# KINEMATIC ANALYSIS OF THE PLANAR TRIANGULAR PARALLEL ROBOT

PHÂN TÍCH ĐỘNG HỌC ROBOT SONG SONG PHẲNG KIỂU TAM GIÁC

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**Abstract -** A planar 3-PRP triangular parallel robot is a special symmetrical closed-loop mechanism which has been proposed by Damien Chablat and Stefan Staicu. A recursive modeling for kinematics of this robot is presented in their paper [1]. In this research, we propose a method, based on the theory of screws for solving the kinematic analysis of this robot. Our study describes the mathematical model of the manipulator which deals with the motion of robot kinematic respect to geometry links, the matter of finding the mathematical relation between the joint variables. Finally, the mathematical modeling is verified by comparing Matlab result and Adams view simulation results.

**Key words -** Planar parallel robot, recursive modeling, kinematics of robot, the theory of screws, geometry links, Adams view

**Tóm tắt -** Robot song song phẳng kiểu tam giác 3-PRP là một cơ cấu cơ khí vòng kín đối xứng được đề xuất bởi Damien Chablat và Stefan Staicu. Hai tác giả trên đã trình bày phương pháp giải bài toán động học robot bằng mô hình đệ quy (recursive model) trong bài báo của họ [1]. Trong nghiên cứu này, chúng tôi đề xuất một phương pháp giải khác, dựa trên lý thuyết vít (Screw theory) để giải quyết bài toán phân tích động học của robot này. Nghiên cứu của chúng tôi mô tả mô hình toán học của các thao tác liên quan tới chuyển động động học của robot trên cơ sở tôn trọng các liên kết hình học, đồng thời tìm ra mối quan hệ toán học giữa các biến khớp. Cuối cùng, mô hình toán học được kiểm chứng bằng cách so sánh kết quả chương trình trên Matlab và kết quả mô phỏng trên phần mềm 3D Adams view.

**Từ khóa -** Robot song song phẳng, mô hình đệ quy, động học robot, lý thuyết vít, liên kết hình học, Adams view

## 1. Introduction

Compared with a serial manipulator, potential advantages of parallel architectures are high stiffness, low inertia, working in high accelerations and accuracies, making the structure suitable for the applications where the serial structure does not provide a suitable performance in practice [2]. A planar parallel robot is constituted of a moving platform connected to the fixed platform by three legs which are composed of joints and the individual actuators. Depending on kind of joints using for legs, these planar manipulators have a different name such as RRR, RPP, PRR, PPP or PRP planar parallel robot where R is denoted by revolute joint and P for a prismatic joint. By using this rule, Jean-Pierre Marlet made a summary of all possible chains for a planar parallel robot in the paper "Direct kinematics of planar parallel manipulators" [3]. Most of these configurations have investigated the kinematic and dynamic problem such as RRR form in [3], [4], [5], PRR [6] or RPR [7].

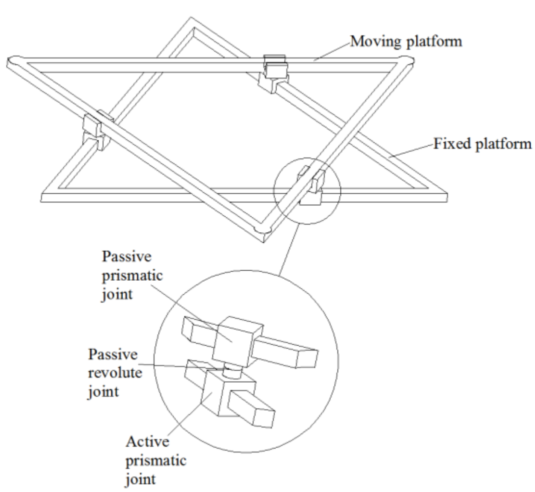


***Figure 1.*** *Triangular planar manipulator*

A planar 3-PRP triangular parallel robot was introduced by Damien Chablat and Stefan Staicu [1]. This planar robot is a special symmetrical parallel mechanism, which is composed of a pair of triangles. One movable is placed on the top of the stationary triangle, connected at three sets of combination of revolute and prismatic joints. The prismatic joints on the fixed platform are actuated, so the upper triangle is moved by changing the length of actuators along the three fixed edges. Thereby, this constitutes three degrees of freedom of planar motion, one degree of orientation freedom and two degrees of translation freedom.

## 2. Kinematic analysis

This section describes the mathematical model of the manipulator which deals with the motion of robot kinematic respect to geometry links, the matter of finding the mathematical relation between the joints variables and that pose. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs. The inverse kinematics problem involves a known position and orientation of the output platform of the robot to a set of input joint variables that will achieve that pose.

Figure 2 shows a general schematic model of the triangular planar manipulator, constructed by connecting a triangular moving platform to the fixed platform with three Prismatic – Revolute – Prismatic (PRP) legs that are displayed in Table 1.

***Table 1.*** *The manipulator platform onfiguration parameters.*

| No. | Position | Connector | Remark |
|---|---|---|---|
| **1** | $R_1$, $R_2$, $R_3$ | Translational joint | Active joint |
| **2** | $R'_1$, $R'_2$, $R'_3$ | Translational joint | Passive joint |
| **3** | $R_1R'_1$, $R_2R'_2$, $R_3R'_3$ | Revolute joint | Passive joint |

In order to implement kinematic analysis, the coordinate axes are fixed to various joints of the robot. The frame $O_{xyz}$ is fixed to the base (the fixed platform) with O at the center of the triangle $B_1B_2B_3$ with Z axis perpendicular to the platform. The frame coordinate of $O'_{x'y'z'}$ is attached to the moving platform $C_1C_2C_3$ with their Z' axis pointing upward and being normal to the platform as shown in Figure 2. The actuated joint variables are the three link lengths $B_2R_1=q_1$, $B_3R_2=q_2$, $B_1R_3=q_1$ and the distance from $C_2R'_1=l_1$, $C_3R'_2=l_2$, $C_1R'_3=l_3$ are the passive joint variables.

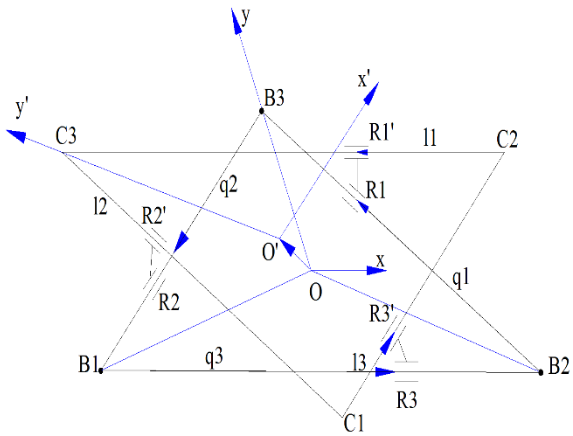


***Figure 2.*** *Kinematic structure of the planar manipulator*

Assume the end-effector of the manipulator coincides with the centroid $O'_{x'y'z'}$ of the equilateral triangle $C_1C_2C_3$ of the moving platform. Denoted by x, y, z, its coordinates in the base coordinate system are attached to the center point O of the base platform with the plane xy coinciding with the base platform. The x'y'z' is the coordinate system attached to the moving platform with origin at point O'. Consider the coordinates of the vertices $B_1$, $B_2$, $B_3$ of the fixed platform of the manipulator.

The coordinates of the vertices $B_1$, $B_2$, $B_3$ are in the fixed Oxyz frame:

$$\overrightarrow{OB_1}=\begin{bmatrix}-\frac{\sqrt{3}}{2}r\\ -\frac{1}{2}r\\ 0\end{bmatrix}\overrightarrow{OB_2}=\begin{bmatrix}\frac{\sqrt{3}}{2}r\\ -\frac{1}{2}r\\ 0\end{bmatrix}\overrightarrow{OB_3}=\begin{bmatrix}0\\ r\\ 0\end{bmatrix} \quad (1)$$

Where r is radius of circumcircle of triangle $B_1$ $B_2$ $B_3$ and the coordinates of $C_1$, $C_2$, and $C_3$ in frame O'x'y'z':

$$\overrightarrow{O'C_1}=\begin{bmatrix}-\frac{\sqrt{3}}{2}r\\ -\frac{1}{2}r\\ 0\end{bmatrix}\overrightarrow{O'C_2}=\begin{bmatrix}\frac{\sqrt{3}}{2}r\\ -\frac{1}{2}r\\ 0\end{bmatrix}\overrightarrow{O'C_3}=\begin{bmatrix}0\\ r\\ 0\end{bmatrix} \quad (2)$$

The length of the individual links is denoted by $q_i$ (*i= 1,2,3*).

Assuming that the position of the origin of the center O' of the moving platform in the frame Oxyz:

$$\overrightarrow{OO'}=\begin{bmatrix}x_{O'}\\ y_{O'}\\ d\end{bmatrix} \quad (3)$$

Then the position of $R'_i$ is expressed relative to $C_i$:

$$\overrightarrow{C_2R_1}=\begin{bmatrix}l_1\\ 0\\ 0\end{bmatrix}\overrightarrow{C_3R_2}=\begin{bmatrix}l_2\\ 0\\ 0\end{bmatrix}\overrightarrow{C_1R_3}=\begin{bmatrix}l_3\\ 0\\ 0\end{bmatrix} \quad (4)$$

The position of the passive prismatic joints $R'_i$ is in the frame O'-x'y'z':

$$\overrightarrow{O'R'_1}=\overrightarrow{O'C_2}+Rz(\pi-B_2).\overrightarrow{C_2R_1} \quad (5)$$

$$\overrightarrow{O'R'_2}=\overrightarrow{O'C_3}+Rz(\pi+B_1).\overrightarrow{C_3R_2} \quad (6)$$

$$\overrightarrow{O'R'_3}=\overrightarrow{O'C_1}+Rz(0).\overrightarrow{C_1R_3} \quad (7)$$

Where Rz denotes the rotation matrix around the z-axis. Therefore, the position the passive prismatic joints $R'_i$ in the frame O-xyz:

$$\overrightarrow{OR_i}=\overrightarrow{OO'}+Rz(\frac{\pi}{3}+\varphi).\overrightarrow{O'R'_i} \quad (8)$$

By applying similar approaches as presented previously, the coordinates of the active joints $R_i$ are defined in the frame O-xyz:

$$\overrightarrow{R_1O}=\overrightarrow{O'B_2}+Rz(\pi-B_2).\overrightarrow{B_2R_1} \quad (9)$$

$$\overrightarrow{R_2O}=\overrightarrow{O'B_3}+Rz(\pi+B_1).\overrightarrow{B_3R_2} \quad (10)$$

$$\overrightarrow{R_3O}=\overrightarrow{O'B_1}+Rz(0).\overrightarrow{B_1R_3} \quad (11)$$

And for the coordinates constraints of revolute joints:

$$x^0_{R_i}=x^0_{R'_i} \quad (12)$$

Based on equations (11), total 3 variables of the coordinates will be determined by using MATLAB software.

The screw theory is a powerful tool for the kinematic analysis of robotic mechanics, mechanical design, and multibody dynamics. In this research, the authors use this tool for studying the velocity of the manipulator. The velocity of the center O' of the moving platform respect to the coordinate Oxyz is obtained by using the theory of screws. Assume $\omega_{O'2}=[0, 0, \omega_{2z}]$ is the angular velocity of the moving platform, respect to the fixed platform. The $v_{O'2}=[v_{O'x}, v_{O'y}, 0]$ is the translational velocity of point O' fixed on the moving platform. In screw theory, the Plucker coordinates of the infinitesimal screws notated as \$ are a six-dimensional vector given by \$ = $(s,s_0)$, where s is primal of the screw. The velocity of twist screw $V_{O'}=[\omega_{O'2}; v_{O'2}]$, of the moving platform, respect to the fixed platform can be written:

$$V_{O'}= \dot{q}_1\,{}^0\$^1_1+\dot{\varphi}\,{}^1\$^2_1+\dot{l}_1\,{}^2\$^3_1 \quad (13)$$

$$V_{O'} = \dot{q}_2\,{}^0\$_2^1 + \dot{\varphi}\,{}^1\$_2^2 + \dot{l}_2\,{}^2\$_2^3 \quad (14)$$

$$V_{O'} = \dot{q}_3\,{}^0\$_3^1 + \dot{\varphi}\,{}^1\$_3^2 + \dot{l}_3\,{}^2\$_3^3 \quad (15)$$

The relationship between the active joints velocity $(\dot{q}_1, \dot{q}_2, \dot{q}_3)$ and the angular velocity of the moving platform, respect to the fixed platform $(\dot{\varphi}_3)$ and the translational velocity of the center of the moving platform O' represents frame {O} $(\dot{x}_{O'/o}, \dot{y}_{O'/o}, 0)$.

$$J_a \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = J_b \cdot \begin{bmatrix} \dot{v}_{O'x} \\ \dot{v}_{O'y} \\ \omega_{2z} \end{bmatrix} = J_b \cdot \begin{bmatrix} \dot{x}_{O'/o} \\ \dot{y}_{O'/o} \\ \dot{\varphi} \end{bmatrix} \quad (16)$$

Where $J_a$ and $J_b$ are the Jacobian of the planar parallel manipulator.

## 3. Simulation and results

To verify the kinematic characteristic of 3-DOF planar parallel manipulator has one rotation and two translations as well as to evaluate the performance of the manipulator, we have applied some numerical softwares and modules such as Matlab and Adams View to conduct simulation before the real platform is available. In Adams View software, we need to locate some markers on the manipulator whose coordinates are shown Table 2.

***Table 2.*** *The virtual manipulator platform coordinates*

| No. | Parameters | Coordinate |
|---|---|---|
| 1 | Base_Origin (O) | (0,0,0) |
| 2 | Origin_Platform_1 (O') | (0,0,30) |
| 10 | $B_1$ | (-188.53, -108.84, 0) |
| 11 | $B_2$ | (188.53, -108.84, 0) |
| 12 | $B_3$ | (0, 217.7, 0) |
| 13 | $R_1$, $R_2$, $R_3$ and $R'_1$, $R'_2$, $R'_3$ | Using CM markers |
| 14 | Revolute_Joint_1, 2, 3 | Using default marker |

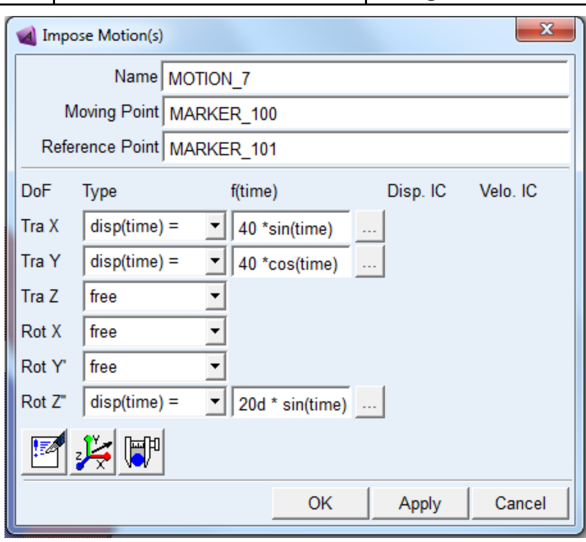


***Figure 3.*** *User windows of general motion of the moving platform*

First, we will check the inverse kinematic formula by assuming the moving platforms in general motion. There are two translation movements in X and Y displacement considered by time function 40 sinus and cousins. The rotation movement respecting to Z axis is with the time function 20° sinus

We will find that the result derived from Matlab program and Adams view software is the same (Figure 4), for example: at t = 4s, $q_1$ = 187.9485 (Adams view) and $q_1$ = 186.5 (Matlab), t = 7s, $q_2$ = 228.8396 (Adams view) and $q_3$ = 227.7 (Matlab) and at t = 1s, $q_3$ = 304.4744 (Adams view) and $q_3$ = 303.4 (Matlab).

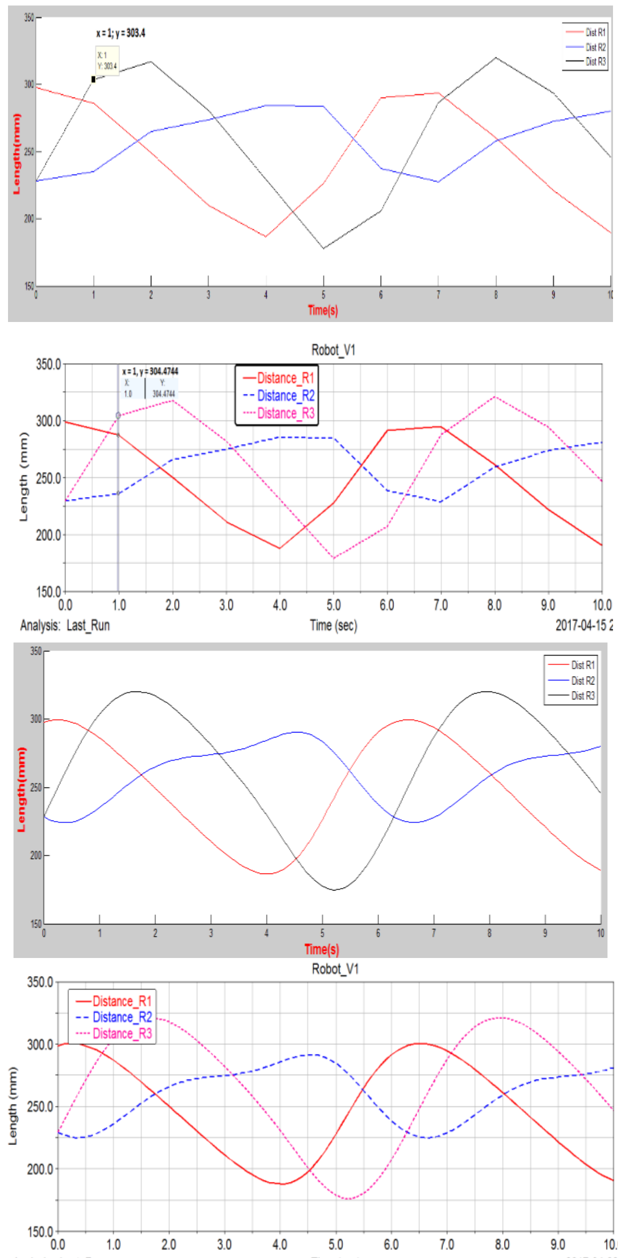


***Figure 4.*** *Comparison of position of the actuated joint $q_1$, $q_2$, $q_3$ between result derived from Matlab and Adams view with step period = 0.1 s*

Similarly, we will add motion for active joints for checking the forward kinematic as shown on Figure 5 where $q_1 = 50*\sin(time)$; $q_2 = 30*\text{sine}(time)$ and $q_3 = 50*\sin(time)$.

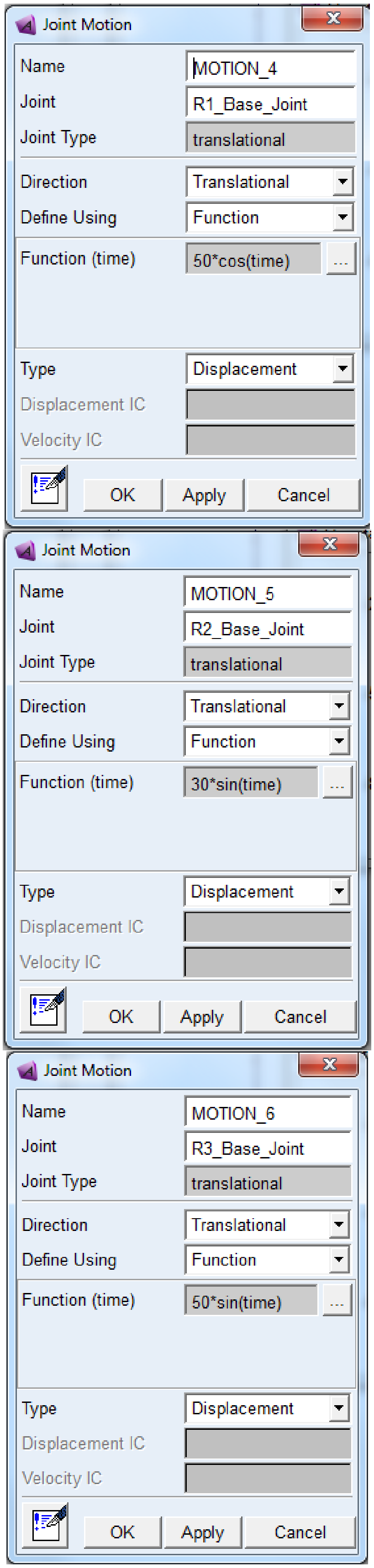


***Figure 5.*** *User windows of actuator motion*

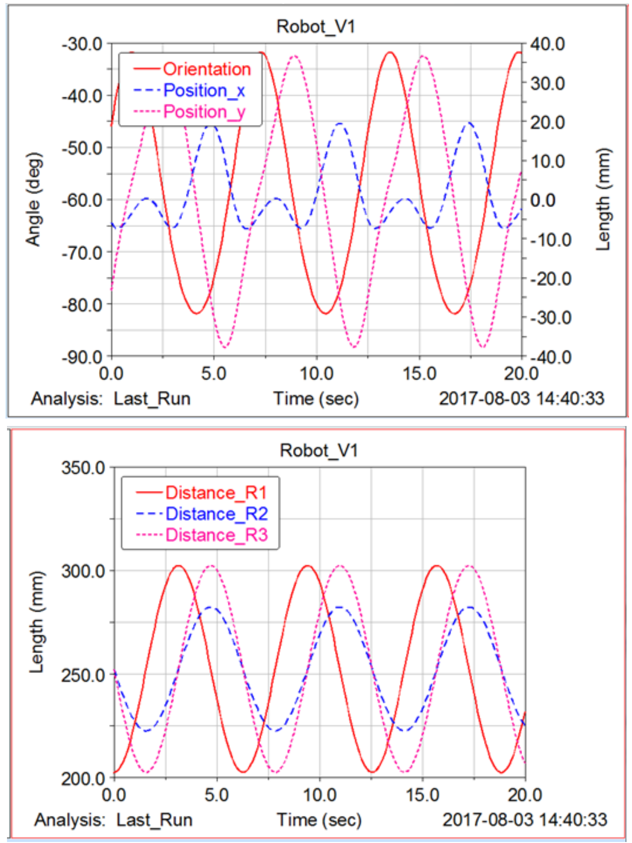


***Figure 6.*** *The position of prismatic joints and position, the orientation of end effector*

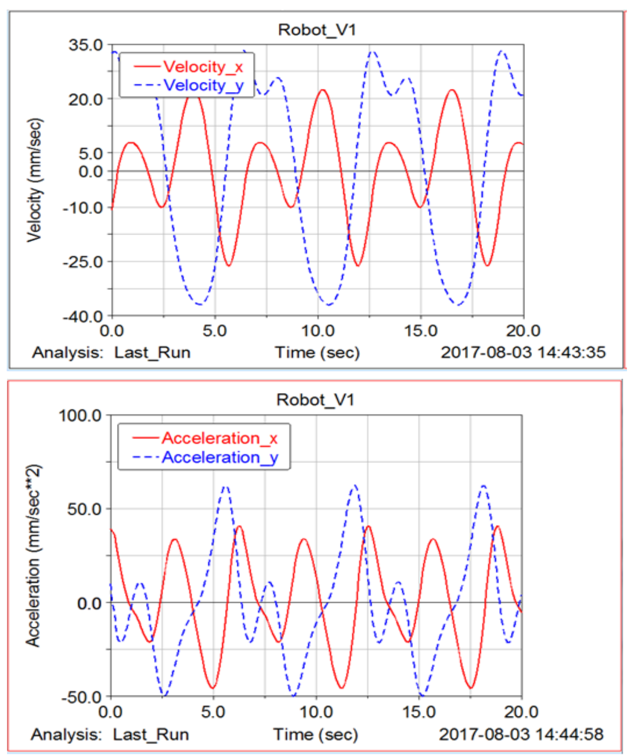


***Figure 7.*** *Simulation forward kinematic - Velocity and Acceleration of end effector*

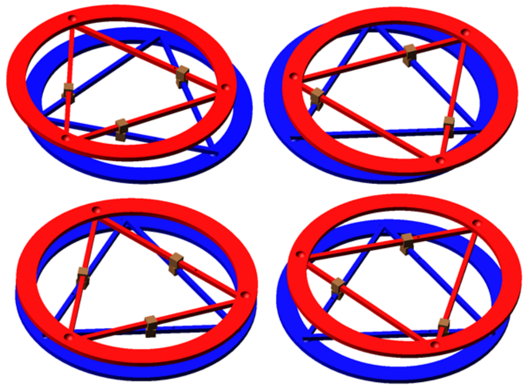

***Figure 8.*** *Snapshot of simulation of manipulator on ADAMS VIEW*

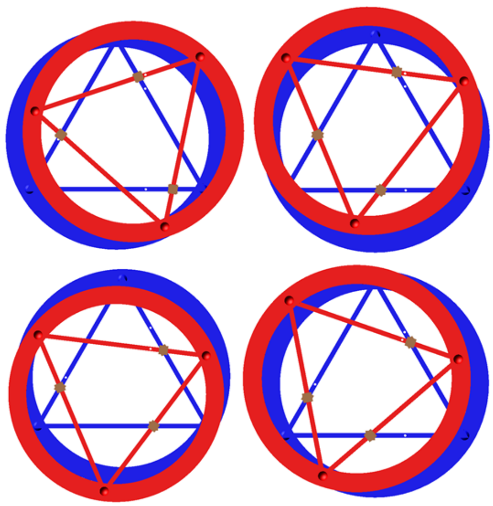

***Figure 9.*** *Sectional view of manipulator simulation*

## 4. Conclusions

This study proposes a new method to solve kinematic of the 3-DOF planar parallel manipulator. The 3D model construction and performance of the manipulator is verified by using Matlab software to find the solutions, then make a comparison with the result of Adams view software. Although there still exist small errors between those two softwares because of some assumption used during solving equations in Matlab to minimize the calculation time, this result is acceptable.

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