DEVELOPING A-Φ MAGNETODYNAMICFORMULATIONS FOR MODELLING OF MASSIVE INDUCTORS BY A FINITE ELEMENT APPROACH

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Abstract - Modeling of massive inductors in the magnetodynamic problems is one of very important problems in electrical systems in general and electrical equipments in particular. Therefore, the calculation and simulation of electromagnetic problems are more and more concerned, and are always topical subjects for researchers and designers worldwide. In this article, magnetodynamic formulations with the magnetic flux density are developed for electromagnetic problems (where massive inductors are considered as sources instead of using stranded inductors. The aim is to compute magnetic field distributions, electric fields and eddy current losses taking skin depths and skin effecs into account due to the different frequencies. The validation of the mehod is illustrated on a practical test.

Key words - Eddy current loss; magnetic field; magnetodynamics; skin effect; finite element approach

1. Introduction

Nowadays, magnetodynamic problems are one of the most important electrical devices of electrical systems in general and electrical equipments in particular. Hence, modeling of massive inductors in magnetodynamic problems is really an important problem and is always a topical subject for researchers and designers in Vietnam and in the world as well. Generally, sources for these problems can be either stranded inductors or massive inductors. Many authors have been recently developed the magnetic vector potential formulations via a numerical technique (finite element method, moment method and integral method) for solving magnetodynamic problems where sources are stranded inductors [2-6]. In these methods, the finite element method (FEM) is the most popular technique for analyzing and simulating field distributions in conducting regions, which do not include any thin regions [1, 3-9].

In this paper, the FEM with the magnetic vector potential formulation is presented for massive inductors, where either a global current or voltage is fixed to analyze and compute magnetic field distributions, electric fields and eddy current losses taking skin depths and skin effects into account due to high frequencies.

2. Magnetodynamic formulations

A canonical magnetodynamic problem is defined in a domain Ω, with boundary $\partial\Omega = \Gamma = \Gamma_h \cup \Gamma_e$. The equations and behavior laws are written in Euclidean space $\mathbb{R}^3$ [2-6, 8]

$$\operatorname{curl}\mathbf{H} = \mathbf{J}_s, \operatorname{curl}\mathbf{E} = -j\omega\,\mathbf{B}, \operatorname{div}\mathbf{B} = 0. \qquad \text{(1a-b-c)}$$

$$\mathbf{B} = \mu\mathbf{H}, \mathbf{J} = \sigma\mathbf{E}, \qquad \text{(2a-b)}$$

where fields **B** and **H** are respectively the magnetic flux density (T) and the magnetic field (A/m); **E** and $\mathbf{J}_s$ are respectively the electric field (V/m) andthe current density (A/m2); μ and σ are respectively the relative permeability, and the electric conductivity (S/m).

The boundary conditions (BCs) are defined on Γ,

$$\boldsymbol{n} \times \mathbf{H}|_{\Gamma_h} = 0, \boldsymbol{n} \cdot \mathbf{B}|_{\Gamma_e} = 0, \qquad \text{(3a-b)}$$

where $\boldsymbol{n}$ is the unit normal exterior to Ω [3-8]. It should be noted that the domain Ω is defined in complementary parts Ω_c and Ω_c^C, where Ω_c is the conducting region containingmassive inductors taking into account the eddy currents and the skin effects and Ω_c^C is the non-conducting region. The fields presented in Maxwell's equations are considered in the frequency domain.

The equations (1 a) and (1 b) are solved with boundary conditions (BCs) taking the tangential component of **H** in (3a) and the normal component of **B** in (3b) into account.

The fields **H, B, E, J** are defined to satisfy Tonti's diagram [10, 11]. This means that $\mathbf{H} \in \boldsymbol{H}_h(\operatorname{curl};\Omega)$, $\mathbf{E} \in \boldsymbol{H}_e(\operatorname{curl};\Omega)$, $\mathbf{J} \in \mathbf{H}(\operatorname{div};\Omega)$ và $\mathbf{B} \in \boldsymbol{H}_e(\operatorname{div};\Omega)$, where $\boldsymbol{H}_h(\operatorname{curl};\Omega)$ and $\boldsymbol{H}_e(\operatorname{dive};\Omega)$ are function spaces containing BCs and the fields defined on Γ_h and Γ_e of studied domain Ω.

3. Weak finite element formulations

3.1. Magnetic vector potential weak form

The weak formulation for **A**-Φ is written based on the set of Maxwell's equations (1a-b-c) and the constitutive laws (2a-b). In order to exactly satisfy the Ampere's law (1a) and the behavior law (2a), the fields $\mathbf{H} \in \boldsymbol{H}_h(\operatorname{curl};\Omega)$, $\mathbf{J} \in \boldsymbol{H}_h(\operatorname{div};\Omega)$. This means that the field **B** has to be in a function space $\boldsymbol{H}_h(\operatorname{curl};\Omega)$, i.e. ($\mathbf{B} \in \boldsymbol{H}_h(\operatorname{curl};\Omega)$, and thus does let only Faraday's law be weakly formulated.

For **A**-Φ formulation, there are two quantities of degree of freedoms needed to be defined: the circulation of magnetic vector potential **A** on the edges of the conductive elements and the electric scalar potential Φ on the nodes.

In addition, the global condition on currents (I) in massive inductors is considered through surface Γ_g, i.e. [8, 10]

$$\oint_{\Gamma_g^-}^{\Gamma_g^+} n \cdot \boldsymbol{j} ds = \mathrm{I}, \qquad \text{(4a-b)}$$

where Γ_g^+ and Γ_g^- are the paths in Ω_g connecting two real or imaginary electrodes.

From the equation (1b), the field **B** can be achieved from a magnetic vector potential **A**, i.e.

$$\mathbf{B} = \operatorname{curl}\mathbf{A}. \qquad (5)$$

By substituting (5) into (1c), one gets curl $(\mathbf{E} + j\omega\,\mathbf{A}) = 0$, this means that the field **E** can be defined via an electric scalar potential Φ, i.e.

$$\mathbf{E} = -j\omega\,\mathbf{A} - \operatorname{grad}\Phi. \qquad (6)$$

From the above analysis, the weak formfor **A**-Φ is now written based on the Ampere's equation (1 a) and the constitutive law (2 a), i.e. [1, 3, 5]

$$\int_{\Omega} \mu^{-1}\mathbf{B}\cdot \mathrm{curl}\mathbf{A}' d\Omega + \int_{\Omega} (\sigma \mathbf{E}\cdot \mathbf{A}')d\Omega_c + + \int_{\Gamma_h} (\boldsymbol{n}\times \mathbf{H})\cdot \mathbf{A}' d\Gamma_h = \int_{\Omega} (\mathbf{J}_s \cdot \mathbf{A}')d\Omega_s\,,$$

$$\mathbf{A}' \in \boldsymbol{H}_h^1(\mathrm{curl};\Omega) \qquad (7)$$

By substituting (5) and (6) into (7), one has

$$\int_{\Omega} \mu^{-1}\mathrm{curl}\,\mathbf{A}\cdot \mathrm{curl}\mathbf{A}' d\Omega + j\omega \int_{\Omega} (\sigma \mathbf{A}\cdot \mathbf{A}')d\Omega_c + j\omega \int_{\Omega} (\sigma \mathrm{grad}\Phi\cdot \mathbf{A}')d\Omega_c + \int_{\Gamma_h} (\boldsymbol{n}\times \mathbf{H})\cdot \mathbf{A}' d\Gamma_h = \int_{\Omega} (\mathbf{J}_s \cdot \mathbf{A}')d\Omega_s\,, \mathbf{A}' \in \boldsymbol{H}_e^0(\mathrm{curl};\Omega) \qquad (8)$$

where the function space $\boldsymbol{H}_e^0(\mathrm{curl};\Omega)$ is a defined on Ω containing the test function $\mathbf{A}'$ and the basis functions **A** as well. It should be noted that at the discrete level, this space is defined by edge finite elements [10].

In addition, the field Φ is determined in Ω_c. Thus, the decretized formulation (8) where $\mathbf{A}' = \mathrm{grad}\,\Phi'$ is considered as a test function, i.e. [10, 11]

$$j\omega \int_{\Omega} (\sigma \mathbf{A}\cdot \mathrm{grad}\,\Phi')d\Omega_c + \int_{\Omega} (\sigma \mathrm{grad}\,\Phi \cdot \mathrm{grad}\,\Phi')d\Omega_c = \int_{\Gamma_g} (\boldsymbol{n}\cdot \mathbf{J})\cdot \Phi' d\Gamma_g,\ \Phi' \in \boldsymbol{H}_e^{10}(\Omega) \qquad (9)$$

where Γ_g is defined in the boundary Ω imposing electric currents in the massive inductors. Hence, equation (9) actually presents the weak form of div **J** = 0 in conducting region Ω_c.

3.2. Global quantities in massive inductors

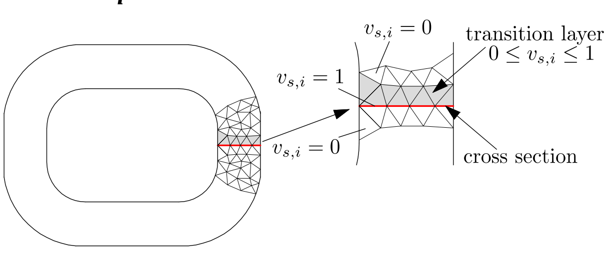

Figure 1. *Transition layer and cross section*

In the weak form (8), the occurrence of total current in stranded inductors is considered as a weak sense, e.g. as a a global constraint, because it appears from Ampere's equation. By solving the equation (9), the current I_i flowing in the electrode of Γ_g of massive inductors can be achieved with Φ' equal to the source scalar potential Φ_s. Thus, with $\Phi = \Phi_s$, the LHS term in (9) is written for the massive inductor Ω_m [10]

$$\int_{\Gamma_g} (\boldsymbol{n}\cdot \mathbf{J})\cdot \Phi_s d\Gamma_g = \int_{\Gamma_g} (\boldsymbol{n}\cdot \mathbf{J})\cdot 1 d\Gamma_g = I_i \qquad (10)$$

and thus (9) transforms

$$j\omega \int_{\Omega_m} (\sigma \mathbf{A}\cdot \mathrm{grad}\,\Phi_s)d\Omega_m + \int_{\Omega_m} (\sigma \mathrm{grad}\,\Phi \cdot \mathrm{grad}\,\Phi_s)d\Omega_m = I_i, \qquad (11)$$

or, for $\Phi = V_i\,\Phi_s$,

$$\omega \int_{\Omega_m} (\sigma \mathbf{A}\cdot \mathrm{grad}\,\Phi_s)d\Omega_m + V_i \int_{\Omega_m} (\sigma \mathrm{grad}\,\Phi_s \cdot \mathrm{grad}\,\Phi_s)d\Omega_m = I_i \qquad (12)$$

Equation (12) is the circuit equation linked to the inductor Ω_m, where I_i can be imposed

3.3. Discretization of fields A and Φ

The fields **A** and Φ in the weak forms (from (9) to (12)) is discretized by edge and node elements, i.e. [11]

$$\mathbf{A} = \sum_{e\in E(\Omega_c)} \mathrm{A_e} s_e + \sum_{e\in E(\Omega_c^C)\setminus E(\partial\Omega_c)} \mathrm{A_e} s_e\,, \qquad (13)$$

$$\Phi_s = \sum_{n\in N(\Gamma_g)} \Phi_n \qquad (14)$$

The elements $E(\Omega_c)$ and $E(\Omega_c^C)$ in (13) are the set of edges of Ω, s_e is the edge basic function linked to edge *e*, and $\mathrm{A_e}$ is the circulation of **A** along the edge *e* of studied domain.The element $N(\Gamma_g)$ in (14) is all the nodes of the cross-section Γ_g, and is only presented in a transition layer including all the elements in one side of the cross-section presented in Figure 1. This means that electric potential Φ_s is equal to 1 on the electrode Γ_g^+ and is equal to 0 for other. This varies continuously in massive inductors Ω_m.

Now, by substituting (13) and (14) into (12), one is rewritten as:

$$\int_{\Omega_m} \left(\sigma \sum_{e\in E(\Omega_c)} \mathrm{A_e} s_e \cdot \mathrm{grad}\,\Phi_s\right) d\Omega_m + \int_{\Omega_m} \left(\sigma \sum_{e\in E(\Omega_c^C)\setminus E(\partial\Omega_c)} \mathrm{A_e} s_e \cdot \mathrm{grad}\,\Phi_s\right) d\Omega_m + V_i \int_{\Omega_m} \left(\sigma \sum_{n\in N(\Gamma_g)} \mathrm{grad}\,\Phi_n \cdot \mathrm{grad}\,\Phi_s\right) d\Omega_m = I_i \qquad (15)$$

4. Application test

The example problem is a practical problem consisting of a plate (made of aluminum) located in the middle of massive inductors shown in Figure 2. The massive inductors carry imposed electric currents of 50 A. The electric conductivity and relative magnetic permeability of the plate and massive inductors are respectively given σ = 2.5 MS/m and μ_r = 1. The problem at hand is considered in 2-D case.

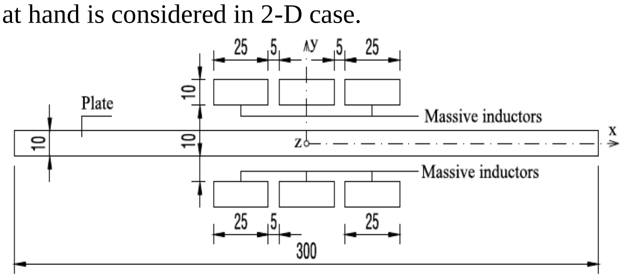

Figure 2. *Geometry of the plate and massive inductors (all dimensions are in mm)*

Figure 3 shows 2-D detailed mesh of the plate and massive inductors, where the plate uses rectangular meshes and triangle meshes for massive inductors. The solutions on the magnetic vector potential generated by imposed massive currents are presented in Figure 4 (*top*) and the raised flux (Figure 4, *bottom*), for μ_r = 1, σ = 2.5 MS/m and f = 50 Hz.

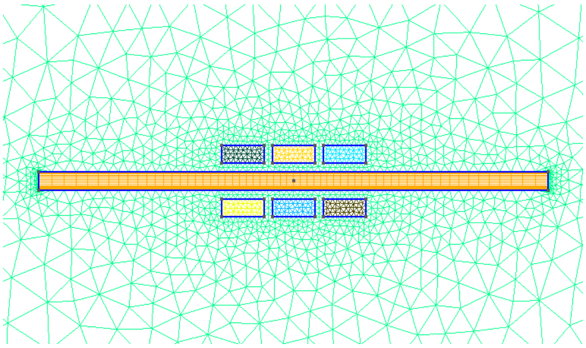

Figure 3. *2-D mesh of the plate and massive inductors*

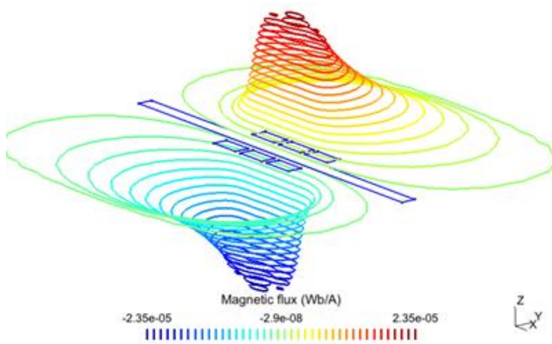

Figure 4. *Flux line distribution due to imposed massive inductors of 20A (top) and the raised flux (bottom), for μ_r = 1, σ = 2.5 MS/m and f = 50 Hz*

The distribution of magnetic flux density due to "curl **A**"is depicted in Figure 5. It can be seen that the field almost focuses on the massive inductor regions, and reduces to the plate ends. Significant magnetic fields along the plate are presented in Figure 6, with effects of different frequencies. The skin-depth (δ) is equal to 3.18 mm with f= 1000Hz, or 6.3 mm with f = 250 Hz, for μ_r = 1, σ = 2.5 MS/m in both cases.

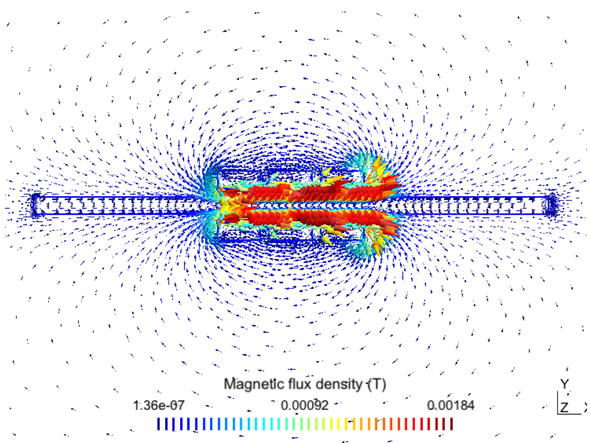

Figure 5. *Magnetic field distribution due to B = $curl$ A, for μ_r = 1, σ = 2.5 MS/m and f = 50 Hz*

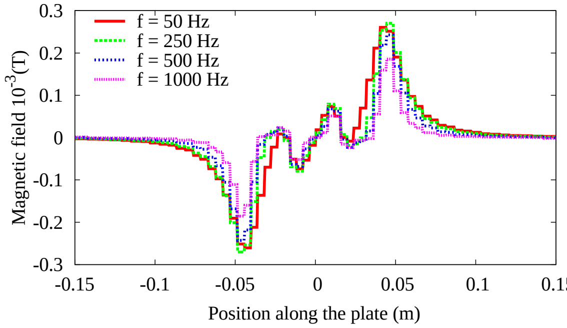

Figure 6. *Magnetic field distribution with effects of different frequencies for μ_r = 1, σ = 2.5 MS/m*

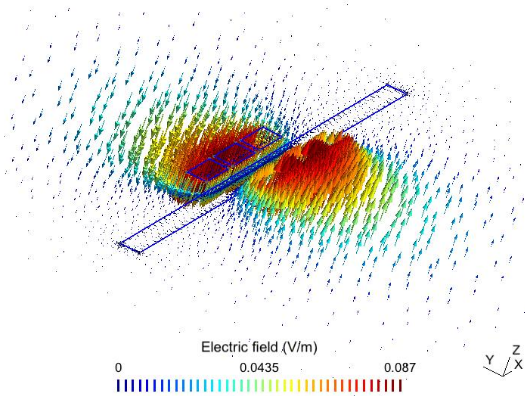

Figure 7. *Electric field distribution due to $E = -j\omega\ A - grad\ \Phi$, for μ_r = 1, σ = 2.5 MS/m and f = 50 Hz*

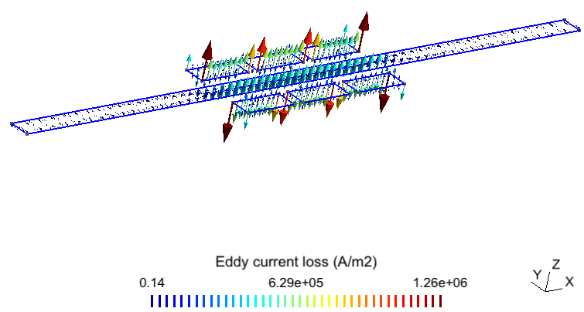

Figure 8. *Skin effect maps on eddy current distribution in massive inductors and the plate, for μ_r = 1, σ = 2.5 MS/m and f = 1000 Hz*

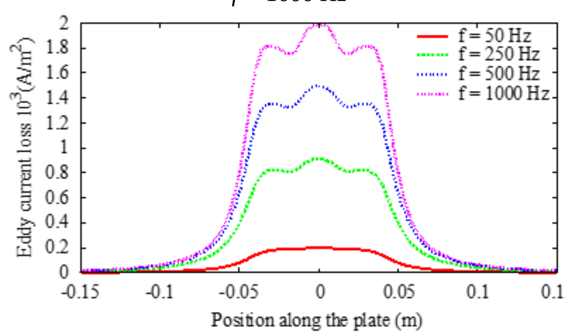

Figure 9. *Eddy current values along the plate with different thicknesses, for μ_r = 1, σ = 2.5 MS/m*

The electric field distribution due to $\mathbf{E} = -j\omega\mathbf{A} - \mathrm{grad}\ \Phi$ is pointed in Figure 7, for μ_r = 1, σ = 2.5 MS/m and f = 50 Hz.

Skin effect maps on the eddy current distribution in massive inductors and the plate are indicated in Figure 8, for μ_r = 1, σ = 2.5 MS/m and f = 1000 Hz. Significant eddy current values along the plate with effects of different frequencies (μ_r = 1, σ = 2.5 MS/m) are presented in Figure 9. For a higher frequency (e.g. f = 1 kHz), the skin-depth is small, the skin effect is great, the eddy current mainly focuses on the surface of the plate. It can reach 2000A/m^{2}for δ = 3.18 mm and f = 1 kHz, or lower than 200A/m^2 for f = 50 Hz.

5. Conclusions

The finite element approach has been successfully extended with magnetic vector potential formulation **A**-Φ**,** where sources are massive inductors. The developed method allows us to compute and simulate fields in and around the plate (magnetic flux, magnetic field, electric field and eddy current loss) with different frequencies. The obtained results have shown that there is a very good validation of the extend method in the calculation of local and global fields taking skin effects into account. In particular, the paper has shown the regions where the fields appear. This will help reseachers to see skin effects appearing massive inductors in magnetodynmic problems, considered with high frequencies.The validation of the proposed method has been successfully applied to the practical problem.

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