BI-LEVEL OPTIMIZATION MODEL BASED EVALUATION OF WHOLESALE ELECTRICITY PRICE INTERVALS CONSIDERING THE WIND POWER UNCERTAINTY AND ELASTIC DEMAND

Pham Nang Van[1], Nguyen Duy Cung[2], Nguyen Duc Huy[1]

[1]Hanoi University of Science and Technology (HUST); van.phamnang@hust.edu.vn, huy.nguyenduc1@hust.edu.vn

[2]Student at Hanoi University of Science and Technology (HUST)

Abstract - In the electricity market operation, wholesale electricity prices or Locational Marginal Prices (LMP) vary according to electric demand (including consumption power consumed, bidding prices and the level of price-sensitivity) as well as the penetration level of the wind power. The variable domain identification of LMP plays a very important role for market participants to assess and mitigate the risk on account of the uncertainty of wind power output forecasting. Traditionally, the Monte Carlo simulation (MCS) method can be used in order to determine the variable intervals of LMP. However, in this paper, the authors deploy a bi-level optimization model to calculate the upper and lower bounds of LMP when considering the uncertainty of wind power and elastic demand. The objective function of the upper-level optimization problem is to maximize (or minimize) LMP at a node whereas the objective function of the lower-level optimization problems is to calculate the optimal power generation of the units participating in supplying the load.

Key words - Wholesale electricity market; mathematical program with equilibrium constraints (MPEC); mixed-integer linear programming (MILP); wind power uncertainty; elastic demand.

1. Introduction

Currently, many countries around the world, including Vietnam, have been operating wholesale electricity markets. In the wholesale electricity market, the market participants are generation companies (GENCOS) and distribution companies (DISCOS). The market operator collects generating offers by producers, load bids by consumers and clears the market by maximizing the social welfare [1]-[2].

The uncertainty from wind output has brought unprecedented challenges to the optimal operation of the electricity market. The power system operation has been dealing with the uncertainty of load, different from load uncertainty; however, wind output is characterized with large uncertainties and low prediction precision [3]. On the other hand, load demand has an intrinsic pattern and thus the load prediction, especially, in short-term, has a significantly high forecast accuracy [3]. Therefore, the optimal operation and dispatching model considering stochastic wind power output has been a hot topic for research.

Reference [4] studied the effect of wind integration and wind uncertainty on power system reliability, using an ARMA model to analyze short-term wind forecast. Reference [5] studied the impact of stochastic wind power on the unit commitment (UC) problem and constructed a UC stochastic optimization problem with the objective to minimize the expected operation cost. In reference [6], the influence of distribution generation on a heavily loaded distribution system with a wind forecast model based on statistics is tackled. A mixed-integer stochastic optimization model is established in [7] where the wind uncertainty is modeled with ARMA as well as Latin hypercube sampling and a scenario reduction method is adopted to simplify the computation.

The first step to investigate the effect of uncertainty is to model the uncertain wind output by using a variety of methods, for instance, probability distribution model [8], fuzzy model [9] and interval number model [10]. In the next steps, different optimization models are applied to find the solution.

To make payments in the electricity market, locational marginal prices (LMP) are calculated. The difference in LMPs between two nodes of a branch depends upon the congestion and losses on that branch [2]. The locational marginal pricing methodology is widely used in electricity markets to determine the electricity prices and to evaluate the transmission congestion cost [11]-[12]. Step change characterizes of LMP under system load variation has been identified and discussed [13]. Moreover, the concept of critical load level (CLL) is defined and employed for load frequency control [13]. Based on a similar idea, the investigation of the impact of variable wind power outputs on LMPs must be worth launching. It is important to find a method to efficiently obtain the wholesale electricity price intervals under the variation of wind power output and elastic demand.

This paper proposes an approach to determine the intervals of LMP using a bi-level optimization model, which is similar to the interval number-based optimization model regarded as the optimization of optimization. In addition, the impact of the uncertainty of wind power as well as the level for demand-bid price sensitivity is also analyzed.

The next sections of the article are organized as follows. In section 2, the authors present bi-level optimization model to determine LMP intervals. In addition, the authors also describe the solution to this bi-level optimization problem including the procedure of transferring it into a Mathematical Program with Equilibrium Constraints (MPEC) problem and the conversion from MPEC to a Mixed-Integer Linear Programming (MILP). Section 3 demonstrates the simulation results and numerical analyses of PJM 5-bus system. Some conclusions are given in section 4.

2. LMP interval under wind power uncertainty

2.1. Market clearing model

Economic Dispatch (ED) considering elastic demands in wholesale electricity market is carried out by Independent System Operators (ISOs) to clear market as well as determine LMPs and output of generating units. In this paper, the DCOPF-based approach without losses is employed to model the electricity market and estimate LMPs. This DCOPF is a linear programming (LP) problem presented as follows:

$$\min \sum_{i=1}^{N}\left(c_{Gi}P_{Gi}+c_{Wi}P_{Wi}-c_{Di}P_{Di}^{E}\right) \quad (1)$$

$$s.t \ \sum_{i=1}^{N}\left(P_{Gi}+P_{Wi}\right)=\sum_{i=1}^{N}\left(P_{Di}^{F}+P_{Di}^{E}\right):\lambda \quad (2)$$

$$-Limit_l \le \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}-P_{Di}^{E}\right) \le Limit_l : \mu_l^{\min}, \mu_l^{\max}, \forall l=\overline{1,M} \quad (3)$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} : \omega_i^{\min}, \omega_i^{\max}, \forall i=\overline{1,N} \quad (4)$$

$$0 \le P_{Wi} \le P_{Wi}^{\max} : \varphi_i^{\min}, \varphi_i^{\max}, \forall i=\overline{1,N} \quad (5)$$

$$0 \le P_{Di}^{E} \le P_{Di}^{E\max} : \phi_i^{\min}, \phi_i^{\max}, \forall i=\overline{1,N} \quad (6)$$

where N is the number of buses; M is the number of lines; c_{Gi} and c_{Wi} are energy prices offered by conventional generation and wind power, respectively; P_{Gi} and P_{Wi} are power outputs of the conventional generating unit and wind power, respectively; c_{Di} is the price bid by demand i; P_{Di}^{E} *and* P_{Di}^{F} are the elastic power and fixed power of demand i, respectively; GSF is the generation shift factor matrix; $P_{Gi}^{\min}$ and $P_{Gi}^{\max}$ are the upper and lower bounds of the convention generation output; $P_{Wi}^{\max}$ is the maximum available wind power output; $P_{Di}^{E\max}$ is the maximum price-sensitivity demand at bus i; and the variables on the right side of the colon are the dual variables associated with the equality and inequality constraints on the left.

The LMP at bus i can be calculated from the Lagrange function of the above ED problem. This function and LMP are given by

$$\begin{aligned}\psi = &\sum_{i=1}^{N}\left(c_{Gi}P_{Gi}+c_{Wi}P_{Wi}-c_{Di}P_{Di}^{E}\right)\\ &-\lambda\sum_{i=1}^{N}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}+P_{Di}^{E}\right)\\ &-\sum_{l=1}^{M}\mu_l^{\min}\left(\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}-P_{Di}^{E}\right)+Limit_l\right)\\ &-\sum_{l=1}^{M}\mu_l^{\max}\left(Limit_l-\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}-P_{Di}^{E}\right)\right)\\ &-\sum_{i=1}^{N}\omega_i^{\min}\left(P_{Gi}-P_{Gi}^{\min}\right)-\sum_{i=1}^{N}\omega_i^{\max}\left(P_{Gi}^{\max}-P_{Gi}\right)\\ &-\sum_{i=1}^{N}\varphi_i^{\min}P_{Wi}-\sum_{i=1}^{N}\varphi_i^{\max}\left(P_{Wi}^{\max}-P_{Wi}\right)\\ &-\sum_{i=1}^{N}\phi_i^{\min}P_{Di}^{E}-\sum_{i=1}^{N}\phi_i^{\max}\left(P_{Di}^{E\max}-P_{Di}^{E}\right)\end{aligned} \quad (7)$$

$$LMP_i=\frac{\partial\psi}{\partial P_{Di}^{F}}=\lambda+\sum_{l=1}^{M}GSF_{l-i}\left(\mu_l^{\min}-\mu_l^{\max}\right) \quad (8)$$

2.2. LMP interval and its bi-level optimization form

Traditionally, the intervals of LMP are usually evaluated using Monte Carlo Simulation (MCS) approach. However, this approach requires a huge amount of computation time in comparison with the bi-level optimization approach in terms of the same level of accuracy.

The problem for calculation LMP interval is formulated as follows:

$$Upper\ Level: \max LMP_i \left(or \min LMP_i\right) \quad (9)$$

$$s.t\ Lower\ level: ED\ optimization\ \mathrm{mod}el\ \left\{(1)-(6)\right\} \quad (10)$$

$$\underline{P_{wf,i}} \le P_{Wi}^{\max} \le \overline{P_{wf,i}} \quad (11)$$

where $\underline{P_{wf,i}}$ and $\overline{P_{wf,i}}$ is the forecast upper and lower bounds of the maximum wind power output. In other words, interval constraints are used to model the maximum wind power output in the upper level.

In the above formulation, we see that if there are N buses in the power system, $2N$ optimization runs should be carried out.

2.3. Formulation as a MPEC

Given that the lower level ED is an LP problem, the bi-level can be transformed into a Mathematical Program with Equilibrium Constraints (MPEC) by recasting the lower level problem as its Karush-Kuhn-Tucker (KKT) optimality conditions, which are added to the upper level problem as the additional complementary constraints [14] - [16]:

OBJ: (9) (12)

s.t. Constraints in (2) and (11) (13)

$$c_{Gi}=\lambda+\sum_{l=1}^{M}GSF_{l-i}\left(\mu_l^{\min}-\mu_l^{\max}\right)+\left(\omega_i^{\min}-\omega_i^{\max}\right) \quad (14)$$

$$c_{Wi}=\lambda+\sum_{l=1}^{M}GSF_{l-i}\left(\mu_l^{\min}-\mu_l^{\max}\right)+\left(\varphi_i^{\min}-\varphi_i^{\max}\right) \quad (15)$$

$$c_{Di}=\lambda+\sum_{l=1}^{M}GSF_{l-i}\left(\mu_l^{\min}-\mu_l^{\max}\right)-\left(\phi_i^{\min}-\phi_i^{\max}\right) \quad (16)$$

$$0 \le \mu_l^{\min} \perp Limit_l+\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}-P_{Di}^{E}\right) \ge 0 \quad (17)$$

$$0 \le \mu_l^{\max} \perp Limit_l-\sum_{i=1}^{N}GSF_{l-i}\left(P_{Gi}+P_{Wi}-P_{Di}^{F}-P_{Di}^{E}\right) \ge 0 \quad (18)$$

$$0 \le \omega_i^{\min} \perp P_{Gi}-P_{Gi}^{\min} \ge 0 \quad (19)$$

$$0 \le \omega_i^{\max} \perp P_{Gi}^{\max}-P_{Gi} \ge 0 \quad (20)$$

$$0 \le \varphi_i^{\min} \perp P_{Wi} \ge 0 \quad (21)$$

$$0 \le \varphi_i^{\max} \perp P_{Wi}^{\max}-P_{Wi} \ge 0 \quad (22)$$

$$0 \le \phi_i^{\min} \perp P_{Di} \ge 0 \quad (23)$$

$$0 \le \phi_i^{\min} \perp P_{Di}^{E\max}-P_{Di}^{E} \ge 0 \quad (24)$$

2.4. Mixed-Integer Linear Programming (MILP)

The MPEC model depicted in (12) – (24) is nonlinear on account of the slack complementarity constraints (17) – (24). These slack complementary constraints are compactly written as $0 \le \mathrm{F}(\mathrm{x}) \perp \mathrm{x} \ge 0$, which is stated equivalently in vector form as:

$$\mathrm{F}(\mathrm{x}) \ge 0,\ \mathrm{x} \ge 0,\ \mathrm{F}(\mathrm{x})^{\mathrm{T}}\mathrm{x} = 0 \tag{25}$$

With the method in [15], however, this MPEC problem can be converted to a mixed-integer linear programming (MILP), which can be solved by CPLEX [18]. The MILP model is presented as follows:

$$\text{OBJ: (9)} \tag{26}$$

$$\text{s.t. Constraints in (13), (14), (15) and (16)} \tag{27}$$

$$0 \le \mu_l^{\min} \le M_\mu^{\min} \nu_{\mu,l}^{\min} \tag{28}$$

$$0 \le Limit_l + \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi} + P_{Wi} - P_{Di}^F - P_{Di}^E\right) \le M_\mu^{\min}\left(1 - \nu_{\mu,l}^{\min}\right) \tag{29}$$

$$0 \le \mu_l^{\max} \le M_\mu^{\max} \nu_{\mu,l}^{\max} \tag{30}$$

$$0 \le Limit_l - \sum_{i=1}^{N} GSF_{l-i}\left(P_{Gi} + P_{Wi} - P_{Di}^F - P_{Di}^E\right) \le M_\mu^{\max}\left(1 - \nu_{\mu,l}^{\max}\right) \tag{31}$$

$$0 \le \omega_i^{\min} \le M_\omega^{\min} \nu_{\omega,\mathrm{i}}^{\min} \tag{32}$$

$$0 \le P_{Gi} - P_{Gi}^{\min} \le M_\omega^{\min}\left(1 - \nu_{\omega,i}^{\min}\right) \tag{33}$$

$$0 \le \omega_i^{\max} \le M_\omega^{\max} \nu_{\omega,\mathrm{i}}^{\max} \tag{34}$$

$$0 \le P_{Gi}^{\max} - P_{Gi} \le M_\omega^{\max}\left(1 - \nu_{\omega,i}^{\max}\right) \tag{35}$$

$$0 \le \varphi_i^{\min} \le M_\varphi^{\min} \nu_{\varphi,\mathrm{i}}^{\min} \tag{36}$$

$$0 \le P_{Wi} \le M_\varphi^{\min}\left(1 - \nu_{\varphi,i}^{\min}\right) \tag{37}$$

$$0 \le \varphi_i^{\max} \le M_\varphi^{\max} \nu_{\varphi,\mathrm{i}}^{\max} \tag{38}$$

$$0 \le P_{Wi}^{\max} - P_{Wi} \le M_\varphi^{\max}\left(1 - \nu_{\varphi,i}^{\max}\right) \tag{39}$$

$$0 \le \phi_i^{\min} \le M_\phi^{\min} \nu_{\phi,\mathrm{i}}^{\min} \tag{40}$$

$$0 \le P_{Di}^E \le M_\phi^{\min}\left(1 - \nu_{\phi,i}^{\min}\right) \tag{41}$$

$$0 \le \phi_i^{\max} \le M_\phi^{\max} \nu_{\phi,\mathrm{i}}^{\max} \tag{42}$$

$$0 \le P_{Di}^{\mathrm{E}\max} - P_{Di}^E \le M_\phi^{\max}\left(1 - \nu_{\phi,i}^{\max}\right) \tag{43}$$

where $M_\mu^{\min}, M_\mu^{\max}, M_\omega^{\min}, M_\omega^{\max}, M_\varphi^{\min}, M_\varphi^{\max}, M_\phi^{\min}$, $M_\phi^{\max}$ are large enough constants and $\nu_{\mu,l}^{\min}, \nu_{\mu,l}^{\max}, \nu_{\omega,i}^{\min}$, $\nu_{\omega,i}^{\max}, \nu_{\varphi,i}^{\min}, \nu_{\varphi,i}^{\max}, \nu_{\phi,i}^{\min}, \nu_{\phi,i}^{\max}$ are the auxiliary binary variables [14].

3. Results and discussions

In this section, the bi-level optimization approach is performed on the modified PJM 5-bus system [17]. The MILP problem is solved by CPLEX 12.7 under MATLAB environment.

3.1. System data

The test system is modified from the PJM 5-bus system [10], as shown in Figure 1. Two wind plants (WF1 and WF2) with the same capacity are added into the system at buses A and C while one original generator is removed from bus A. The total fixed and maximum elastic demand is 1200 MW equally distributed among buses B, C and D.

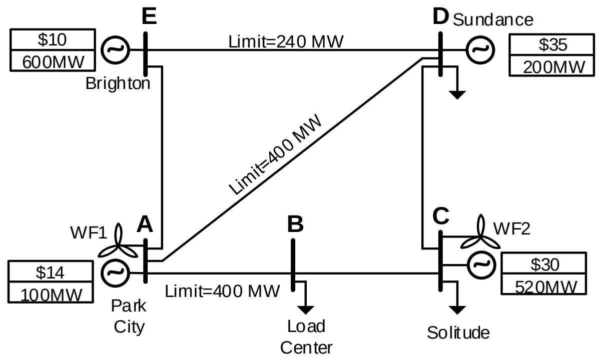

Figure 1. *PJM 5-bus system with two wind farms*

3.2. Impact from wind power uncertainty

This subsection shows the impact of wind power forecast uncertainty on LMP interval. The simulation parameters are shown in Table 1. In addition, the only fixed demand is considered in this subsection.

Table 1. *Uncertain parameters of wind power*

Wind Power Model	Normal Distributed
WF wind power mean (MW)	180
WF2 wind power mean (MW)	180
Wind power standard deviation range (%)	0-30%

It should be emphasized that the findings calculated in this work are exactly the same in comparison with the MCS method (with 5000 samples), while the simulation time for bi-level optimization-based approach is dramatically lower than that of MCS. Table 2 shows results achieved across all buses from both approaches when the standard deviation equals 15% from the mean.

Table 2. *LMP result intervals from MCS method and Bi-level optimization method*

Bus	Bi-vel optimization method	MCS method
A	[15.24, 16.98]	[15.24, 16.98]
B	[23.68, 28.18]	[23.68, 28.18]
C	[26.70, 30.00]	[26.70, 30.00]
D	[35.00, 39.94]	[35.00, 39.94]
E	[10, 10]	[10, 10]

Table 3 shows LMP intervals when the indicator of wind forecast uncertainty (σ) changes. According to the results shown in Table 3, in general, when the standard deviation of forecasting wind power increases, the difference between the upper and lower bound of LMP at every bus also rises. Moreover, these results also reveals that despite the variation of wind power uncertainty, the LMP intervals at bus C remain unchanged.

3.3. Impact from Demand-Bid Price Sensitivity

To investigate the influence of variation in the level of demand price sensitivity, [11] proposed the coefficient which is defined as Eq. (43).

$$R_i = \frac{P_{Di}^{E\max}}{P_{Di}^{F} + P_{Di}^{E\max}} \qquad (44)$$

From the formula (43), the R values range from 0.0 (100% fixed demand) to 1.0 (100% price-sensitive demand).

Figure 2 illustrates the construction of R for the special cases R = 0.0, R = 0.5 and R = 1.0.

In this paper, it is assumed that the ratio R of four demands is similar. Additionally, the forecast mean value of both wind powers is 180 MW and it follows a normal distribution with a standard deviation of 10% from the mean.

Table 4 presents the LMP internal results with the various R values from the proposed approach. According to this Table, the R ratio markedly affects the locational marginal prices at every bus in terms of the maximum and minimum values as well as the difference between these two items. In particular, when R varies from 0.4 to 0.6, there is a decline in the maximum and the minimum values of LMP at bus D; however, these values of LMP at bus E grows considerably. Furthermore, when the R ratio equals 0.6, 0.8 and 1.0, respectively, the LMP intervals remain stable at all buses.

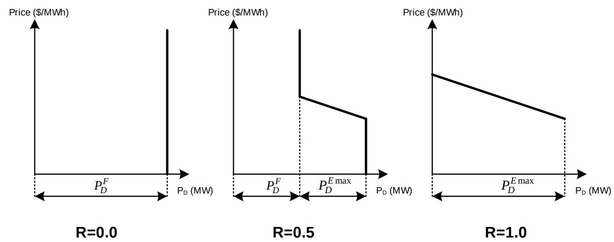

Figure 2. *Illustration of the R ratio construction for the experimental control of demand-bid price sensitivity*

Table 3. *LMP interval results with the various standard deviation*

Bus	LMP interval						
	σ = 0%	σ = 5%	σ = 10%	σ = 15%	σ = 20%	σ = 25%	σ = 30%
A	[15.24, 15.24]	[15.24, 15.83]	[15.24, 16.98]	[15.24, 16.98]	[15.24, 23.45]	[15.24, 23.45]	[15.24, 23.45]
B	[28.18, 28.18]	[23.68, 28.18]	[23.68, 28.18]	[23.68, 28.18]	[23.68, 28.18]	[23.68, 28.18]	[23.68, 28.18]
C	[30, 30]	[26.70, 30.00]	[26.70, 30.00]	[26.70, 30.00]	[26.70, 30.00]	[26.70, 30.00]	[26.70, 30.00]
D	[35, 35]	[35, 35]	[35.00, 39.94]	[35.00, 39.94]	[35.00, 39.94]	[35.00, 39.94]	[35.00, 39.94]
E	[10, 10]	[10, 10]	[10, 10]	[10, 10]	[10.00, 19.94]	[10.00, 19.94]	[10.00, 19.94]

Table 4. *LMP interval results with R different demand-bid price sensitivity*

Bus	LMP interval					
	R = 0	R = 0.2	R = 0.4	R = 0.6	R = 0.8	R = 1
A	[15.24, 16.98]	[15.65, 15.83]	[14.00, 15.23]	[14, 14]	[14, 14]	[14, 14]
B	[23.68, 28.18]	[23.68, 25.00]	[19.39, 22.29]	[18.51, 25.00]	[18.51, 25.00]	[18.51, 25.00]
C	[26.70, 30.00]	[26.70, 27.67]	[21.47, 25.00]	[20.24, 25.00]	[20.24, 25.00]	[20.24, 25.00]
D	[35.00, 39.94]	[35, 35]	[27.17, 32.46]	[25, 25]	[25, 25]	[25, 25]
E	[10, 10]	[10, 10]	[10, 10]	[10.66, 11.68]	[10.66, 11.68]	[10.66, 11.68]

4. Conclusion

This paper presents an approach to determine the intervals of locational marginal prices (LMPs) based on bi-level optimization model. Moreover, authors also present the conversion of this model to a mathematical program with equilibrium constraints (MPEC), then to a mixed-integer linear programming (MILP), which can be easily solved by available software tools. The results of this bi-level optimization problem reveal that the wind uncertainty and the demand-bid price sensitivity level have a remarkable impact on LMP intervals. In the computational aspect, the bi-level optimization-based method is more efficient compared with Monte-Carlo simulations although the calculated results using both approaches are identical.

REFERENCES

[1] Pham Nang Van, Nguyen DucHuy, Nguyen Van Duong, Nguyen The Huu, "A tool for unit commitment schedule in day-ahead pool-based electricity markets", *Journal of Science and Technology, The University of Danang,* no. 6, pp. 21-25, June, 2016.

[2] Pham Nang Van, Nguyen Dong Hung, Nguyen Duc Huy, "The impact of TCSC on transmission costs in wholesale power markets considering bilateral transactions and active power reserves", *Journal of Science and Technology, The University of Danang*, no. 12, pp. 24-28, December, 2016.

[3] Zou J, Lai X, Wang N, Time series model of stochastic wind power generation, *Power System Technology,* pp. 2416-2421, 2014.

[4] Zhang Y, Ka WC, "The impact of wind forecasting in power system reliability", *Third international electric utility deregulation and restructuring and power technologies,* Nanjing.

[5] Tuohy A, Meibom P, Denny E, "Unit commitment for systems with significant wind penetration", *IEEE Trans. Power Syst*, vol. 24, pp. 592-601, 2009.

[6] Kroposki B, Sen PK, Malmedal K, “Selection of distribution feeders for implementing distributed and renewable energy applications”, *IEEE Rural electric power conference,* Fort Collins, CO, 2009.

[7] Yan Y, Wen F, Yang S, “Generation scheduling with fluctuating wind power”, *Automation of Electric Power Systems,* vol. 34, pp. 79-88, 2010.

[8] Juan M. Morales, Antonio J. Conejo and Juan Perez-Ruiz, “Simulating the impact of wind production on locational marginal prices”, *IEEE Trans. Power Systems*, vol. 26, no. 2, pp. 820-828, May 2011.

[9] Hu Y, *Study on wind power storage technology and the intelligent dispatch model,* Master thesis, Lanzhou University of Technology, 2013.

[10] Xin Fang, Qinran Hu, Fangxing Li, Beibei Wang and Yang Li, “Coupon-based demand response considering wind power uncertainty: a strategic bidding model for load serving entities”, *IEEE Trans. Power Syst.,* vol. 31, no. 5, pp.1025-1037, May 2016.

[11] Hongyan Li, Leigh Tesfatsion, “ISO Net surplus collection and allocation in wholesale power markets under LMP”, *IEEE Trans. Power Systems,* vol. 26, pp. 627-641, April 2011.

[12] V. Sarkar and S. A. Khaparde, “Optimal LMP Decomposition for the ACOPF Calculation”, *IEEE Trans. Power Systems*, vol. 26, pp. 1714-1723, May 2011.

[13] F. Li and R. Bo, “Congestion and price prediction under load variation”, *IEEE Trans. Power Syst.,* vol. 24, no. 2, pp. 911-922, May 2009.

[14] L.Baringo and A. J. Conejo, “Strategic offering for a wind power producer”, *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp.1645-1654, November 2013.

[15] Zhi-Quan Luo, Jong-Shi Pang and Daniel Ralph, *Mathematical Programs with Equilibrium constraints,* Cambridge University Press, 2004.

[16] R.C.Bansal, *Optimization Methods for Electric Power Systems: An Overview,* International Jounal of Emerging Electric Power Systems, 2005.

[17] Fangxing Li, Rui Bo, “Small test systems for Power System Economic Studies”, *IEEE PES General Meeting,* pp. 1-4, 2010.

[18] CPLEX optimization studio. [Online] Available: https://www.ibm.com/bs-en/marketplace/ibm-ilog-cplex

(The Board of Editors received the paper on 25/4/2017, its review was completed on 15/5/2017)