DYNAMIC BEHAVIORS OF A SINGLE FLEXIBLE LINK MANIPULATOR UNDER DIFFERENT DRIVING RULES

Bien Xuan Duong[1], My Anh Chu[1], Lac Van Duong[2]

[1]*Military Technical Academy; xuanbien82@yahoo.com*

[2]*Hanoi University of Science and Technology; duonglacbk@gmail.com*

Abstract - In this article, nonlinear dynamic modeling and investigation into the effects of different driving rules on dynamic behavior of a single flexible link manipulator are presented. Dynamic equations are derived from finite element method based on Lagrange approach. The effects of elastic displacements on robot motion make dynamic modeling and control become complicated by highly nonlinear characteristics. Besides, most studies have not clearly determined driving rule or the reason why the driving rule is selected. The results have important meanings to select reasonable driving rule for single flexible link manipulator with payload or not. Dynamic behaviors of the system are simulated like a specific example on Matlab/Simulink software.

Key words - Flexible link manipulator; nonlinear dynamic; elastic displacements; driving rules; dynamic behaviors.

1. Introduction

The elastic displacements engendered by flexible manipulator's motion are the main reason for position accuracy, velocity, stability and others which are hard to determine exactly because of nonlinear characteristics of model. A number of researchers tried to study on the behavior of the flexible manipulator by using linearization methods assumed as small deflections, small hub angle. Tokhi [1] presented theoretical and experimental investigations into the dynamic characterization of a single flexible link manipulator system with dynamic model developed using finite element method. Gavriloiu [2] proved that linear control theory is not generally suitable for manipulator system with flexible link. On the other hand, many researchers focused on building control system with optimal parameters. Based on the dynamic model, Wang [3, 4] proposed a technique to investigate the maximum load carrying capacity of a robot manipulator given a dynamic robot trajectory. Korayem [5] studied on the dynamic load carrying capacity of a two-link flexible manipulator using finite element method and Pontryagin's minimum principle. Gee and Lee [6] discussed the modeling and nonlinear dynamic equations of a one-flexible link manipulator based on finite element method in Lagrange approach with varied payload. Reddy [7] presented the nonlinear modeling of a one-link flexible manipulator based on AMM and LS-DYNA. The results of both are compared. The numerical simulation shows that the LS-DYNA model gives the smooth hub-angle profile. In fact, the dynamic equations of systematic complex under effect of hub inertial, mass of rotors, payload, structural damping, coriolis forces and driving rules. It is noteworthy that effects of driving rules on dynamic behavior of flexible link manipulator are not much mentioned. Most studies have not clearly determined driving rule or the reason why any driving rule is selected. The dynamic characteristics of model are not general.

In this work, the nonlinear dynamic equations of one-link flexible manipulators which motion on horizontal plane are presented by using finite element method in Lagrange approach. The factors could be taken into account as hub inertia, payload, structural damping, coriolis and centrifugal in the dynamic equations. The effect of Coulomb friction and gravity are ignored because manipulator motions on horizontal plane. Dynamic behaviors of single flexible link manipulators are studied under different driving rules.

2. Dynamic modeling

2.1. Finite element method

The finite element method is a numerical method which can be used for the accurate solution of complex engineering problems. The FEM has also been utilized to describe the behaviour of flexible manipulators [6]. The flexible link is considered as an assemblage of a finite number of small elements. The elements are assumed interconnected at certain points, known as nodes. For each finite element, the scalar kinetic and potential energy functions are formulated as functions of the generalized coordinate. The dynamic model of a one-link flexible manipulator is depicted in Figure 1.

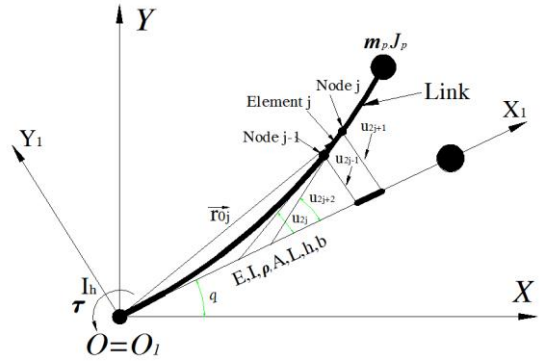

Figure 1. *Schematic diagram of a one-link flexible manipulator*

In Figure 1, the symbol q is the angle of rotation at the hub. The link is assumed as Euler-Bernoulli's beam and is divided into n elements. Each element has two nodes. Each node of element j has 2 elastic displacement variables which are the flexural (u_{2j-1}, u_{2j+1}) and the slope displacements (u_{2j}, u_{2j+2}). The coordinate system $\mathbf{X}_1\mathbf{O}_1\mathbf{Y}_1$ is attached to link. Symbols E, I and ρ are mass density, Young's modulus, inertial moment of area of link and total length of link (L), thickness (h), width (b) and cross sectional area (A) of link. The symbol τ is the applied

torque at the joint and motor inertial moment is I_h. The vector from O to a point on j element in the coordinate system is $\boldsymbol{r}_{01j}$. Symbols m_P and J_P are the mass and inertial moment of payload on the end point of link. The material of link is assumed homogeneity. Link is divided into n elements, the length of any element (l_j). The lengths of elements are equal because the cross-sectional areas of links are constant along length. Total elastic displacements of j element with (x_j, y_j) coordinate on $\mathbf{X_1O_1Y_1}$ [1].

$$\mathbf{w}_j(x_j,t) = \boldsymbol{N}_j(x_j)\boldsymbol{Q}_j(x_j,t) \quad (1)$$

Where vector of shape function $N_j(x_j)$ and $Q_j(x_j,t)$ is vector of elastic displacement of element j in FEM theory.

$$\boldsymbol{N}_j(x_j) = \begin{bmatrix}\phi_1(x_j) & \phi_2(x_j) & \phi_3(x_j) & \phi_4(x_j)\end{bmatrix} \quad (2)$$

$$\boldsymbol{Q}_j(x_j,t) = \begin{bmatrix}u_{2j-1}(t) & u_{2j}(t) & u_{2j+1}(t) & u_{2j+2}(t)\end{bmatrix}^T \quad (3)$$

Shape functions of j element $\phi_1(x_j)$ are given by [4]

$$\phi_1(x_j) = 1 - \frac{3x_j^2}{l_j^2} + \frac{2x_j^3}{l_j^3}; \phi_2(x_j) = x_j - \frac{2x_j^2}{l_j} + \frac{x_j^3}{l_j^2} \quad (4)$$

$$\phi_3(x_j) = \frac{3x_j^2}{l_j^2} - \frac{2x_j^3}{l_j^3}; \phi_4(x_j) = \frac{x_j^3}{l_j^2} - \frac{x_j^2}{l_j} \quad (5)$$

Position vector $\boldsymbol{r}_{1j}$ on $\mathbf{X_1O_1Y_1}$ and vector $\boldsymbol{r}_{0j}$ of j element on **XOY** are expressed as

$$\mathbf{r}_{1j} = \begin{bmatrix}(j-1)l_j + x_j \\ w_j(x_j,t)\end{bmatrix} \quad (6)$$

$$\mathbf{r}_{0j} = \mathbf{T}_0^1 \mathbf{r}_{1j} \quad (7)$$

Where transformation matrix $\mathbf{T}_0^1$ form $\mathbf{X_1O_1Y_1}$ to **XOY** is given by [1]

$$\mathbf{T}_0^1 = \begin{bmatrix}\cos q & -\sin q \\ \sin q & \cos q\end{bmatrix} \quad (8)$$

The elastic kinetic energy of j element is determined as

$$\boldsymbol{T}_j = \frac{1}{2}\int_0^{l_j} \rho A \left[\frac{\partial \boldsymbol{r}_{0j}}{\partial t}\right]^2 dx_j \quad (9)$$

If vector $\boldsymbol{Q}_j$ is generalized vector of elastic displacement of element and as defined

$$\boldsymbol{Q}_j(t) = \begin{bmatrix}q & u_{2j-1} & u_{2j} & u_{2j+1} & u_{2j+2}\end{bmatrix}^T \quad (10)$$

Then Eq. (6) can be rewritten as

$$\boldsymbol{T}_j = \frac{1}{2}\dot{\boldsymbol{Q}}_j^T \boldsymbol{M}_j \dot{\boldsymbol{Q}}_j \quad (11)$$

And each element of inertial mass matrix $\boldsymbol{M}_j$ can be computed as

$$\boldsymbol{M}_j(m,e) = \int_0^{l_j} \rho A \left[\frac{\partial \boldsymbol{r}_{0j}}{\partial \boldsymbol{Q}_{jm}}\right]^T \left[\frac{\partial \boldsymbol{r}_{0j}}{\partial \boldsymbol{Q}_{je}}\right] dx_j \quad (12)$$

$(m,e = 1,2,..,5)$

Besides, the elastic potential energy of j element is shown as $\mathbf{P}_j$ and presented as

$$\mathbf{P}_j = \frac{1}{2}\int_0^{l_j} EI \left[\frac{\partial^2 \mathbf{w}_j(x_j,t)}{\partial x_j^2}\right]^2 dx_j = \frac{1}{2}\mathbf{Q}_j^T(t)\mathbf{K}_j\mathbf{Q}_j(t) \quad (13)$$

The stiffness matrix $\mathbf{K}_j$ is defined as [4]

$$\boldsymbol{K}_j = \frac{EI}{l_j^3}\begin{bmatrix}0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 6l_j & -12 & 6l_j \\ 0 & 6l_j & 4l_j^2 & -6l_j & 2l_j^2 \\ 0 & -12 & -6l_j & 12 & -6l_j \\ 0 & 6l_j & 2l_j^2 & -6l_j & 4l_j^2\end{bmatrix} \quad (14)$$

Vector $\mathbf{Q}$ represents the generalized coordinate overall system and is given as

$$\mathbf{Q} = \begin{bmatrix}q_1 & u_1 & u_2 & . & . & u_{2n+1} & u_{2n+2}\end{bmatrix}^T \quad (15)$$

The total kinetic and potential energy of elastic on link can be written as

$$\boldsymbol{T}_{dh} = \sum_{j=1}^{n}\boldsymbol{T}_j = \frac{1}{2}\dot{\boldsymbol{Q}}^T(t)\boldsymbol{M}_{dh}\dot{\boldsymbol{Q}}(t) \quad (16)$$

$$\boldsymbol{P} = \sum_{j=1}^{n}\boldsymbol{P}_j = \frac{1}{2}\boldsymbol{Q}^T(t)\boldsymbol{K}\boldsymbol{Q}(t) \quad (17)$$

Where $\boldsymbol{M}_{dh}$ and $\boldsymbol{K}$ are total inertial mass and stiffness matrices. Each element of them is computed by FEM theory [1].

2.2. Dynamic equations

Fundamentally, the method relies on the Lagrange equations with Lagrange function $\mathbf{L} = \mathbf{T} - \mathbf{P}$ are given by

$$\frac{d}{dt}\left(\frac{\partial \mathbf{L}}{\partial \dot{\mathbf{Q}}}\right) - \frac{\partial \mathbf{L}}{\partial \mathbf{Q}} = \mathbf{F}(t) \quad (18)$$

Where $\mathbf{T}$ is the kinetic energy overall system. It is given by

$$\boldsymbol{T} = \boldsymbol{T}_{dh} + \boldsymbol{T}_{dc} + \boldsymbol{T}_P = \frac{1}{2}\dot{\boldsymbol{Q}}^T(t)\boldsymbol{M}\dot{\boldsymbol{Q}}(t) \quad (19)$$

Kinetic energy of rotor $\boldsymbol{T}_{dc}$ is determined as

$$\boldsymbol{T}_{dc} = \frac{1}{2}I_h\dot{q}(t) = \frac{1}{2}\dot{\boldsymbol{Q}}^T(t)\boldsymbol{M}_{dc}\dot{\boldsymbol{Q}}(t) \quad (20)$$

Payload kinetic energy $\boldsymbol{T}_P$ is computed as

$$\boldsymbol{T}_P = \frac{1}{2}\dot{\boldsymbol{Q}}^T(t)\boldsymbol{M}_P\dot{\boldsymbol{Q}}(t) \quad (21)$$

Where matrices $\mathbf{M}_{dc}$ and $\mathbf{M}_P$ are determined from variables $\mathbf{Q}$ vector, respectively. The total inertial mass matrix $\mathbf{M}$ can be written as

$$M = M_{dh} + M_{dc} + M_P \tag{22}$$

The total potential energy can be computed by summing over all the potential energy of each element following Eq. (17). Vector $\boldsymbol{F}(t)$ is the external generalized force and presented as

$$\mathbf{F}(t) = \begin{bmatrix} \tau & 0 & 0 & . & . & 0 \end{bmatrix}^T \tag{23}$$

Thus, by implementation of Lagrange principle, the nonlinear dynamic equations of the system are summarized as follows

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{C}\dot{\mathbf{Q}} + \mathbf{D}\dot{\mathbf{Q}} + \mathbf{F_c} + \mathbf{K}\mathbf{Q} + \mathbf{G} = \mathbf{F} \tag{24}$$

Where matrix $\mathbf{F_c}$ is coulomb friction at the joints and $\mathbf{G}$ represents forces from gravity. In this paper, coulomb friction and gravity effects are ignored as the manipulator movement is confined to the horizontal plane. The centrifugal $\mathbf{C}$ and structural damping $\mathbf{D}$ terms are calculated by using Eq. (25) and Eq. (26).

$$\mathbf{C}\dot{\mathbf{Q}} = \dot{\mathbf{M}}\dot{\mathbf{Q}} - \frac{1}{2}\left(\frac{\partial}{\partial \mathbf{Q}}(\dot{\mathbf{Q}}^T\mathbf{M}\dot{\mathbf{Q}})\right)^T \tag{25}$$

$$\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{26}$$

Where α and β are the damping ratios and are determined by experiences [1]. Depending on the system, it is possible to simplify Eq. (21). The variation in $\mathbf{M}$ depends on the geometry and $\mathbf{Q}$ variables. This is different in [1, 2] with assumption that the flexible manipulator motions in small workspace, small angular of joints and small elastic displacements. So, matrix $\mathbf{M}$ is constant. The dynamic equations (24) become in compact from as

$$\mathbf{M}\ddot{\mathbf{Q}} + \mathbf{C}\dot{\mathbf{Q}} + \mathbf{D}\dot{\mathbf{Q}} + \mathbf{K}\mathbf{Q} = \mathbf{F} \tag{27}$$

It must be further noticed that the first node of link is coincided on the joint of the link. Thus, this node has elastic displacements which are zero ($u_1(t) = 0; u_2(t) = 0$). In this work, as the link modeled by one element, $\mathbf{Q} = \begin{bmatrix} q & u_3 & u_4 \end{bmatrix}^T$ is the generalized coordinate vector of the system, where u_3 and u_4 are flexural and slope displacement of the end point of link, respectively. The external generalized force is $\mathbf{F}(t) = \begin{bmatrix} \tau & 0 & 0 \end{bmatrix}^T$. Matrices $\mathbf{M}$ and $\mathbf{K}$ can be compacted by elimination 2nd, 3rd rows and 2nd, 3rd columns, respectively. Matrices $\mathbf{C}$ and $\mathbf{D}$ are determined by using Eq. (25, 26). The dynamic nonlinear equations of one-link flexible manipulator can be derived as the following Eq. (27).

3. Dynamic behaviors of system

The initial values of dynamic model are given by [4]

$L = 1(m); b = 0.02(m); h = 0.003(m); A = b.h(m^2);$

$I = 4.5\times10^{-11}(m^4); \rho = 2710(kg/m^3); E = 7.11\times10^{10}(N/m^2);$

$I_h = 5.86\times10^{-5}(kg.m^2); m_p = 0.1(kg); \alpha = 0.76; \beta = 5.6\times10^{-5};$

The driving rules are shown in Figure 2. They are Bang-Bang [1], trapezoidal, triangular and sinusoidal, respectively. The Bang-Bang and trapezoidal rule highly consume energy. Minimum energy consumption of triangular rule is the smallest. Torque impact of Sinusoidal rule is smooth.

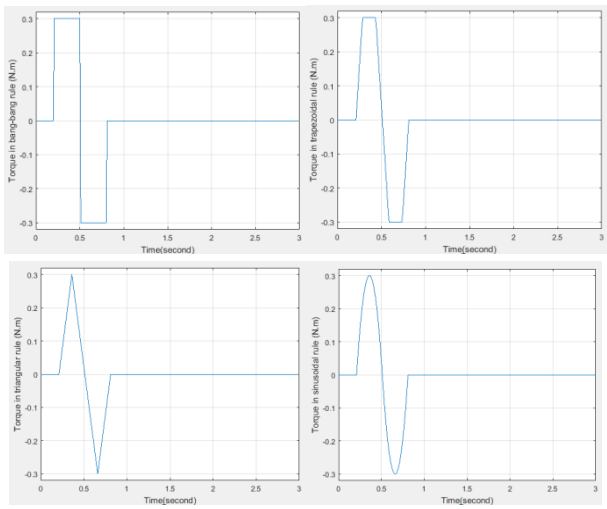

Figure 2. *Bang-Bang, Trapezoidal, Triangular and Sinusoidal driving rules*

The time for torque transmission is $0,6(s)$, simulation time is $3(s)$. The dynamic equations (27) solved on MATLAB/SIMULINK software by using ODE45 function. The Figure 3 is example with Bang-Bang driving rule.

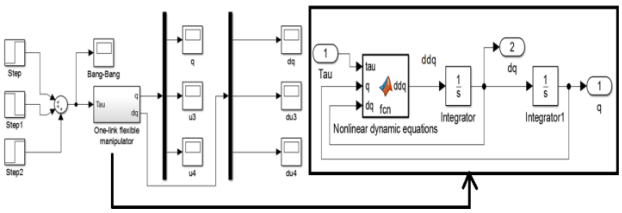

Figure 3. *System block diagram in SIMULINK with Bang-Bang driving rule*

The effects of different driving rules on dynamic behavior of one-link flexible manipulators are show through the values of joint variable, its velocity, elastic displacements and their velocity at the end-point of link. Two cases are presented to consider dynamic behavior. The first case is given without payload and the other case is opposite.

Case 1: One-link flexible manipulator without payload ($m_P = 0(kg)$)

The Figure 4 shows the values of joint variable, flexural and slope displacement. The maximums are $0.5(rad)$, $0.06(m)$ and $0.1(rad)$ with Bang-Bang driving rule, respectively. The minimums are given by triangular driving rule. The system appears vibrant and stable after 1.2(s) with 0.6(s) torque impact. Figure 5 shows the velocity of joint variable, flexural and slope displacement.

Figure 6 shows that Bang-Bang driving rule creates maximum distance on X and Y axis.

Case 2: One-link flexible manipulator with payload ($m_P = 0.1(kg)$)

In case 2, the dynamic behaviors of system are shown more clearly than case 1. The system vibrates longer especially with Sinusoidal driving rule. Results are shown in Figure 7. The velocity in Bang-bang driving rule has changed hugely in Figure 8. In general, it is easily recognized that system with payload vibrates longer than case which is without payload. The values of hub-angle and distance on X, Y axis of end-effectors in case 2 are smaller than case 1. Furthermore, velocity of joint variable increases fast while elastic displacement values are not too different from, other rules and dynamic behaviors of system still are quite fully expressed.

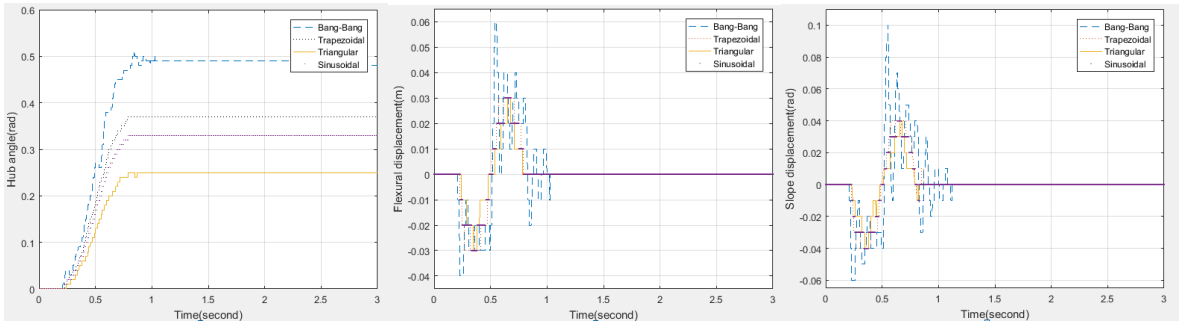

Figure 4. *Hub angle, flexural and slope displacement values (q, u_3, u_4)*

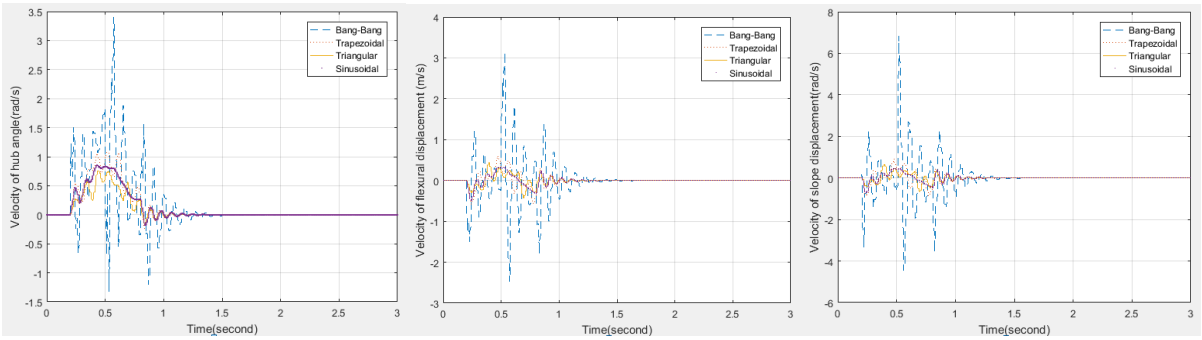

Figure 5. *Velocity of hub angle, flexural and slope displacement values (dq, du_3, du_4)*

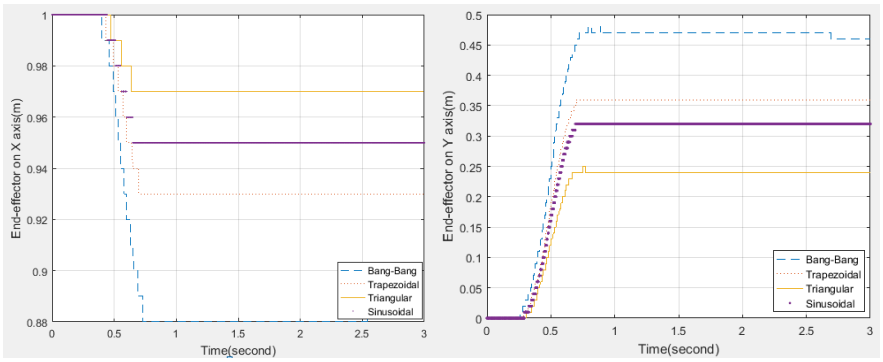

Figure 6. *The X and Y coodinates of end-effector on* ***XOY***

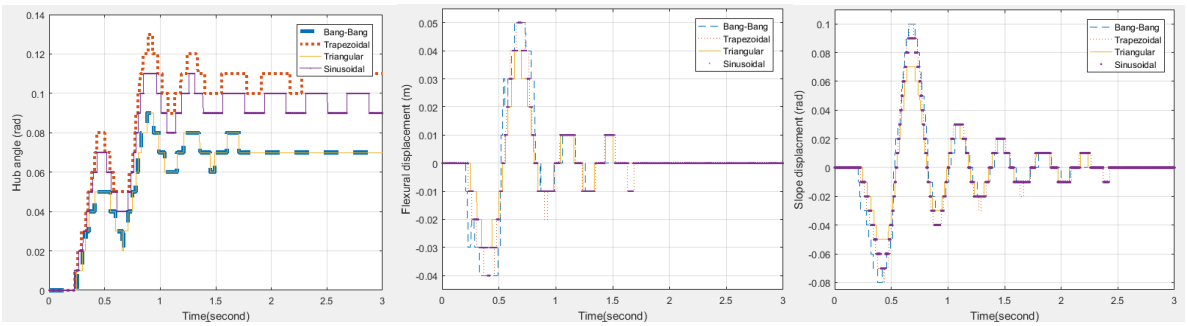

Figure 7. *Hub angle, flexural and slope displacement values*

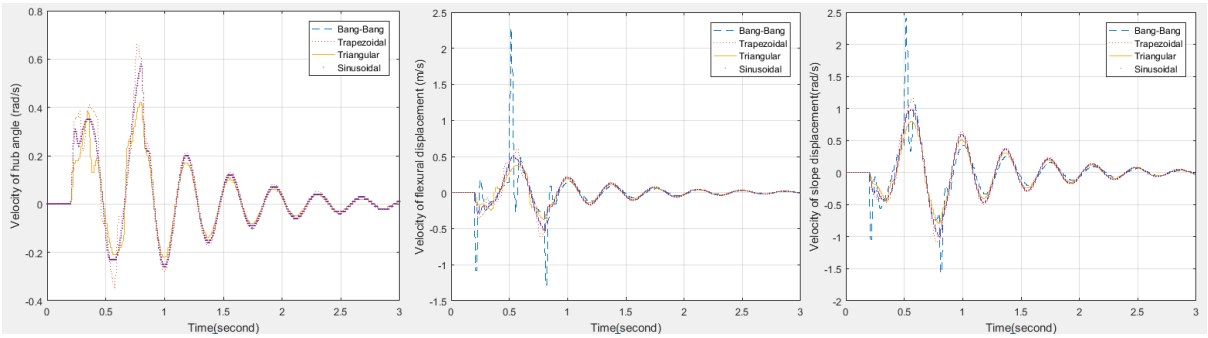

Figure 8. *Velocity of hubangle, flexural and slope displacement values*

4. Conclusion

In this paper, the nonlinear dynamic model of a one-link flexible manipulator is presented with or without payload. The motor inertial moment, centrifugal coriolis and the damping are considered. The nonlinear dynamic equations are clear and full by using FEM in Lagrange approach. The effects of different driving rules on dynamic behaviors of system are studied. Bang-Bang law can be a reasonable driving rule which is used in most studies for analyzing dynamic behaviors of flexible manipulators.

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