Using a Bi-Level Optimization Model for Assessing the Impact of Demand Forecast Uncertainty on the Maximum Profit of Power Distribution Companies Owning Energy Storage Systems

Dang Vu Kien, Le Thi Minh Chau, Pham Quang Phuong, Pham Nang Van*

Abstract—This paper proposes a bi-level optimization model to maximize the net revenue of a power distribution company owning energy storage systems. Furthermore, the proposed model also takes into account the uncertainty of load forecast based on a set of multiple scenarios in order to assess its impact on the maximum net revenue of the power distribution company. This bi-level optimization model is transformed into the single-level mixed-integer linear programming model by using the Karush-Kuhn-Tucker optimality conditions and strong duality theorem. This single-level optimization formulation can be effectively solved by using standard commercial solvers such as CPLEX. The proposed model and the effects of the load forecast uncertainty on the net revenue of the power distribution company are validated and analyzed on an IEEE 24-bus meshed transmission grid.

Index Terms—Bi-level optimization; energy storage systems; the demand forecast uncertainty; power distribution companies; electricity markets.

1. Introduction

WITH renewables increasingly integrated into power networks, there is a growing need for enhancing operational flexibility. One of the solutions to achieve this demand is to deploy energy storage (ES) systems due to their capability of storing and releasing electricity energy [1]. There are numerous contributions devoted to the deployment of ES devices in power systems. Authors [2] addressed the problem of scheduling ES systems using the mean-variance optimization, in which the price uncertainty in day-ahead and balance markets is taken into account. The strategy for optimally bidding ES devices in power markets incorporating battery cycle life and fast regulation capability was put forward in [3]. The method of robust optimization was adopted in reference [4] to evaluate the scheduling strategy of ES systems in energy markets combined with ancillary service markets. In [5], the optimal operation of synchronous generators and battery energy storage in the energy market coupled with reserve markets was proposed. The operation cost model of ES systems taking account of degradation cost was introduced in [6]. This paper will investigate the influence of the utilization of ES devices on the net revenue or profit of the power distribution company (PDC) in the energy markets.

The power distribution company purchases electricity from the wholesale markets and sells it to customers at retail prices. Conventionally, the PDC forecasts the electricity consumed and the spot price at the wholesale electricity markets and uses them as input parameters in order to calculate the operation schedule of ES systems. This means that, initially, the PDC forecasts those parameters without the presence of ES devices. Then, the power output of ES devices is determined based on the forecasted spot price. Finally, the PDC calculates its electricity bid for 24 hours of the day-ahead markets and submits this to the market operator. However, because the power output of ES devices changes the value of the electricity bid, the actual spot price will be vastly different from the forecasted value. This is due to the fact that in the market-clearing process, the spot price is

Dang Vu Kien is with School of Electrical and Electronic Engineering, Hanoi University of Science and Technology and Institute of Energy, Vietnam (e-mail: kien.dv1997@gmail.com).

Le Thi Minh Chau is with School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam.

Pham Quang Phuong is with School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam.

Pham Nang Van is with School of Electrical and Electronic Engineering, Hanoi University of Science and Technology, Vietnam (e-mail: van.phamnang@hust.edu.vn).

**Corresponding author: Pham Nang Van (e-mail: van.phamnang@hust.edu.vn)*
Manuscript received March 05, 2022; revised May 19, 2022; accepted June 10, 2022.
Digital Object Identifier 10.31130/ud-jst.2022.107E

the result of the economic dispatch (ED) problem, which takes the electricity bid as an input. Consequently, the value of the output power of ES systems may not help the PDC to achieve maximum revenue.

One way to resolve this problem is to recalculate the spot price after the first set of power output values of ES systems is achieved by solving the ED problem (mimicking the market-clearing process of market operators). With the new spot price, new values of the power output of ES devices are also obtained. The process is iterated until it reaches a predefined tolerance. This method is usually time-consuming and ineffective. Therefore, this paper proposes a bi-level optimization model for the strategic operation of ES devices to maximize the net revenue of the PDC considering the uncertainty of demand forecasting. Recently, there has been a variety of works pertaining to the adoption of bi-level optimization for energy storage in the technical literature. Authors [7] coped with the problem of maximizing the yearly revenue of the energy storage in the electricity markets with considerable penetration of wind power. The optimal placement and capacity of storage systems constrained by recovering the investment cost were addressed in [8]. The bidding strategy of ES systems in the day-ahead markets was developed in [9]. Authors [10] suggested a tool for a merchant owning energy storage with the aim of making trading decisions in the day-ahead and real-time markets. This research aims at utilizing the bi-level optimization model to evaluate the effects of the demand forecast uncertainty on the maximum profit of the power distribution company that possesses the energy storage devices. The main contributions of this work encompass:

- Convert the bi-level optimization formulation into the single-level mixed-integer linear programming (MILP) model that can be effectively solved by the standard commercial solvers;
- The impact of the demand forecast uncertainty on the PDC's maximum profit is analyzed and compared.

2. Bi-level optimization formulation

In the day-ahead market, the PDC, as a retailer, purchases electricity from the market at spot price $\pi_{i,t}$ for each bus *i* that it owns (set *A*) and for every period *t* (hour). This spot price is the result of the market-clearing process based on the economic dispatch (ED) problem, which is implemented by the market operator. Then, the PDC sells the electricity to the customers at a retail price $\eta_{i,t}$. For each hour, the electricity purchased at each bus *i* is the amount bid in DA market $(P_{\mathrm{D}i,t})$, while the electricity sold to the customers at each bus *i* is the forecast demand of PDC $\left(P^0_{\mathrm{D}i,t}\right)$. Thus, the net revenue (profit) of a PDC is

$$R_n = \sum_{t=1}^{24}\left[\sum_{i\in A}\left(\eta_{i,t}\times P^0_{Di,t} - \pi_{i,t}\times P_{Di,t}\right)\right] \quad (1)$$

Without ES systems, the electricity bid $P_{Di,t}$ should be equal to the forecasted demand $P^0_{Di,t}$. If the PDC decides to install some ES devices at some of its buses and these devices are regarded as power sources, the electricity bids at ES buses can be calculated as

$$P_{\mathrm{D}i,t} = P^0_{\mathrm{D}i,t} - S_{i,t},\ \forall i\in SC \quad (2)$$

where, $S_{i,t}$ is the power output of ES devices on bus *i* at time *t*, and *SC* is the set of buses where ES devices are installed. Then, the net revenue of a PDC installing ES systems can be expressed as

$$R_n = \sum_{t=1}^{24}\left[\sum_{i\in A}\left(\eta_{i,t}-\pi_{i,t}\right)\times P^0_{Di,t} + \sum_{i\in SC}\pi_{i,t}\times S_{i,t}\right] \quad (3)$$

By adjusting the power output of ES devices (consisting of discharging power $P^d_{Si,t}$ and charging power $P^c_{Si,t}$), the PDC can change its net revenue and ultimately maximize the net revenue. Therefore, the calculation of PDC's profit becomes an optimization problem with the installation of ES devices. In other words, the PDC must schedule the operation of ES systems in order to obtain maximum profit.

In this paper, two optimization problems are solved simultaneously, namely: maximizing the PDC's net revenue as the upper-level optimization problem and; market clearing as the lower-level one. The diagram of the bi-level optimization model is depicted in Figure 1. From Figure 1, the spot price is obtained as the result of the lower-level optimization problem and used as input for the upper-level one. Furthermore, ES systems' power output is the result of the upper-level problem and is also used as input for the lower-level problem.

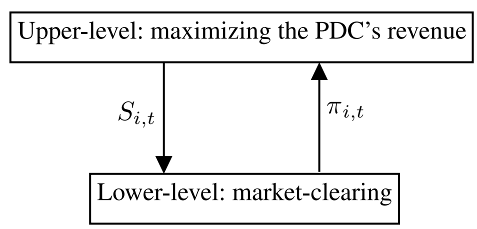

Fig. 1: The bi-level optimization model

The bi-level optimization model shown in Figure 1 can be expanded to integrate the uncertainty of the demand forecast. It is assumed that the uncertainty of load is described by a set of multiple scenarios (*s*=1 *S*) with a probability set of p_s. The load is assumed to follow the Gaussian distribution $N(u,\sigma)$. As such, for each scenario *s* of the load $P^{0,s}_{Di,t}$, a market-clearing problem must be solved, making the total number of the lower-level problems to be *S*. Then, the PDC's expected revenue is calculated across all scenarios. This means that in this new bi-level model, there are one upper-level problem and *S* lower-level ones, which is sketched in Figure 2.

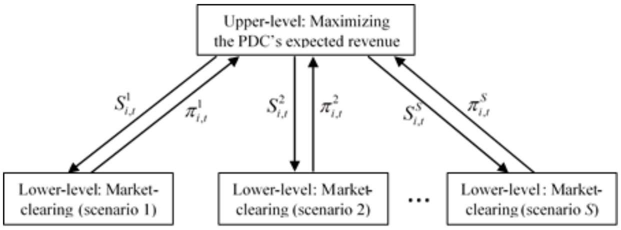

Fig. 2: The bi-level optimization model integrating load uncertainty

2.1. Upper-level optimization formulation

In the upper-level problem, the objective is to maximize the PDC's expected revenue, constrained by the technical specifications of the ES devices.

$$\text{maximize} \left\{ R_n = \sum_{s=1}^{S} p_s \times \sum_{t=1}^{24} \left[\sum_{i \in A} \left(\eta_{i,t} \times P_{Di,t}^{0,s} - \pi_{i,t}^{s} \times P_{Di,t}^{0,s} \right) \right] + \sum_{i \in SC} \pi_{i,t}^{s} \times S_{i,t}^{s} \right\} \tag{4}$$

$$\text{s.t.} \quad P_{Di,t}^{s} = P_{Di,t}^{0,s} - S_{i,t}^{s}, \ \forall i \in SC \tag{5}$$

$$S_{i,t}^{s} = P_{Si,t}^{d,s} - P_{Si,t}^{c,s} \tag{6}$$

$$\left(E_i^{\min} - E_{i0}\right) \le \sum_{\tau=1}^{t} \left(\varsigma_c P_{Si,\tau}^{c,s} - \frac{1}{\varsigma_d} P_{Si,\tau}^{d,s} \right) \tag{7}$$

$$\sum_{\tau=1}^{t} \left(\varsigma_c P_{Si,\tau}^{c,s} - \frac{1}{\varsigma_d} P_{Si,\tau}^{d,s} \right) \le \left(E_i^{\max} - E_{i0}\right), \tag{8}$$

$$\forall i \in SC, \ t = 1, \ldots, 24, \ s = 1, \ldots, S$$

$$0 \le P_{Si,t}^{c,s} \le P_{Si}^{c(\max)} \alpha_{i,t}^{c,s} \tag{9}$$

$$0 \le P_{Si,t}^{d,s} \le P_{Si}^{d(\max)} \alpha_{i,t}^{d,s} \tag{10}$$

$$\alpha_{i,t}^{c,s} + \alpha_{i,t}^{d,s} \le 1 \tag{11}$$

where $E_i^{\max}$ and $E_i^{\min}$ are maximum and minimum energy of the ES device at bus i, respectively; $P_{Si}^{c(\max)}$ and $P_{Si}^{d(\max)}$ are the maximum charging/discharging power of the ES device at bus i; E_{i0} is the the initial state of ES device at node i; ς_c and ς_d are charging and discharging effiency of ES, respectively; $\alpha_{i,t}^{c,s}$ and $\alpha_{i,t}^{d,s}$ denote binary variables.

Equation (4) is the objective function of the upper-level problem, which is to maximize the expected net revenue of the PDC across all scenarios. Mathematical expressions (5) are the demand bid at ES buses. Statements (6) are the power output of the ES at bus i, which is positive for discharging and negative for charging. The energy of the ES system in hour t is limited according to (7) and (8). Constraints (9) and (10) are the limits of charging and discharging power, respectively. Constraint (11) ensures that the ES cannot discharge and charge at the same time (only one status is allowed to be active at a specific time).

2.2. Lower-level optimization formulation for scenario *s*

Each lower-level problem is a market-clearing process corresponding to scenario s of the load. The objective of the problem is to minimize the production cost of conventional power plants while satisfying the constraints of the power system (the ED problem). The ED problem for scenario s is as follows:

$$\text{minimize} \sum_{t=1}^{24} \sum_{i=1}^{N} c_{i,t} \times P_{Gi,t}^{s} \tag{12}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_{Gi,t}^{s} = \sum_{i=1}^{N} P_{Di,t}^{s} : \lambda_t^s \tag{13}$$

$$-\,Limit_l \le \sum_{i=1}^{N} GSF_{l-i} \times \left(P_{Gi,t}^{s} - P_{Di,t}^{s}\right) \le Limit_l : \mu_{l,t}^{\min,s}, \mu_{l,t}^{\max,s}, \ \forall l = 1, 2, \ldots, M \tag{14}$$

$$P_{Gi}^{\min} \le P_{Gi,t}^{s} \le P_{Gi}^{\max} : \omega_{i,t}^{\min,s}, \omega_{i,t}^{\max,s} \tag{15}$$

$$RR_i^{\min} \le P_{Gi,t+1}^{s} - P_{Gi,t}^{s} \le RR_i^{\max} : \xi_{i,t}^{\min,s}, \xi_{i,t}^{\max,s}, \quad \forall i = 1, 2, \ldots, N, \ \forall t = 1, 2, \ldots, 24 \tag{16}$$

where (12) is the objective function; (13) guarantees that generation must meet demand at any given time; (14) enforces the power flow on transmission lines to be within upper and lower limits (power flow is expressed as a function of the nodal power); (15) is the maximum and minimum capacity of the power plant at bus i; (16) is the ramp-rate limits of the power plant at bus i. The variables on the right side of the colons are the dual variables related to the corresponding equality or inequality constraints on the left side of the colons.

2.3. Locational marginal price for scenario *s*

The spot price $\pi_{i,t}$ is the electricity price at each bus which is calculated at the time of transaction. This nodal price is called the locational marginal price (LMP) and is one of the results of the ED problem. Mathematically, the LMP is the partial derivative of the Lagrangian function of the ED problem with respect to the nodal demand. It can be formulated from the dual variables in (13) and (14). The Lagrangian function and the LMP for scenario s are expressed as follows.

$$\mathcal{L}(\mathrm{x}) = \sum_{t=1}^{24} \Bigg\{ \sum_{i=1}^{N} c_{i,t} \times P_{Gi,t}^{s} - \lambda_t^s \times \left[\sum_{i=1}^{N} P_{Gi,t}^{s} - \sum_{i=1}^{N} P_{Di,t}^{s} \right] - \sum_{l=1}^{M} \mu_{l,t}^{\max,s} \times \left[Limit_l - \sum_{i=1}^{N} GSF_{l-i} \times \left(P_{Gi,t}^{s} - P_{Di,t}^{s}\right) \right] - \sum_{l=1}^{M} \mu_{l,t}^{\min,s} \times \left[Limit_l + \sum_{i=1}^{N} GSF_{l-i} \times \left(P_{Gi,t}^{s} - P_{Di,t}^{s}\right) \right]$$

$$- \sum_{i=1}^{N} \begin{bmatrix} \omega_{i,t}^{\max,s} \times \left(P_{Gi}^{\max} - P_{Gi,t}^{s}\right) \\ + \omega_{i,t}^{\min,s} \times \left(P_{Gi,t}^{s} - P_{Gi,t}^{\min}\right) \end{bmatrix} \Bigg\}$$
$$- \sum_{t=1}^{23} \sum_{i=1}^{N} \begin{bmatrix} \xi_{i,t}^{\max,s} \times \left(RR_{i}^{\max} - P_{Gi,t+1}^{s} + P_{Gi,t}^{s}\right) \\ + \xi_{i,t}^{\min,s} \times \left(P_{Gi,t+1}^{s} - P_{Gi,t}^{s} - RR_{i}^{\min}\right) \end{bmatrix} \quad (17)$$

$$\pi_{i,t}^{s} = \lambda_{t}^{s} + \sum_{l=1}^{M} GSF_{l-i} \left(\mu_{l,t}^{\min,s} - \mu_{l,t}^{\max,s}\right), \quad (18)$$
$$\forall i = 1, 2, \ldots, N, \ \forall t = 1, 2, \ldots, 24$$

where (17) states the Lagrangian function of the ED problem; (18) is the formulation of the LMP $\pi_{i,t}^{s}$, which consists of the system marginal price component and the congestion component. The loss component of the LMP is ignored since (12)-(16) utilize the DC power flow model.

3. Single-level MPEC formulation

Currently, there is no direct method to solve a bi-level optimization problem. Therefore, the bi-level problem must be transformed to be solvable. In the lower-level problem, DC-Optimal Power Flow (DC-OPF) is utilized. DC-OPF is a linear model; hence, there exists only one solution to the lower-level problem, and the solution must satisfy the Karush-Kuhn-Tucker (KKT) optimality conditions. Then, the bi-level problem can be transformed into a single-level MPEC (Mathematical programming with equilibrium constraints) one by converting the lower-level problem into its KKT conditions. Eventually, the MPEC problem consists of the upper-level problem and complementary constraints representing the lower-level one.

$$\text{maximize } (4) \quad (19)$$

$$\text{s.t. } (5) - (11) \quad (20)$$

$$c_{i,t} = \lambda_{t}^{s} + \sum_{l=1}^{M} GSF_{l-i}(\mu_{l,t}^{\min,s} - \mu_{l,t}^{\max,s}) + \omega_{i,t}^{\min,s} - \omega_{i,t}^{\max,s} + \xi_{i,t}^{\max,s} - \xi_{i,t}^{\min,s}, \ \forall t = 1 \quad (21)$$

$$c_{i,t} = \lambda_{t}^{s} + \sum_{l=1}^{M} GSF_{l-i}(\mu_{l,t}^{\min,s} - \mu_{l,t}^{\max,s}) + \omega_{i,t}^{\min,s} - \omega_{i,t}^{\max,s} + \xi_{i,t}^{\max,s} - \xi_{i,t}^{\min,s} + \xi_{i,t-1}^{\min,s} - \xi_{i,t-1}^{\max,s}, \ \forall t = 2, \ldots, 23 \quad (22)$$

$$c_{i,t} = \lambda_{t}^{s} + \sum_{l=1}^{M} GSF_{l-i}(\mu_{l,t}^{\min,s} - \mu_{l,t}^{\max,s}) + \omega_{i,t}^{\min,s} - \omega_{i,t}^{\max,s} + \xi_{i,t-1}^{\min,s} - \xi_{i,t-1}^{\max,s}, \ \forall t = 24 \quad (23)$$

$$\sum_{i=1}^{N} P_{Gi,t}^{s} = \sum_{i=1}^{N} P_{Di,t}^{s}, \ \forall t = 1, \ldots, 24 \quad (24)$$

$$0 \leq \mu_{l,t}^{\min,s} \perp Limit_{l} + \sum_{i=1}^{N} GSF_{l-i} \times \left(P_{Gi,t}^{s} - P_{Di,t}^{s}\right) \geq 0 \quad (25)$$

$$0 \leq \mu_{l,t}^{\max,s} \perp Limit_{l} - \sum_{i=1}^{N} GSF_{l-i} \times \left(P_{Gi,t}^{s} - P_{Di,t}^{s}\right) \geq 0 \quad (26)$$
$$\forall l = 1, \ldots M, \ \forall t = 1, \ldots, 24$$

$$0 \leq \omega_{i,t}^{\min,s} \perp P_{Gi,t}^{s} - P_{Gi}^{\min} \geq 0, \ \forall t = 1, \ldots, 24 \quad (27)$$

$$0 \leq \omega_{i,t}^{\max,s} \perp P_{Gi}^{\max} - P_{Gi,t}^{s} \geq 0, \ \forall t = 1, \ldots, 24 \quad (28)$$

$$0 \leq \xi_{i,t}^{\min,s} \perp P_{Gi,t+1}^{s} - P_{Gi,t}^{s} - RR_{i}^{\min} \geq 0, \ \forall t = 1, \ldots, 23 \quad (29)$$

$$0 \leq \xi_{i,t}^{\max,s} \perp RR_{i}^{\max} - P_{Gi,t+1}^{s} + P_{Gi,t}^{s} \geq 0, \ \forall t = 1, \ldots, 23 \quad (30)$$
$$\forall s = 1, \ldots S$$

where the perpendicular sign $\perp$ represents a zero cross-product of of the corresponding variables in vector form.

4. MILP formulation

The non-linearity of the MPEC model (19)-(30) lies in the product term $\pi_{i,t}^{s} \times S_{i,t}^{s}$ in the objective function and the complementary constraints (25)-(30). This makes it difficult to find the solution using available commercial software. However, the model can be transformed into a MILP model, which is then easily solved. According to the strong duality theory, the objective of the primal problem is equal to that of the dual one [11]. Then, the equality between the objective of the ED problem and that of the corresponding dual one is expressed as

$$\sum_{t=1}^{24} \Bigg[\lambda_{t}^{s} \times \sum_{i=1}^{N} P_{Di,t}^{s} + \sum_{l=1}^{M} \mu_{l,t}^{\max,s} \times \left(-Limit_{l} - \sum_{i=1}^{N} GSF_{l-i} \times P_{Di,t}^{s}\right) + \sum_{l=1}^{M} \mu_{l,t}^{\min\, s} \times \left(-Limit_{l} + \sum_{i=1}^{N} GSF_{l-i} \times P_{Di,t}^{s}\right) + \sum_{i=1}^{N} \left(\omega_{i,t}^{\min\, s} \times P_{Gi}^{\min} - \omega_{i,t}^{\max\, s} \times P_{Gi}^{\max}\right) \Bigg] + \sum_{t=1}^{23} \sum_{i=1}^{N} \left(\xi_{i,t}^{\min\, s} \times RR_{i}^{\min}\right) - \sum_{t=1}^{23} \sum_{i=1}^{N} \left(\xi_{i,t}^{\max\, s} \times RR_{i}^{\max}\right) = \sum_{t=1}^{24} \sum_{i=1}^{N} c_{i,t} \times P_{Gi,t}^{s} \quad (31)$$

From the LMP expression in (18), the term $\pi_{i,t}^{s} \times S_{i,t}^{s}$ in (19) can be rewritten as

$$\sum_{t=1}^{24} \Bigg[\lambda_{t}^{s} \times \sum_{i \in SC} S_{i,t}^{s} + \sum_{l=1}^{M} \sum_{i \in SC} GSF_{l-i} \left(\mu_{l,t}^{\min,s} - \mu_{l,t}^{\max,s}\right) \times S_{i,t}^{s} \Bigg] \quad (32)$$
$$= \sum_{t=1}^{24} \sum_{i \in SC} \pi_{i,t}^{s} \times S_{i,t}^{s}$$

Replace the term $P^s_{Di,t}$ in (31) with the term in (5), then substitute (32) into (31), equation (33) is obtained.

$$\sum_{t=1}^{24}\sum_{i=SC}\pi^s_{i,t}\times S^s_{i,t}=\sum_{t=1}^{24}\left[\lambda^s_t\times\begin{pmatrix}\sum\limits_{i\in SC}P^{0,s}_{Di,t}\\ +\sum\limits_{i\notin SC}P^s_{Di,t}\end{pmatrix}\right.$$
$$+\sum_{l=1}^{M}\mu^{\max,s}_{l,t}\times\begin{pmatrix}-Limit_l-\sum\limits_{i\in SC}GSF_{l-i}\times P^{0,s}_{Di,t}\\ -\sum\limits_{i\notin SC}GSF_{l-i}\times P^s_{Di,t}\end{pmatrix}$$
$$+\sum_{l=1}^{M}\mu^{\min,s}_{l,t}\times\begin{pmatrix}-Limit_l+\sum\limits_{i\in SC}GSF_{l-i}\times P^{0,s}_{Di,t}\\ +\sum\limits_{i\notin SC}GSF_{l-i}\times P^s_{Di,t}\end{pmatrix}$$
$$+\sum_{i=1}^{N}\left(\omega^{\min,s}_{i,t}\times P^{\min}_{Gi}-\omega^{\max,s}_{i,t}\times P^{\max}_{Gi}\right)$$
$$\left.-\sum_{i=1}^{N}c_{i,t}\times P^s_{G,t}\right]$$
$$+\sum_{t=1}^{23}\sum_{i=1}^{N}\left(\xi^{\min,s}_{i,t}\times RR^{\min}_i-\xi^{\max,s}_{i,t}\times RR^{\max}_i\right)\tag{33}$$

Then, the objective function in (19) is converted to a linear function in equation (34).

$$R_n=\sum_{s=1}^{S}p_s\times\left\{\sum_{t=1}^{24}\left[\begin{matrix}\sum\limits_{i\in A}\left(\eta_{i,t}-\pi^s_{i,t}\right)\times P^{0,s}_{Di,t}\\ +\lambda^s_t\times\left(\sum\limits_{i\in SC}P^{0,s}_{Di,t}+\sum\limits_{i\notin SC}P^s_{Di,t}\right)\end{matrix}\right.\right.$$
$$+\sum_{l=1}^{M}\mu^{\max,s}_{l,t}\times\begin{pmatrix}-Limit_l-\sum\limits_{i\in SC}GSF_{l-i}\times P^{0,s}_{Di,t}\\ -\sum\limits_{i\notin SC}GSF_{l-i}\times P^s_{Di,t}\end{pmatrix}$$
$$+\sum_{l=1}^{M}\mu^{\min,s}_{l,t}\times\begin{pmatrix}-Limit_l+\sum\limits_{i\in SC}GSF_{l-i}\times P^{0,s}_{Di,t}\\ +\sum\limits_{i\notin SC}GSF_{l-i}\times P^s_{Di,t}\end{pmatrix}$$
$$+\sum_{i=1}^{N}\left(\omega^{\min,s}_{i,t}\times P^{\min}_{Gi}-\omega^{\max,s}_{i,t}\times P^{\max}_{Gi}\right)$$
$$\left.-\sum_{i=1}^{N}c_{i,t}\times P^s_{Gi,t}\right]$$
$$\left.+\sum_{t=1}^{23}\sum_{i=1}^{N}\left(\xi^{\min,s}_{i,t}\times RR^{\min}_i-\xi^{\max,s}_{i,t}\times RR^{\max}_i\right)\right\}\tag{34}$$

Finally, the MPEC model is transformed into a MILP model below.

$$\text{maximize } 34\tag{35}$$

$$\text{s.t. } (20)-(24)\tag{36}$$

$$0\le\mu^{\min,s}_{l,t}\le M^{\min}_{\mu}\nu^{\min,s}_{\mu,l,t}\tag{37}$$

$$0\le Limit_l+\sum_{i=1}^{N}GSF_{l-i}\left(P^s_{Gi,t}-P^s_{Di,t}\right)$$
$$Limit_l+\sum_{i=1}^{N}GSF_{l-i}\begin{pmatrix}P^s_{Gi,t}\\ -P^s_{Di,t}\end{pmatrix}\le M^{\min}_{\mu}\left(1-\nu^{\min,s}_{\mu,l,t}\right)\tag{38}$$

$$0\le\mu^{\max,s}_{l,t}\le M^{\max}_{\mu}\nu^{\max,s}_{\mu,l,t}\tag{39}$$

$$0\le Limit_l-\sum_{i=1}^{N}GSF_{l-i}\left(P^s_{Gi,t}-P^s_{Di,t}\right)$$
$$Limit_l-\sum_{i=1}^{N}GSF_{l-i}\begin{pmatrix}P^s_{Gi,t}\\ -P^s_{Di,t}\end{pmatrix}\le M^{\max}_{\mu}\left(1-\nu^{\max,s}_{\mu,l,t}\right)\tag{40}$$

$$0\le\omega^{\min,s}_{i,t}\le M^{\min}_{\omega}\nu^{\min,s}_{\omega,i,t}\tag{41}$$

$$0\le P^s_{Gi,t}-P^{\min}_{Gi}\le M^{\min}_{\omega}\left(1-\nu^{\min,s}_{\omega,i,t}\right)\tag{42}$$

$$0\le\omega^{\max,s}_{i,t}\le M^{\max}_{\omega}\nu^{\max,s}_{\omega,i,t}\tag{43}$$

$$0\le P^{\max}_{Gi}-P^s_{Gi,t}\le M^{\max}_{\omega}\left(1-\nu^{\max,s}_{\omega,i,t}\right)\tag{44}$$

$$0\le\xi^{\min,s}_{i,t}\le M^{\min}_{\xi}\nu^{\min,s}_{\xi,i,t}\tag{45}$$

$$0\le P^s_{Gi,t+1}-P^s_{Gi,t}-RR^{\min}_i\le M^{\min}_{\xi}\left(1-\nu^{\min,s}_{\xi,i,t}\right)\tag{46}$$

$$0\le\xi^{\max,s}_{i,t}\le M^{\max}_{\xi}\nu^{\max,s}_{\xi,i,t}\tag{47}$$

$$0\le RR^{\max}_i-P^s_{Gi,t+1}+P^s_{Gi,t}\le M^{\max}_{\xi}\left(1-\nu^{\max,s}_{\xi,i,t}\right)\tag{48}$$

where $M^{\min}_{\mu}$, $M^{\max}_{\mu}$, $M^{\min}_{\omega}$, $M^{\max}_{\omega}$, $M^{\min}_{\xi}$ and $M^{\max}_{\xi}$ are sufficiently large constants [12] and $\nu^{\min,s}_{\mu,l,t}$, $\nu^{\max,s}_{\mu,l,t}$, $\nu^{\min,s}_{\omega,i,t}$,$\nu^{\max,s}_{\omega,i,t}$,$\nu^{\min,s}_{\xi,i,t}$ and $\nu^{\max,s}_{\xi,i,t}$ are the auxiliary binary variables. In this model, the retail price $\eta_{i,t}$ is treated as a constant.

5. Results and discussions

In this section, the proposed model is validated using the IEEE 24-bus electrical system with some modifications [13]- [14]. The calculation results are implemented using GAMS 25.1.3 software [15], and the MILP model is solved using CPLEX solver.

System load profiles that correspond to this system are shown in Fig. 3. In Fig. 3, the standard deviation σ is 5% of mean load u corresponding to scenario 3 (*s3*). A total of five scenarios are selected to denote the load uncertainty. The load values of each scenario *s1*, *s2*, *s3*, *s4* and *s5* are $[u+2\sigma; u+\sigma; u; u-\sigma; u-2\sigma]$, respectively, and the corresponding probabilities are [0.023; 0.135; 0.684; 0.135; 0.023]. The more load scenarios there are, the more appropriate the probability of each scenario is. However, more number of load scenarios mean more computational time to solve the problem. Thus, it is important to choose the suitable number of load scenarios to ensure the efficiency of the method.

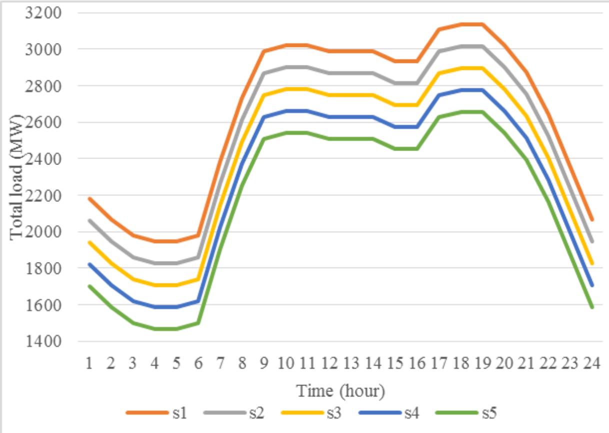

Fig. 3: System demand profile

The PDC provides electricity for demands at buses 4, 5, 6 and 8, and it installs an ES device with a capacity of 50 MWh at bus 8. The parameter of the ES device is shown in Table 1. The retail price $\eta_{i,t}$ is set to 18 \$/MWh.

TABLE 1: Parameters of ES device

$\mathbf{E}^{(\max)}$ (MWh)	$\mathbf{E}^{(\min)}$ (MWh)	$\mathbf{E_0}$ (MWh)	ς_c	ς_d	$P^{c(\max)}$ (MW)	$P^{d(\max)}$ (MW)
45	5	10	0.9	0.9	15	15

Firstly, to assess the effectiveness of the proposed model, the PDC's revenue is compared among three cases: the revenue without the ES device (base case); the actual revenue (after considering the impact of ES's charging/discharging power) received by employing the traditional model and; the revenue by employing the bi-level model. The results are shown in Table 2.

TABLE 2: PDC's revenue under different cases

	Base case	**Traditional**	**Bi-level**
Revenue (\$)	15050.9	16173.5	18983.6

The bi-level model yields a much higher revenue than the traditional model. Specifically, the revenue of the bi-level model exceeds that of the traditional model by 17.4%. This is due to the fact that the traditional model does not reassess the impact of the ES's power on the LMP, which eventually makes the model unable to achieve the maximum revenue. On the other hand, the bi-level model successfully takes into account the correlation between ES's power and the LMP, thereby yielding a higher revenue. This is further confirmed by examining the LMP at bus 8 and the ES's power in each model, as shown in Fig. 4 and Fig. 5.

It is evident that the ES device in the bi-level model makes better choices in terms of determining when and how much power to charge or discharge. At 1AM and 2AM when the LMPs are relatively low, the ES device in the bi-level model charges more power than that in the traditional model to later discharge at 4AM and 5AM, reducing the LMPs at these hours. Meanwhile, the ES device in the traditional model decides to charge about 13 MW at 4AM and 5AM, which increase the LMP at these hours. Similarly, from 8PM to 12AM, the ES device in the bi-level model makes smarter decisions to charge and discharge, thus reducing the LMPs during this period.

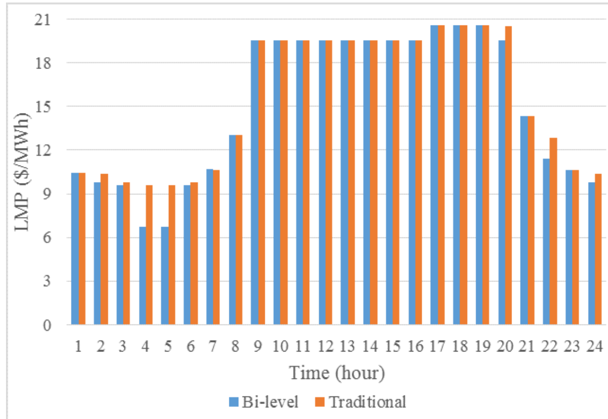

Fig. 4: LMPs at bus 8 in two cases

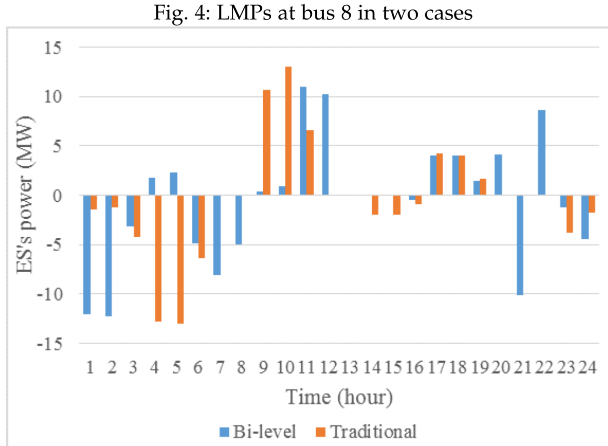

Fig. 5: ES's power in two cases

Secondly, the impact of load uncertainty on the PDC's revenue must be considered. This is done by changing the standard deviation σ and observing the change in revenue. For this purpose, the standard deviation σ will take the following values: 0%, 2.5%, 5%, 7.5% and 10%. The results are summarized in Table 3. It can be seen that the expected revenue is proportional to the standard deviation, with the exception of 7.5% standard deviation.

TABLE 3: Revenue with different standard deviation

σ	0%	2.5%	5%	7.5%	10%
Expected revenue (\$)	14610.3	15550.8	18983.6	18739.0	19514.4

In order to gain insight into this pattern, the LMPs and energy of the ES device will be analyzed as well as the five-scenario revenues for each standard deviation value. The results are visualized in Fig. 6, Fig. 7 and Fig. 8.

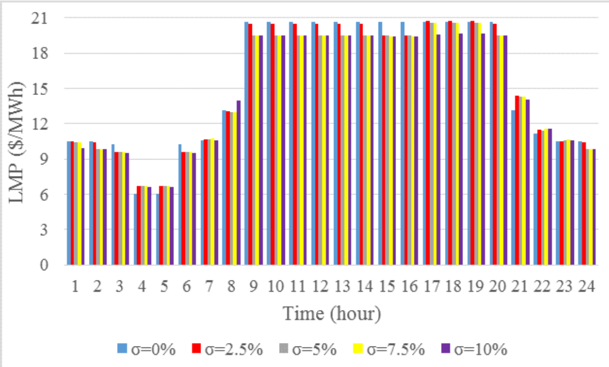

Fig. 6: LMPs with different standard deviation values

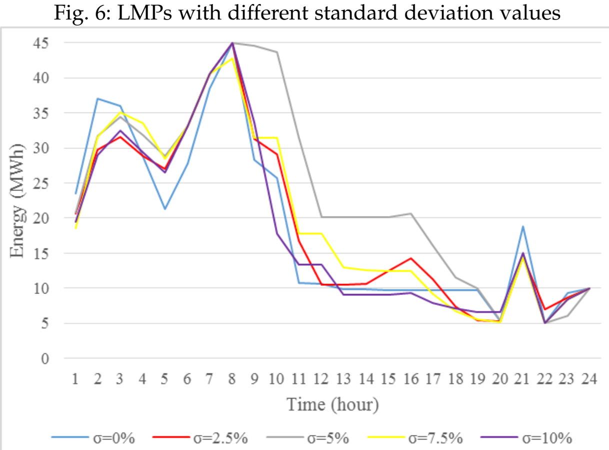

Fig. 7: ES's energy levels with different standard deviation values

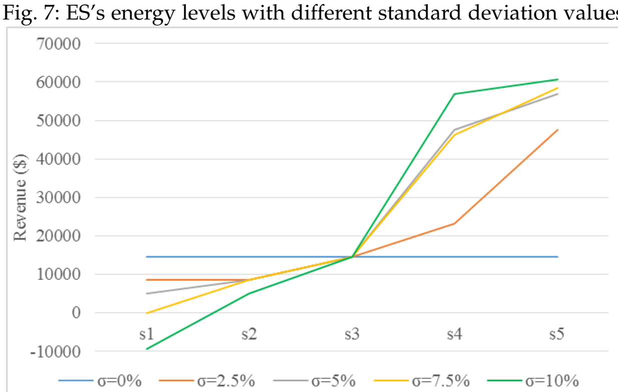

Fig. 8: Revenues with different standard deviation values in five scenarios

In Fig. 6, it can be seen that almost all the LMPs of each hour tend to be inversely proportional to the standard deviation σ. Furthermore, in Fig. 7, the ES's energy levels with different σ somewhat follow the same pattern with little difference in values. This suggests that the ES's behaviour has less to do with the relationship between σ and the expected value. Rather, the change in load profile is the main factor influencing the expected revenue. Indeed, when σ is greater than 0%, scenarios *s1* and *s2* become the high-load scenarios while scenarios *s4* and *s5* become the low-load scenarios. As σ increases, the revenues in *s1* and *s2* decrease, whereas the revenues in *s4* and *s5* rise significantly. In high-load scenarios, most generators with higher costs must generate maximum power to meet the load, which raises the LMP and ultimately reduces the PDC's revenue. Therefore, the ES device cannot reduce the LMP considerably. Consequently, when σ = 10%, the revenue in *s1* falls to roughly -9500$, which indicates that the LMP is higher than the retail price. On the contrary, in low-load scenarios, the LMP is lower, making room for the ES device to function more effectively. As a result, the revenues in low-load scenarios skyrocket, with the revenue in *s5* reaching more than 19500$ when σ = 10%. Because the revenues in low-load scenarios are remarkably higher than those in high-load scenarios, the expected revenue goes up as the standard deviation increases. However, if the load value equals the sum of the minimum power of all generators, the LMP will be at the lowest, and thus the ES device can no longer reduce the LMP. This translates into the revenues in low-load scenarios slowly increasing as the load profile decreases.

Furthermore, by looking at Fig. 8, it is evident that the objective function's values (which are the revenues) across all scenarios are not linear functions. Therefore, if more load scenarios are added, the objective function can be plotted more accurately. This along with a more appropriate probability for each load scenario will yield a better result for the problem.

6. Conclusion

A bi-level optimization model is proposed in this paper. This model simultaneously optimizes an upper-level problem and multiple weighted lower-level problems to obtain the optimal solution. By considering the correlation between the LMP and the ES's power output, the proposed model is able to accurately maximize the revenue of the power distribution company. Moreover, since the model integrates a set of load scenarios, it can help the power distribution company carefully assess its expected revenue and effectively schedule the operation of energy storage systems. The validation of the proposed model is executed using an IEEE 24-bus meshed transmission grid.

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Dang Vu Kien received an Engineer's Degree in Electrical Engineering from Hanoi University of Science and Technology (HUST) in 2020. He is currently a researcher at Institute of Energy (IE). His current research topic is energy storage systems (ESSs) in the power system.

Le Thi Minh Chau received a Master's Degree (2008) and her Ph.D. degree (2012) in Electrical Engineering from Grenoble INP – Universite Grenoble Alpes of French. Currently, she is a lecturer at Hanoi University of Science and Technology. Her research interests focus on Optimization of Power system Operation, Integration of Renewable Energy sources in the network, Solar energy.

Pham Quang Phuong received the Undergraduate Degree in Electrical Engineering at Hanoi University of Science and Technology (HUST) in 2007, the Master Degree in 2008 and Ph.D. in 2011 at Grenoble Polytechnic Institute (France). From 2008 to 2014 he worked as an R&D engineer at CEDRAT SA (now ALTAIR). From 2014 to 2020 he worked as an R&D engineer at EVN National Load Dispatch Centre (Viet Nam). Since 2021, he is a lecturer at HUST.

Pham Nang Van received a Master of Science in Electrical Engineering from Hanoi University of Science and Technology (HUST) in 2010. He is a lecturer at Hanoi University of Science and Technology. His current research topic is new business models in retail markets with the integration of distributed renewables.