analysis for ASER performance of AF FSO/SC-QAM systems over atmospheric turbulence and misalignment fading and a good agreement between theoretical and simulation results has been confirmed.

REFERENCES

[1] Majumdar AK, Ricklin JC., "Free-space laser communications: principles andadvances", *New York: Springer*, 2008.

[2] L. Andrews, R. Phillips, and C. Hopen, "Laser Beam Scintillation with Applications", *Bellingham, WA: SPIE Press*, 2001.

[3] T. A. Tsiftsis, H. G. Sandalidis, G. K. Karagiannidis, N. C. Sagias, "Multihop Free-Space Optical Relaying Communications Over Strong Turbulence Channels", *IEEE Int. Conf. on Communications*, no. 6, pp. 2755-2759, 2006.

[4] C. K. Datsikas, et. al., "Serial Free-Space Optical Relaying Communications Over Gamma-Gamma Atmospheric Turbulence Channels", *IEEE/OSA J. of Optical Communications and Networking*, no. 2, pp. 576-586, 2010.

[5] Rajbhandari S, Ghassemlooy Z, Angelova M., "A study of discrete wavelet transform based denoising to reduce the effect of artificial light interferences for indoor optical wireless communication", *In: Proc. CSNDSP*, pp. 610–614, 2010.

[6] X. Song, M. Niu, and J. Cheng, "Error rate of subcarrier intensity modulations for wireless optical communications", *IEEE Commun. Lett.*, no. 16, pp. 540-543, 2012.

[7] K. P. Peppas, C. K. Datsikas, "Average symbol error probability of general-order rectangular quadrature amplitude modulation of optical wireless communication systems over atmospheric turbulence channels", *J. Opt. Commun. Netw.*, pp. 102-110, 2012.

[8] H. D. Trung and T. A. Pham, "Performance Analysis of MIMO/FSO Systems using SC-QAM Over Atmospheric Turbulence Channels", *IEICE Trans. on Fundamentals of Elec., Commun. and Computer Sciences*, no. 1, pp. 49-56, 2014.

[9] Hassan M.Z., Song X, Cheng J., "Subcarrier intensity mod-ulated wireless optical communications with rectangular QAM", *IEEE/OSA J Opt Commun Netw*, vol. 4, no. 6, pp. 522–532, 2012.

[10] Lee E, Ghassemlooy Z, Ng WP, Uysal M., " Performance analysis of free space optical links over turbulence and misalignment induced fading channels", *In: Proc. IET International Symposium on Communication Systems, Networks and Digital Signal Processing*, p. 1–6, 2012.

[11] D. H. Ai, H. D. Trung, and D. T. Tuan, "Pointing Error Effects on Performance of Amplify-and-Forward Relaying MIMO/FSO Systems Using SC-QAM Signals Over Log-Normal Atmospheric Turbulence Channels", *Asian Conference on Intelligent Information and Database Systems*, vol. 9622, pp. 607-619, Mar. 2016.

[12] Ahmed A, Hranilovic S., " Outage capacity optimization for freespace optical links with misalignment fading", *S. J Lightw Technol*, vol. 25, no. 7, pp. 1702–1710, 2007.

[13] M. Safari, and M. Uysal, "Relay-Assisted Free-Space Optical Communication", *IEEE Trans. Wireless Communication*, vol. 7, pp. 5441-5449, 2008.

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