ONE-DIMENSION CONSOLIDATION ANALYSIS OF SOFT SOILS UNDER EMBANKMENT LOADED WITH VARIABLE COMPRESSIBILITY AND PERMEABILITY

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Abstract - Terzaghi's 1D consolidation theory is commonly used for evaluation of consolidation characteristics of soft soils. Several simplifying assumptions have been made to resolve differential equation for one-dimension consolidation. Particularly, the assumption of constant value for coefficient of consolidation C_v during consolidation process is one of the major limitations in Terzaghi's theory; it is not entirely consistent with reality. In this paper a one-dimensional nonlinear partial differential equation is derived for prediction of consolidation characteristics of soft clays considering variable values for C_v based on linear relationships for e-Log(σ′) and e-Log(k). The nonlinear partial differential equation has been solved by a finite different method. An example has been implemented to show that the result of average degree of consolidation is different from calculating nonlinear consolidation theory and Terzaghi's theory.

Key words - Terzaghi's 1D consolidation; permeability; compressibility; pore water pressure; nonlinear consolidation theory.

1. Introduction

In order to predict the progress of consolidation with time in cohesive soils, the oedometer test is performed to determine consolidation characteristic of soil and Terzaghi's linear theory is commonly used for evaluation of the result. In this approach the coefficient of consolidation is assumed to be constant. In reality, it varies as the coefficient of volume compressibility m_v and permeability k change during the consolidation process. Thus, the assumption of coefficient of consolidation C_v being constant is not exact.

Furtherrmore, the coefficient of consolidation C_v obtains different results for different methods and different experiments (Terzaghi & Peck, 1967). The upper limit, leading to the results of average degree of consolidation predicted by Terzaghi's theory unlike in measurement results (Ducan, 1993). To solve this problem, many researchs have been done to improve and overcome the limitations of consolidation test. Among them, the theoretical study about non-linear consolidation with coefficient of consolidation C_v changes during consolidation process can be considered (Evance, 1998; Lekha et al., 2003; Zhuang, 2004; Abbasi et al., 2007; Fattah, 2012).

The nonlinear consolidation theory for clay was first proposed by Davis và Raymond (1965). Lekha et al,. (2003) derived a theory for consolidation of a compressible medium of finite thickness neglecting the effect of seft-weight of soil and creep effects but considering variation in compressibility and permeability. They proposed an analytical closed form solution to determine the relation between degree of consolidation and time factor. Zhuang (2004) presented a non-linear analysis and a semi-analytical closed form solution for consolidation with variable compressibility and permeability. Although the research results (Lekha et al., 2003; Zhuang, 2004) considered the variation of C_v during consolidation progess, but their solution give the relation between degree of consolidation with time factor. Where, time factor T_v determined via real time and coefficient of consolidation. Thus, these limitations concering the determination of C_v have still remained. Abbasi et al., (2007) had developed nonlinear defferential equation of consolidation by using linear relation for e-log(σ′) and e-log(k). Finite difference method was used for the solution of the proposed non-linear differential equation.

This paper presents a generalized theory for one-dimensional consolidation of soft soil with variable compressibility and permeability. Two coefficients (C_n and α) are used to describe changes in soil characteristics and take into consideration the changes in coefficient C_v during the consolidation. Using finite difference method, the differential equation of nonlinear one-dimensional consolidation is solved to determine the variations of excess pore water pressure and C_v in time and space.

2. Theory of one-dimensional consolidation

2.1. Terzaghi's 1D consolidation equation

The one-dimentional consolidation theory was first proposed by Terzaghi and become basic theory for all study of consolidation process for soft soil. The assumptions in the derivation of the mathematical equations are:

(i) The clay layer is homogenous;

(ii) The clay layer is fully saturated (Sr=100%);

(iii) The compression of the soil layer is due to the change in volume only, wich in turn is due to the squeezing out of water from the void spaces;

(iv) The process of pore water drainage occurs only vertically;

(v) Permeability process through Dacrcy's permeability law;

(vi) The coefficient of volume compressibility (m_v) and permeability (k) is constant during the consolidation process;

The basic differential equation of Terzaghi's 1D consolidation theory

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \qquad (1)$$

Coefficient of consolidation (C_v) can be determined from Eq (2):

$$C_v = \frac{k}{m_v \gamma_w} \quad (2)$$

where: m_v – coefficient of volume compressibility; γ_w- unit weight of water; k – coefficient of perrneability.

2.2. Solution of the Terzaghi's consolidation equation according to Taylor's series

Pore water pressures at any times t and depth z, can be obtained from Eq (3):

$$u_{(t,z)} = 1 - \sum_{m=0}^{m=\infty} \frac{2u_0}{M} \text{Sin}\frac{Mz}{H} \exp(-M^2.T_v) \quad (3)$$

The average degree of consolidation for the entire layer can be determined from Eq (4):

$$U_{ave} = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2.T_v) \quad (4)$$

Where: M=(2m+1)π/2; T_v – time factor ($T_v = C_v.t/H^2$); H – length of drainage path.

3. The nonlinear theory of one-dimensional consolidation considering variable compressibility and permeability

3.1. The differential equation of nonlinear consolidation theory

The differential equation of nonlinear consolidation describes the variation of pore water pressure with time and space for clay layer during consolidation process, using linear relationships for e-log(σ′) and e-log(k) (Evance, 1998; Gibson et al., 1967). This equation was first proposed by Davis and Raymond (1965) and subsequent developed by Gibson (1981) and Abbasi et al., (2007).

$$e = b + C_k.\log(k) \quad (5)$$

$$e = a - C_c.\log(\sigma') \quad (6)$$

Eq. (5) presents a linear relationships between the void ratio (e) and coefficient of permeability k (with k on a logarithmic scale). In this equation, C_k and b are the slope and intercept of the line respectively; b is the void ratio at unit coefficient of permeability (k=1).

Eq. (6) defines a straight line representing variation of void ratio (e) with effective stress (σ'). C_c is compressibility index, defined as the slope of the straight line; a is the void ratio at unit effective stress (σ'=1).

Combining equations (5), (6) and substituting into Equ.(1) will result:

$$\frac{\partial u}{\partial t} = \frac{\ln 10(1+e_0)}{C_c.\gamma_w} 10^{(a-b)/c_k}.(\sigma')^{\left(1-\frac{C_c}{C_k}\right)} \frac{\partial^2 u}{\partial z^2} \quad (7)$$

Assuming:

$$\alpha = 1 - \frac{C_c}{C_k} \quad (8)$$

$$C_n = \frac{\ln 10.(1+e_0)}{\gamma_w.C_c}.10^{(a-b)/C_k} \quad (9)$$

Eq (7) can be written as:

$$\frac{\partial u}{\partial t} = C_n (\sigma')^{\alpha} \frac{\partial^2 u}{\partial z^2} \quad (10)$$

Where $\sigma' = \sigma_t - u$ (11)

Non-linear differential equation (10) has form the same as Terzaghi's equation (1) with the coefficient of consolidation defined as Eq (12):

$$C_v = C_n.(\sigma_t - u)^{\alpha} \quad (12)$$

In equation (12), the coefficient of consolidation C_v is not constant, and varies during consolidation as the excess pore water pressure (u) changes. Coefficient α determind by Eq. (8), depends on compressibility and permeability characteristic (C_c and C_k). Coefficient C_n determined by Eq. (9), depends on compressibility and permeability characteristics (a,b,C_k, C_c), initial void ratio (e_0) and unit weight of water (γ_w). In the special case when α=0 (or C_c/C_k=1), C_v will be constant and equal to C_n. This case, Eq (10) will reduce to Terzaghi's equation.

3.2. Solution of the nonlinear differential equation by finite diference method

The nonlinear differential equation (10) can be solved using explicit algorithms of the finite difference method (Evance, 1998). In this procedure, the clay layer will be divided in to n thin layers ($\Delta z = H_t/n$) and the time is divided in to small time step Δt (Figure 1). The coefficient of consolidation C_v determined from the Eq. (12) is assumed to be constant temporarity in given small time step Δt.

At the first time t=Δt, pore water pressure at nodes ($u_{i,j}$) calculated corresponding to $C_v = C_n$. The consolidation equation is solved for new value of pore water pressure at the end t=Δt. Then, the coefficient of consolidation will calculate again corresponding new pore water pressure and it used at next time step.

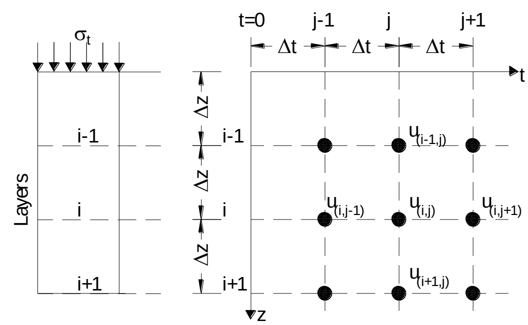

Figure 1. *Divide the soil in to small layers*

Using explicit algorithms of the finite difference method, equation (1) becomes:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = c_v \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta z^2} \quad (13)$$

Symbols numeral i specific for depth z, numeral j specific for time t. So:

$u_{i-1,j}$; $u_{i,j}$; $u_{i+1,j}$ are pore water pressure at point i-1, i and i+1 at time t, (j=t).

$u_{i,j+1}$ are pore water pressure at point i at time t+Δt, (j+1). Since we known water pore pressure $u_{i-1,j}$; $u_{i,j}$; $u_{i+1,j}$, we can compute $u_{i,j+1}$. This is chematically showed on Figure 1

Let: $\beta = c_v \frac{\Delta t}{\Delta z^2}$ (14)

Equation (15) can be written as:

$$u_{i,j+1}=\beta.u_{i-1,j}+[1-2\beta]u_{i,j}+\beta.u_{i+1,j} \qquad (15)$$

Based on Eq. (15) the pore water pressure at nodes can written in a matrix form as follow:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ . \\ u_{n-1} \\ u_n \end{bmatrix}^{j+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & 1-2\beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 1-2\beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 1-2\beta & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ . \\ u_{n-1} \\ u_n \end{bmatrix}^{j}$$

Solving matrix allow to determine pore water pressure excess at nodes at any times.

3.3. Calculation of average degree of consolidation:

The average degree of consolidation for the entire layer is defined as (18):

$$U_{ave}=\frac{(1/H_t)\int_0^{H_t} u_0 dz-(1/H_t)\int_0^{H_t} udz}{(1/H_t)\int_0^{H_t} u_0 dz}=\frac{A}{H_t.u_0} \qquad (16)$$

Where, u - excess pore water pressure at time t; u_0 - initial excess pore water pressure (t=0); A - area of the diagram pore water pressure dissipated; $H.u_0$ - area of the diagram initial pore water pressure (see Figure 2).

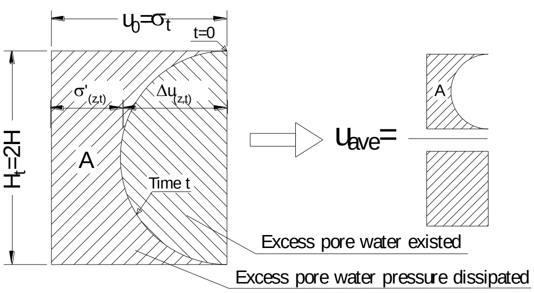

Figure 2. *Average degree of consolidation*

4. Application

In order to compare the averge degree of consolidation caculated with the Terzaghi's theory and non-linear theory, this study performed calculation for two different soft soils using the results of consolidition test by Abbasi et al (2007).

The soil layer has a thickness of 10m (drained at top and bottom), which is applied of uniform surcharge at the ground surface, $\Delta q=\sigma_t= 60kN/m^2$ (Figure 3).

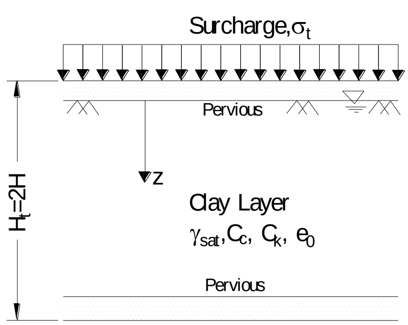

Figure 3. *Model of clay layer subjected to loading*

4.1. Soil properties

Physical and index properties of two types of soil, named S-1 and S-2, are given in Table 1.

Table 1. *Physical properties of samples*

Soil	Applied stress	Grain size distribute (%)			Atterberg limits		USCS
		Sand	Slit	Clay	LL (%)	PL (%)	
S-1	60	0	35	65	71	31	CH
S-2	60	13	67	20	30.5	22	CL

The linear relationships of e-Log(σ') and e-Log(k) for two soil samples tested by Row hydraulic consolidation cell are plotted in Figure 4 and Figure 5. The black diamond symbol expresses the S-1 soil (LL=71) and white triangle symbol expresses the S-2 soil (LL=30.5).

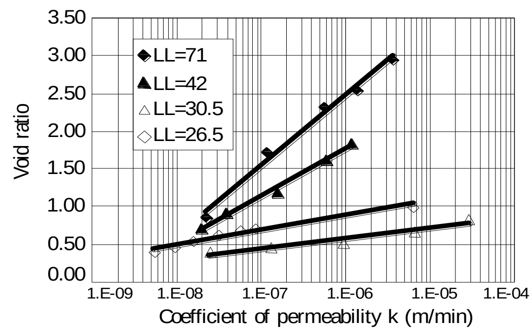

Figure 4. *Void ratio versus permeability*

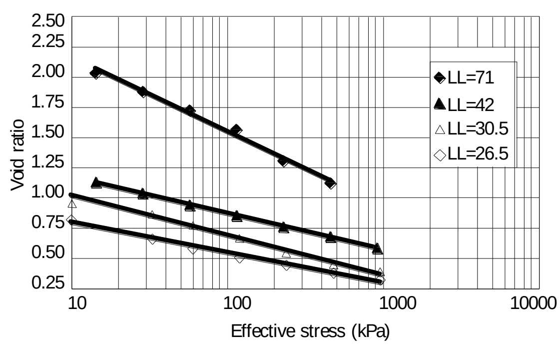

Figure 5. *Void ratio versus effective stress*

The permeability, compressibility and non-linearity charactistics of the studied samples are summarized in Table 2.

Table 2. *Non- linearity charactistic of samples*

Soil	Initial void	Compressibility characteristic		Permeability characteristic		Non-linearity coefficients	
		a	C_c	b	C_k	C_n	α
S-1	2.14	2.77	0.61	8.1	0.92	1.90E-06	0.34
S-2	0.83	1.36	0.33	2.71	0.29	2.82E-05	-0.14

4.2. Variation of coefficient of consolidation C_v with time and depth

Figure 6 shows the variations of excess pore water pressure at different depths over time of the soil named S-1 for a duration of 3000days. Because the layer of clay is free to drain at upper and lower boundaries, then the dissipation of excess pore water pressure at the top and bottom layer is faster than at midheight of the clay layer.

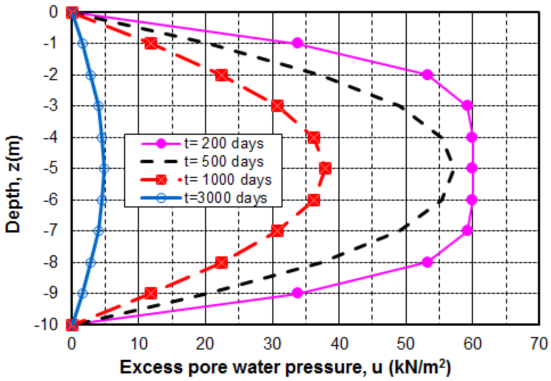

***Figure 6**. Excess pore water pressure variations during the consolidation (S-1 sample)*

According to equation (12), the coefficient of consolidation C_v depends on C_n, α. In adition it depend on variation of excess pore water pressure during consolidation. The Figure 7 shows the variations of C_v with space (depth) and time. The continuous lines represent the soil sample S-1 (with $\alpha=0.34$, $C_n=1.9\times10^{-6}$) and the discontinuous lines represent the soil sample S-2 (with $\alpha=-0.14$, $C_n=2.82\times10^{-5}$).

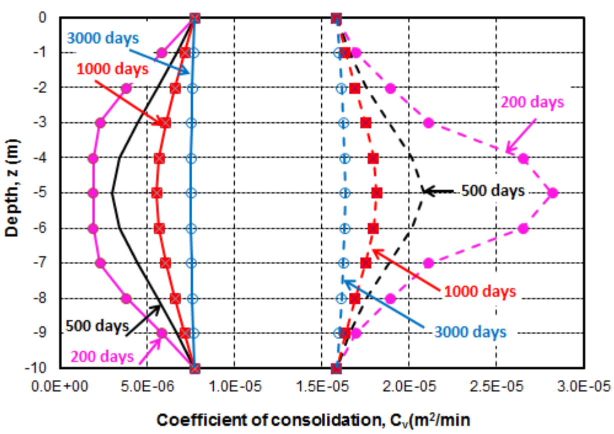

***Figure 7**. Typical variations of C_v with depth at different times*

The coefficient of consolidation (C_v) tends to increase with time for positive value of α (sample S-1) and to decrease for negative value of α (sample S-2). This can be explained by the following: when $\alpha>0$ (or $C_c<C_k$) permeability of soil increases at a faster rate compared to reduction in compressibility. Therefore, the coefficient of consolidation increases as the consolidation progresses. On the other hand, the coefficient of consolidation decreases when $\alpha<0$ (Abbasi et al., 2007).

Along the depth of the soil layer, typical variations of C_v depend the distribution of the excess pore water pressure (u) and applied stress (σ_t). The variation of C_v causes the changes in coefficient of permeability and volume compressibility, resulting from the changes in effective stress due to the decrease of excess pore water pressure in the consolidation process.

Figure 8 represents the typical variation of C_v with time at the midheight of soil layer for positive and negative values of α. In the case of $\alpha=0$, it is evident that the coefficient of consolidation is constant. This is the results of Terzaghi's solution.

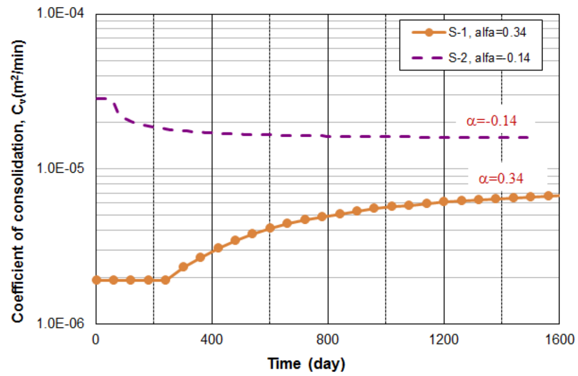

***Figure 8**. Typical variation of C_v with time for negative and positive value of α.*

4.3. Comparison the average degree of consolidation with conventional theory (Terzaghi's theory)

Figure 9 and figure 10 show the results obtained of the average degree of consolidation which are calculated according to the non-linear consolidation theory and Terzaghi's theory on the sample S-1 and S-2, respectively.

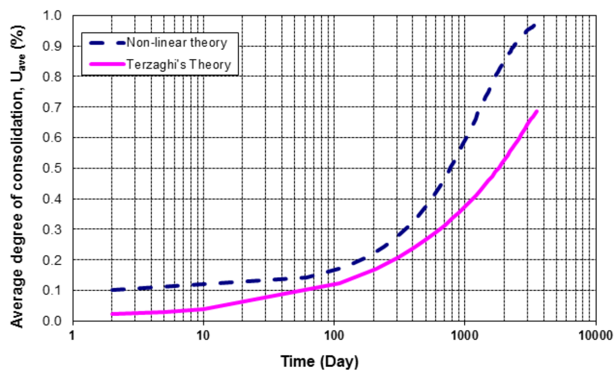

***Figure 9**. Average degree of consolidation (Sample S-1, α=0.34)*

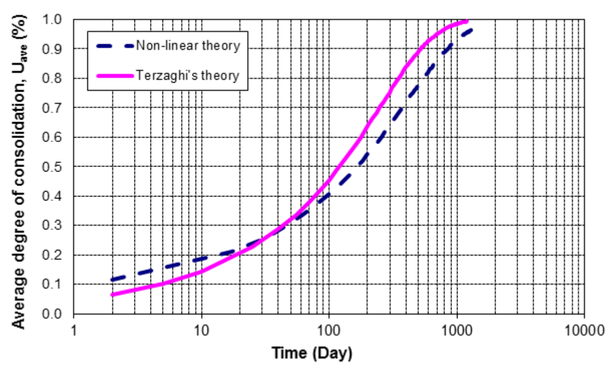

***Figure 11**. Average degree of consolidation (Sample S-2, α=-0.14)*

It has a significant difference between the average degree of consolidation according to the non-linear consolidation theory and Terzaghi's theory. In the case of $\alpha>0$, the U-log(t) curve predicted by non-linear theory is positioned over the curve predited by Terzaghi's theory

(Figure. 10). This implies that the consolidation will be faster than that predicted by Terzaghi's solution and the rate of consolidation increases with increasing α. For the negative value of α, the consolidation will be slower than that predicted by Terzaghi's solution.

5. Conclusion

This paper studies one-dimensional consolidation phenomenon considering the variation of C_v with time and space. Using finite difference method, the excess pore water pressure variations is determined based on solution of nonlinear differential equation for consolidation.

By solving the nonlinear differential equation, the average degree of consolidation can be calculated with real time and does not require time factor (T_v). The results show that the rate of consolidation according to nonlinear consolidation theory may be faster or slower than that caculated by Terzaghi's theory. It depends on ratio of (C_c/C_k) which is determined from the linear relationship e-log(σ′) and e-log(k).

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