STUDYING AND BUILDING PROGRAMME FOR SIMPLIFYING ELECTRICAL NETWORK DIAGRAM USING GAUSSIAN ELIMINATION ALGORITHM

Pham Van Kien[2], Ngo Van Duong[1], Le Dinh Duong[2], Huynh Van Ky[1]

[1]*The University of Danang; nvduong@ac.udn.vn*

[2] *University of Science and Technology, The University of Danang; pvkien@ac.udn.vn*

Abstract - Along with the development of the socio-economy in general and of science and technology in particular, the need for energy is increasing .Particularly, electrical energy plays a very important role. Load of power systems continuously develops and expands, leading to several difficulties in management and operation of power systems. In order to ensure security of power systems, during their operation, parameters of system mode in different operating conditions need to be calculated and assessed with respect to their limits. Thence, security of power systems can be evaluated and measures for enhancing system security can be provided. A methodology for calculating power system that satisfies both computation time and accuracy is extremely necessary. Gaussian elimination algorithm is very helpful in supporting such methods. This paper analyzes mathematical fundamentals and the application of Gaussian elimination algorithm to build programme for simplifying electrical network diagram that is useful to deal with serveral issues in power systems such as short-circuit calculation and stability analysis.

Key words - short-circuit; stability; equivalent diagram; state matrix; Gaussian elimination algorithm.

1. Introduction

In power system calculation and analysis, the physical network diagram is first replaced by impedance and admittance diagram in which each component of power systems is represented by an impedance or an admittance [1]. In order to perform stability analysis according to realistic criteria and perform short-circuit calculation occurring at some nodes in power systems, we have to do equivalent transformation of the diagram into one considered node. This equivalent transformation can be done with many different methods: combining series or parallel impedance; combining in parallel branches with power sources; performing star-delta transformation; combining and separating branches with power sources; duplicating symmetrical diagrams; performing star-mesh transformation; etc [2, 3, 4, 5]. These transformation methods can be applied for simple power systems or power systems with a small number of components. However, for more complicated power systems, the number of components is quite large, and the above mentioned transformation methods are no longer suitable since they are quite computationally challenging and the manual calculation leads to calculating errors exceeding recommended limits. To overcome this problem, Gaussian elimination algorithm is applied to compute for any complicated power systems. The advantage of this method is that it is based on state matrix of power systems to transform the matrix along with transforming the diagram, and hence it is convenient to build computational programme, which helps reduce computation time and increase accuracy.

2. Mathematical foundations of Gaussian elimination algorithm

Consider a simple two-node network as in Figure 1, at steady state, with the assumption that its equivalent diagram is linear and its parameters are constant.

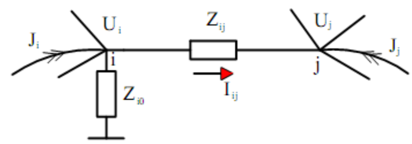

Figure 1. *The power network between nodes i and j*

Then, the current balance equation at node i, according to Kirchhoff I law, can be written as follows [1, 5, 6, 7, 8]:

$$\sum_{\substack{j=0 \\ j\neq i}}^{n} \dot{I}_{ij} = \dot{J}_i \quad \text{with } i=\overline{1,n} \tag{1-1}$$

In which, J_i is the current injection into bus i, and for an intermediate node $J_i = 0$. Applying Ohm's law for the branch impedance on Figure 1, equation (1-1) can be modified as:

$$\sum_{\substack{j=0 \\ j\neq i}}^{n} \dot{I}_{ij} = \sum_{\substack{j=0 \\ j\neq i}}^{n} \frac{\dot{U}_i - \dot{U}_j}{Z_{ij}} = \dot{J}_i, \quad \text{with } i=\overline{1,n}$$

$$\Leftrightarrow \sum_{\substack{j=0 \\ j\neq i}}^{n} \frac{1}{Z_{ij}} \dot{U}_i - \sum_{\substack{j=0 \\ j\neq i}}^{n} \frac{1}{Z_{ij}} \dot{U}_j = \dot{J}_i, \quad \text{with } i=\overline{1,n} \tag{1-2}$$

Let: $$Y_{ii} = \sum_{\substack{j=0 \\ j\neq i}}^{n} \frac{1}{Z_{ij}}; \quad Y_{ij} = -\frac{1}{Z_{ij}} \tag{1-3}$$

Replacing the admittance Y_{ii} and Y_{ij} from *(1-3)* to *(1-2)*, we have:

$$\Rightarrow \sum_{\substack{j=0 \\ j\neq i}}^{n} Y_{ij} \dot{U}_i - \sum_{\substack{j=0 \\ j\neq i}}^{n} Y_{ij} \dot{U}_j = \dot{J}_i, \quad \text{with } i=\overline{1,n} \tag{1-4}$$

where: Y_{ii}: self-admittance of node i;

Y_{ij}: mutual admittance between nodes i and j.

Hence, equation (1-4) is the state equation of the system. From this equation, we can build a set of state equations for any n-node power system which has F source nodes and (n-F) load nodes as follows [1, 5, 6, 7]:

$$\begin{cases} Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 + \ldots + Y_{1F}\dot{U}_F + \ldots + Y_{1n}\dot{U}_n = \dot{J}_1 \\ Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 + \ldots + Y_{2F}\dot{U}_F + \ldots + Y_{2n}\dot{U}_n = \dot{J}_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Y_{F1}\dot{U}_1 + Y_{F2}\dot{U}_2 + \ldots + Y_{FF}\dot{U}_F + \ldots + Y_{Fn}\dot{U}_n = \dot{J}_F \\ Y_{(F+1)1}\dot{U}_1 + Y_{(F+1)2}\dot{U}_2 + \ldots + Y_{(F+1)F}\dot{U}_F + \ldots + Y_{(F+1)n}\dot{U}_n = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Y_{n1}\dot{U}_1 + Y_{n2}\dot{U}_2 + \ldots + Y_{nF}\dot{U}_F + \ldots + Y_{nn}\dot{U}_n = 0 \end{cases} \tag{1-5}$$

The set of equations (1-5) can be used as a foundation for many steady state calculation problems such as calculating current distribution and generation power, calculating equivalent network diagram, determining equivalent admittance matrix of reduced network with only source nodes, etc.

The above equivalent transformation of a power network diagram into a simple diagram consisting of some source nodes and one load node can be done by using Gaussian elimination algorithm. Gaussian elimination algorithm is a process of gradually reducing the number of variables and number of equations. In other words, it is the process of eliminating intermediate nodes in the set of state equations (1-5). Firstly, to eliminate the variables U_n and the last equation, both sides of this equation is divided by Y_{nn}. The resulting equation is multiplied with the last coefficient of each of the remaining equations respectively and then subtracted from the corresponding equation. This subtraction of two equations with the same last component makes the variable U_n eliminated. The resulting set of equations has only (n-1) equations and (n-1) variables as seen in equation (1-6) [1, 5, 6]:

$$\begin{cases} Y'_{11}\dot{U}_1 + Y'_{12}\dot{U}_2 + \ldots + Y'_{1F}\dot{U}_F + \ldots + Y'_{1(n-1)}\dot{U}_{(n-1)} = \dot{J}_1 \\ Y'_{21}\dot{U}_1 + Y'_{22}\dot{U}_2 + \ldots + Y'_{2F}\dot{U}_F + \ldots + Y'_{2(n-1)}\dot{U}_{(n-1)} = \dot{J}_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y'_{F1}\dot{U}_1 + Y'_{F2}\dot{U}_2 + \ldots + Y'_{FF}\dot{U}_F + \ldots + Y'_{F(n-1)}\dot{U}_{(n-1)} = \dot{J}_F \\ Y'_{(F+1)1}\dot{U}_1 + Y'_{(F+1)2}\dot{U}_2 + \ldots + Y'_{(F+1)F}\dot{U}_F + \ldots + Y'_{(F+1)(n-1)}\dot{U}_{(n-1)} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y'_{(n-1)1}\dot{U}_1 + Y'_{(n-1)2}\dot{U}_2 + \ldots + Y'_{(n-1)F}\dot{U}_F + \ldots + Y'_{(n-1)(n-1)}\dot{U}_{(n-1)} = 0 \end{cases} \quad (1\text{-}6)$$

In the set of equations (1-6), the current sources J_1, J_2,… J_F remain the same since the right-hand side of the last equation is equal to zero. The transformation for each admittance Y'_{ij} in (1-6) is determined as [1, 5, 6]:

$$Y'_{ij} = Y_{ij} - \frac{Y_{nj}Y_{in}}{Y_{nn}} \qquad (1\text{-}7)$$

Generally, (1-7) can be rewritten as:

$$Y_{ij}^{k-1} = Y_{ij}^{k} - \frac{Y_{ik}^{k}.Y_{kj}^{k}}{Y_{kk}} \qquad (1\text{-}8) \qquad \text{with } k = \overline{n \rightarrow F}$$

Perform continuously the transformation according to expression (1-8) to eliminate all equations corresponding to all intermediate nodes (with the right-hand side equals to zero). The resulting simplified set of equations will have only F equations of source nodes as can be seen in (1-9), in which all voltage U_i are replaced by the electromotive force E_i [1, 5, 6].

$$\begin{cases} Y''_{11}\dot{E}_1 + Y''_{12}\dot{E}_2 + \ldots + Y''_{1F}\dot{E}_F = \dot{J}_1 \\ Y''_{21}\dot{E}_1 + Y''_{22}\dot{E}_2 + \ldots + Y''_{2F}\dot{E}_F = \dot{J}_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y''_{F1}\dot{E}_1 + Y''_{F2}\dot{E}_2 + \ldots + Y''_{FF}\dot{E}_F = \dot{J}_F \end{cases} \qquad (1\text{-}9)$$

Therefore, by applying continuous transformation according to expression (1-8), the complicated set of state equations (1-5) has been reduced to a much simpler set of state equations (1-9).

3. Applying Gaussian elimination algorithm to compute equivalent network diagram

The self-admittance Y_{ii} and mutual admittance Y_{ij} in the set of state equations (1-5) can be expanded using expression (1-10) and (1-11) as follows:

$$\begin{aligned} Y_{ii}^{(n)} &= \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1}{Z_{ij}} = G_{ii}^{(n)} + jB_{ii}^{(n)} \\ &= \sum_{j=0}^{n} \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} + j\sum_{j=0}^{n} \frac{-X_{ij}}{R_{ij}^2 + X_{ij}^2} \end{aligned} \qquad (1\text{-}10)$$

and

$$\begin{aligned} Y_{ij}^{(n)} &= -\frac{1}{Z_{ij}} = G_{ij}^{(n)} + jB_{ij}^{(n)} \\ &= -\frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} + j\frac{X_{ij}}{R_{ij}^2 + X_{ij}^2} \end{aligned} \qquad (1\text{-}11)$$

With the impedance Z_{ij} determined as: $Z_{ij} = R_{ij} + jX_{ij}$

Using Gaussian elimination algorithm according to (1-8), (n-(F+1)) intermediate nodes in the set of state equations (1-5) can be eliminated, only source nodes and one considered load node remains. The resulting simplified set of equations consists of (F+1) equations of the $(F+1)^{th}$ intermediate node and F source node as in the set of equations (1-12):

$$\begin{cases} Y_{11}^{(F+1)}\dot{U}_1 + Y_{12}^{(F+1)}\dot{U}_2 + \ldots + Y_{1F}^{(F+1)}\dot{U}_F + Y_{1(F+1)}^{(F+1)}\dot{U}_{(F+1)} = \dot{J}_1 \\ Y_{21}^{(F+1)}\dot{U}_1 + Y_{22}^{(F+1)}\dot{U}_2 + \ldots + Y_{2F}^{(F+1)}\dot{U}_F + Y_{2(F+1)}^{(F+1)}\dot{U}_{(F+1)} = \dot{J}_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_{F1}^{(F+1)}\dot{U}_1 + Y_{F2}^{(F+1)}\dot{U}_2 + \ldots + Y_{FF}^{(F+1)}\dot{U}_F + Y_{F(F+1)}^{(F+1)}\dot{U}_{(F+1)} = \dot{J}_F \\ Y_{(F+1)1}^{(F+1)}\dot{U}_1 + Y_{(F+1)2}^{(F+1)}\dot{U}_2 + \ldots + Y_{(F+1)F}^{(F+1)}\dot{U}_F + Y_{(F+1)(F+1)}^{(F+1)}\dot{U}_{(F+1)} = 0 \end{cases} \qquad (1\text{-}12)$$

The simplified matrix $Y^{(F+1)}$ is then formed as in (1-13):

$$Y^{(F+1)} = \begin{vmatrix} Y_{11}^{(F+1)} & Y_{12}^{(F+1)} & \ldots Y_{1F}^{(F+1)} & Y_{1(F+1)}^{(F+1)} \\ Y_{21}^{(F+1)} & Y_{22}^{(F+1)} & \ldots Y_{2F}^{(F+1)} & Y_{2(F+1)}^{(F+1)} \\ \ldots & \ldots & \ldots & \ldots \\ Y_{F1}^{(F+1)} & Y_{F2}^{(F+1)} & \ldots Y_{FF}^{(F+1)} & Y_{F(F+1)}^{(F+1)} \\ Y_{(F+1)1}^{(F+1)} & Y_{(F+1)2}^{(F+1)} & \ldots Y_{(F+1)F}^{(F+1)} & Y_{(F+1)(F+1)}^{(F+1)} \end{vmatrix} \qquad (1\text{-}13)$$

The simplified matrix $Y^{(F+1)}$ can be represented by a reduced equivalent diagram as in Figure 2.

In which, the impedance Z_{ij} are determined as:

$$\begin{aligned} Z_{ij} &= -\frac{1}{Y_{ij}} = R_{ij} + jX_{ij} = \frac{1}{G_{ij} + jB_{ij}} \\ &= \frac{G_{ij}}{G_{ij}^2 + B_{ij}^2} - j\frac{B_{ij}}{G_{ij}^2 + B_{ij}^2} \end{aligned} \qquad (1\text{-}14)$$

where $R_{ij} = -\frac{G_{ij}}{G_{ij}^2 + B_{ij}^2};\ X_{ij} = \frac{B_{ij}}{G_{ij}^2 + B_{ij}^2}$ (1-15)

Accordingly, with the method of calculating equivalent power network diagram using Gaussian elimination algorithm, we can simplify a complicated power network diagram into an equivalent simple diagram consisting of only source nodes and considered load nodes. The simplified diagram as in Figure 2 provides a tool for solving many problems related to short-circuit, stability analysis, etc.

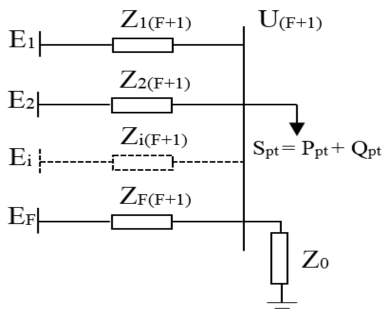

Figure 2. *Equivalent diagram reduced to the considered load node using Gaussian elimination algorithm*

4. Building programme to compute equivalent network diagram using Gaussian elimination algorithm

4.1. Flowchart of the algorithm

From the method of calculating equivalent network diagram using Gaussian elimination algorithm described above, we build the flowchart to calculate for any power system as in Figure 3.

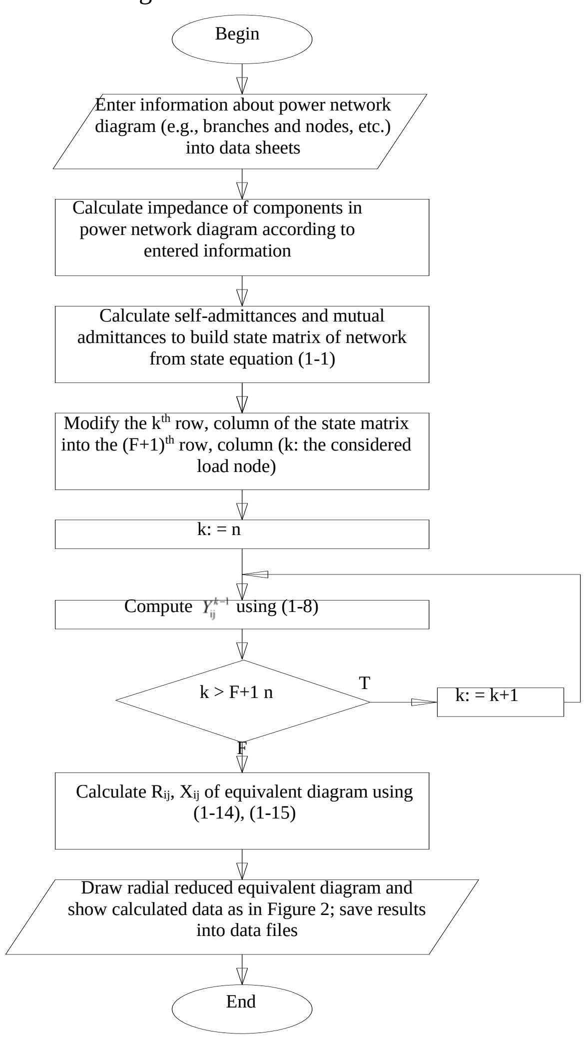

Figure 3. *Flowchart for calculating equivalent network diagram using Gaussian elimination algorithm*

The steps in the flowchart are explained below:

Step 1: Enter all necessary information such as parameters of generators, transformers, transmission lines, compensating capacitors, compensating reactors, and load. These data will be saved by the programme and will become a library of necessary parameters for the calculating process of the programme.

Step 2: From the network diagram, enter detailed information of all nodes into the table “Node information” and information of all branches into the table “Branch information”. From these node and branch data, the programme will automatically calculate all equivalent parameters of transmission lines, capacitors, reactors, transformers, load, etc., and will manage information of these parameters to serve the following calculating steps.

Step 3: From the calculated parameters in Step 2, the programme will automatically perform calculation according to the above expression (1-4) and determine the set of network state equations (1-5).

Step 4: From the set of state equations (1-5), the programme will determine the state matrix Y_{tt} and modify the k^{th} row, column of the matrix into the $(F+1)^{th}$ row, column (k: the considered load node).

Step 5: The programme will perform calculation according to (1-8), (1-14), (1-15), results are shown on the equivalent network diagram and on the corresponding data sheet. Calculated results of the equivalent network diagram will be saved.

4.2. Introduction to the programme

Based on the flowchart in Figure 3, we build the programme to compute equivalent network diagram of power systems with up to thousands nodes into simple radial network diagrams which can be used for solving problems related to short-circuit or stability analysis, etc. The starting interface window of the programme is shown in Figure 4.

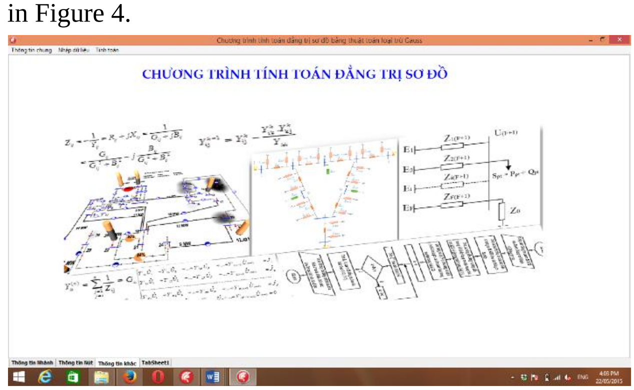

Figure 4. *The interface window of the programme*

The programme needs all the database of power system components such as parameters of generators, transformers, transmission lines, loads, compensating capacitors, compensating reactors. Therefore, to use the equivalent network diagram calculation function, this information of system components has to be entered into data sheets shown in Figure 5. The data is saved with the same database format as Microsoft Access.

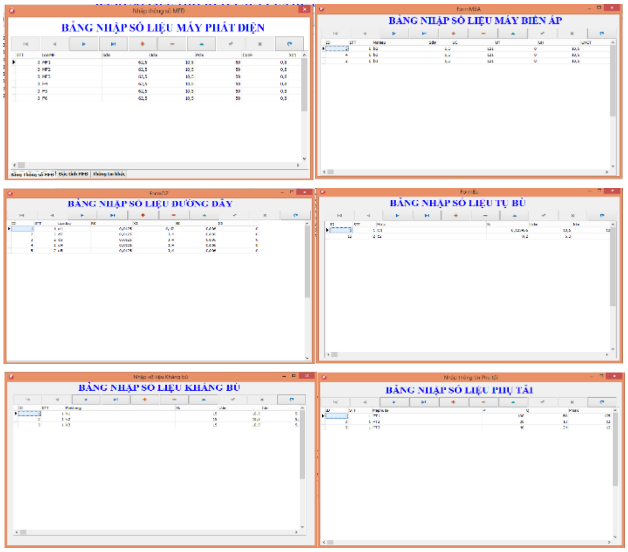

Figure 5. *The interface for entering information of power system components*

With the entered data, we proceed to enter information of branches and nodes of the system into branch information and node information data sheets as in figures 6 and 7.

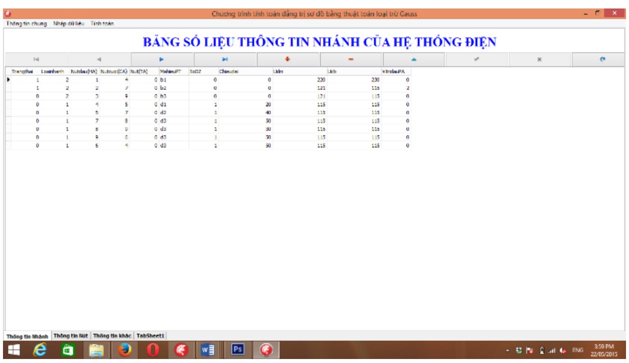

Figure 6. *Interface to enter branch information*

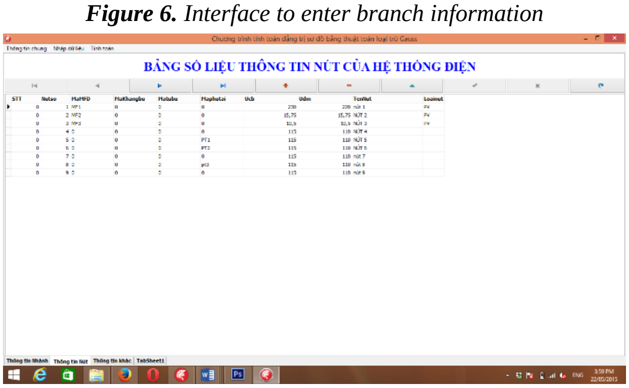

Figure 7. *Interface to enter node information*

Since the programme is built to facilitate users, the information to be entered is mostly codes which already exist in the library entered above. As a result, for data of power system components, only the corresponding codes should be entered and the programme will automatically search for and calculate the remaining parameters. This is a new point in current computational programmes.

After entering all necessary information of the power network, go to menu Calculation ☞ Calculate state matrix (Figure 8). Calculating results are shown as in Figure 9.

Figure 8. *Select the option to calculate equivalent network diagram*

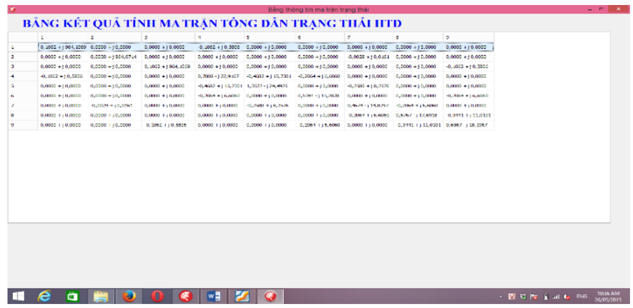

Figure 9. *Result sheet of calculating state matrix of the network*

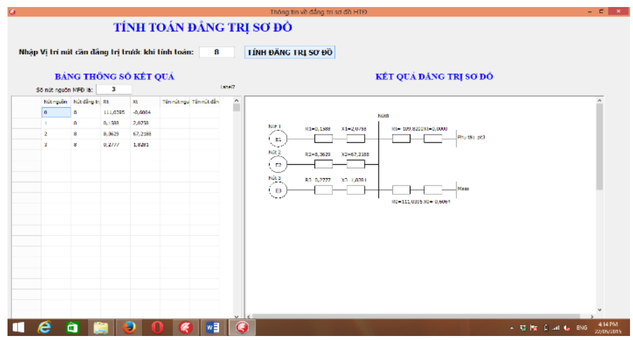

Figure 10. *Interface for calculating and showing results*

To calculate equivalent network diagram, go to menu Calculation ☞ Calculate equivalent network diagram (Figure 8). Enter value of the load node which needs to be calculated as in Figure 10 and press the button Calculate equivalent network diagram. The programme will perform the calculation and show results in the form of data sheets and equivalent network diagram.

5. Conclusions

Gaussian elimination algorithm can be applied to calculate equivalent network diagram for power systems; especially, for complicated power networks with many nodes, applying this algorithm can help systematize the computation process into an order.

This programme has many advantages compared to current calculating programmes:

- Steps to enter input parameters are very simple and fast. Especially, the programme will automatically look for information in the library to calculate equivalent parameters of power system components, hence, intermediate steps of manually processing data in advance are reduced to the maximum compared to other calculating softwares such as PSSE, Power World, PSS/ADEPT.

- The mathematical foundations of the programme are based on simple algebraic expressions, with less loops, thus computational speed is faster. This is a remarkable advantage of this programme for online applications that require high speed.

- The programme is tested on IEEE 9-bus system and gives the same results as those manually calculated with traditional methods.

REFERENCES

[1] Ngo Van Duong (2008), "Studying and building programme for supervising steady state stability for Vietnam Power System 2015", *Journal of Science and Technology, The University of Danang*, (3/2008), p. 32-39.

[2] La Van Ut (2014), *Short-circuit in power systems*, Scientific and Technical Publisher, Ha Noi.

[3] Nguyen Hoang Viet, Phan Thi Thanh Binh, Huynh Chau Duy (2005), *Short-circuit and stability in power systems*, Publisher of Vietnam National University, Ho Chi Minh City.

[4] Tran Bach (2001), *Stability in power systems*, Hanoi University of Science and Technology, Ha Noi.

[5] La Van Ut (2011), *Analysis and control for stability of power systems*, Scientific and Technical Publisher, Ha Noi.

[6] Nguyen Minh Ngoc (2004), *Building operating security region of transmission lines*, Master thesis, The University of Danang.

[7] C. W. Taylor (1994), *Power System Voltage Stability*, McGraw-Hill, New York.

[8] P. Kundur (1994), *Power System Stability and Control*, McGraw-Hill, New York.

(The Board of Editors received the paper on 06/07/2015, its review was completed on 06/23/2015)