Electromechanical Modeling and Simulation of MEMS-Based Piezoelectric Vibration Energy Harvesting Device Using PZT-5H Material

Tuan Ngoc Dao, Phuoc Thanh Quang Le, Tho Quang Than, Son Thanh Nguyen, Tung Thanh Huynh, Phuoc-Anh Le, Phuc Hong Than*, Cong-Kha Pham

Abstract—In this study, we describe the operation of piezoelectric energy converters and electromechanical modeling of piezoelectric energy harvesting (PEH) devices based on microelectromechanical systems (MEMS) for low-power sensors. Consideration is given to a piezoelectric energy harvester based on standard MEMS. The parameters are determined and optimized using a simple MEMS cantilever model. On top of a Brass substrate, the model uses a single layer of piezoelectric material. We utilized the finite element method (FEM) models created with software tools NanoHUB and COMSOL to analyze the electromechanical behavior of MEMS-based piezoelectric energy harvesting (PEH) devices. The electromechanical modeling was applied to predict the modal and harmonic response of the PEH devices. By using a modal analysis, the resonant frequencies are 182 Hz for the FEM models of the PZT-5H PEH device through NanoHUB and COMSOL. The simulated MEMS can provide a voltage between 1.7 and 1.9 mV, 0.074 \( \mu \)W of output power. The produced voltage and output power may be increased by connecting the piezoelectric layers in parallel and series. And the Internet of Things (IoT) sensors might be driven by this array of devices.

Index Terms—Piezoelectric, MEMS, energy Harvesting, piezoelectric materials, finite element analysis.

1. Introduction

RECENTLY, energy harvesting technology has been increased due to the development of IoT and the reduction in power consumption of electronic components such as wireless sensor nodes. A major limitation of IoT sensor nodes is the lifetime of batteries. Energy harvesting has been proposed as a self-power supply for these sensor nodes. There are many different ways to generate usable power, including through human activities, mechanical devices, vehicles, and even environmental energy sources, including solar, thermal, and radio frequency energy. Among environmental energy, vibration energy harvesters can convert vibration energy into electric energy, classified in two types; resonance and non-resonance type. Random vibration such as human movement is the non-resonance type, ambient vibration such as vehicle engine, 3-axis machine tool, railway is resonance type. The demand for self-powered autonomous devices will increase in the period when a lot of IoT sensors are produced and used in various places. Small vibration energy harvesters can be used as the power source of IoT sensor nodes which are installed at a variety of areas. Among vibration energy harvesters, many research efforts have been focused on piezoelectric vibration energy harvesters (pVEHs), energy harvesters with a simple-cantilever structure, high mechanical quality factor, high output power, easy fabrication and and compatibility with microfabrication technology high output power [1]–[12]. Due to its high power density, simplicity of construction, and compatibility with microfabrication technology, piezoelectric energy harvesting concentrates on transferring mechanical vibration energy to electrical energy. Due to their capacity to resonate at low frequencies and produce more power, the majority of piezoelectric energy harvesters take the form of cantilevers. In this paper, we present the successful design and modeling of a piezoelectric vibration energy harvester based on the results of simulations performed in COMSOL Multiphysics and NanoHUB.

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2. Piezoelectric effect

2.1. Piezoelectric Effect

2.1.1. Working principle

The piezoelectric effect was first proposed by Pierre Curie and Jacques Curie in 1880. As illustrated in Fig. 1, crystals become electrically polarized when subjected to a mechanical force. This polarization generates a potential difference, which is inversely proportional to the applied force, through tension and compression. Conversely, when an electric field is applied to the crystal, this behavior is reversed. Depending on the intensity of the electric field, the crystal undergoes mechanical strain along the direction of the field. These phenomena are referred to as the inverse and direct piezoelectric effects, respectively. The forward and reverse piezo effects are normalized by the piezo components as follows [2]:

\[
\begin{bmatrix}
\delta \\
D
\end{bmatrix} =
\begin{bmatrix}
s & d \\
E & \epsilon
\end{bmatrix}
\begin{bmatrix}
\sigma \\
E
\end{bmatrix}
\]

(1)

Where \(\delta\) and \(\sigma\) are the components strain and stress; \(D\) and \(E\) are the electric displacement and electric field components; \(s\), \(\epsilon\), and \(d\) are elastic compliance, dielectric constant and piezoelectric coefficient, respectively. The symbols above \(E\) and \(T\) indicate that the respective constants are evaluated at constant electric field and constant stress, respectively; and the character above \(t\) stands for transpose. In practice, the direct piezoelectric effect is important for sensing and capturing energy when stress effects are used to generate charges on the surface of piezoelectric materials.

2.1.2. Piezoelectric material

The ability of piezoelectric materials to generate an electric charge under external stress or strain when an electric field is applied is a distinct property. Only 20 out of the 32-point groups of limited crystalline materials exhibit piezoelectricity, as illustrated in Fig. 2, based on their structural characteristics. Out of these 20 groups, the 10-point group is classified as \(t\) polar material, where an electric dipole moment occurs without any applied electric field. This is known as spontaneous polarization that changes with temperature, generating thermoelectricity.

2.1.3. Mode-31 and mode-33

A piezoelectric material has to be able to create both an electric charge and a potential difference when mechanical stress is applied in order for it to produce electrical energy. It is crucial to remember that the majority of piezoelectric materials used for energy harvesting have a particular polar axis (\(M\)), and the efficacy of energy harvesting will depend on the direction of the stress applied relative to this axis. The polarization direction determines the polar axis for ferroelectric ceramics or polymers. However, the crystallographic orientation along the \(c\)-axis of the Wurtzite crystal structure determines the polar axis for non-ferroelectric crystalline materials like AlN or ZnO. The "3" direction is seen as being on this polar axis. Due to symmetry, the other directions perpendicular to the polar axis are considered equivalent and referred to as the "1" direction. The applied stress can be either along the polar axis (direction 3) or perpendicular to it (direction 1), leading to two common piezoelectric energy harvesting configurations, known as mode 33 and mode 31, as depicted in Fig. 3. In mode 33, the stress/compression strain is applied parallel to axis 3, while the generated voltage is along the same axis. In mode 31, the stress strain is applied perpendicular to the polar axis and the generated voltage direction is at a right angle to the applied force. The piezoelectric coefficient \((d_{31})\) is used to measure the efficiency of piezoelectric materials, which is defined as the ratio of open-circuit charge density to applied stress (in C/N).
The coefficient \( d_{31} \) is usually higher than the coefficient \( d_{33} \). Mode 31 operation, on the other hand, results in a greater strain in direction 1 and is thus used in vibrating energy harvesting.

\[
\zeta = \frac{e}{2m\omega_n} = C/2\sqrt{MK} \tag{4}
\]

In the case of a beam, the stiffness \( K \) is given by \( K = 3Y_cI/L^2 \), \( Y_c \) is Young’s modulus, \( I \) is the moment of inertia and \( L \) is the length of the beam. The moment of inertia for the rectangular cross-section can be given as \( I = (1/12)bh^3 \), where \( b \) and \( h \) being the beam’s width and height, respectively. For cross-sectional areas and other stiffnesses, the formula can be found in the standard mechanical engineering manual [13], [14]. The ratio between relative displacement \( z(t) \) (i.e. output) and external displacement \( y(t) \) (i.e. input) can be obtained by applying the Laplace transform with zero initial condition in formula (2) is as follows [2]:

\[
\frac{Z(s)}{Y(s)} = \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}
\]

By assuming the external excitation \( y(t) = Y_0\sin(\omega t) \), \( Y_0 \) and the excitation amplitude and frequency, respectively, the time domain \( z(t) \) can be obtained by applying the above inverse Laplace transform (5), which is given as:

\[
z(t) = \frac{Y_0(\omega/\omega_n)^2}{\sqrt{1 - ((\omega/\omega_n)^2)^2 + ((2\zeta\omega/\omega_n)^2)}}\sin(\omega t - \phi) \tag{6}
\]

Where \( z(t) \) is the net shift \( z(t) = Z_0\sin(\omega t - \phi) \), the phase angle between the output and the input can be expressed as:

\[
\phi = \arctan\left(\frac{C\omega}{K - M\omega^2}\right) \tag{7}
\]

Fig. 6 displays an energy conversion diagram for an energy collecting system. From the input vibration energy \( U_{IN} \), the spring mass damper system produces kinetic energy \( U_K \) and spring potential energy \( U_S \). When mechanical and electrical dampers are present, kinetic energy \( U_C \) is dissipated from the system and converted to electrical energy \( U_E \) and loss energy \( U_L \).

\[
U_c = 2C \int_{-Z_0}^{Z_0} \dot{z} dz \tag{8}
\]
Integrating in (8) gives the magnitude $Z_0$ given by (6) and multiplying by the angular frequency gives the expression for the dissipated power:

$$P = \frac{m\zeta Y_0^2 (\omega/\omega_n)^3 \omega^3}{(1 - (\omega/\omega_n)^2 + (2\zeta\omega/\omega_n)^2)}$$  

(9)

Setting the operating frequency as the resonant frequency ($\omega = \omega_n$) in (9) yields the highest power:

$$P_{\text{max}} = \frac{mY_0^2 \omega_n^3}{4\zeta}$$  

(10)

From (10), it can be seen that when the energy receiver operates at a suitable frequency, the power generated can be maximized by reducing damping, increasing natural frequency, mass and excitation amplitude. In principle, zero damping will produce infinite power at resonance; however, in practice this is not possible. Reducing the damping coefficient will increase the displacement of the mass. The maximum available displacement of the mass $Z_L$ is limited by the size and shape of the energy collector as shown in Fig. 9. Therefore, the damping factor must be large enough to prevent the mass from moving beyond the limit. The optimum value of the damping factor $\zeta_{opt}$ exists when the mass displacement falls just below the $Z$ limit $L$ and an unbiased resonance period is reached. Therefore, the limited optimal power is achieved by operating as close to $\zeta_{opt}$ as possible while observing the link displacement [15]–[24]. By rearranging (7), the optimal limiting damping factor $\zeta_{opt}$ is presented as [2]:

$$\zeta_{opt} = \frac{1}{2\omega_c} \sqrt{\omega_c^4 \left(\frac{Y_0}{Z_L}\right)^2 - (1 - \omega_c^2)^2}$$  

(11)

where $\omega_c$ is the ratio between the excitation frequency and the resonant frequency. By substituting (11) for (10), the maximum power wasted under the restricted mode of displacement condition may be found:

$$P_{C_{\text{max}}} = \frac{mY_0^2 \omega_n^3}{2\omega_c^2} \left(\frac{Z_L}{Y_0}\right)^2 \sqrt{\omega_c^4 \left(\frac{Y_0}{Z_L}\right)^2 - (1 - \omega_c^2)^2}$$  

(12)

Therefore, by setting $\omega_c = 1$ (i.e. $\omega = \omega_n$) at (12), the power wasted at resonance can be determined as:

$$P_{C_{\text{res}}} = \frac{1}{2} mY_0^2 \omega_n^3 Z_L$$  

(13)

Equation (13) gives the maximum power dissipated at resonance conditions for the limited mass displacement $Z_L$. Dissipated power can be converted into electric power using piezoelectric, electromagnetic, and electrostatic mechanisms.

### 2.2.2. Equivalent circuit

Piezoelectric MEMS energy receivers with a block at the top have been the most studied and reported configuration. Fig. 7 shows a schematic design and equivalent circuit model of a piezoelectric beam with a single block positioned at the end of the beam.

The piezoelectric thin film on the beam is sandwiched between the bottom and top electrode layers. Fig. 7(b) depicts its equivalent circuit. The mass motion is coupled to the piezoelectric converter in the form of damped oscillations. Thus, the equation of the piezoelectric harvester can be obtained by applying Kirchhoff’s law (KCL).

$$F_M + F_C + F_K + F_E = F_{IN}$$  

(14)

Where $F_{IN}$ denotes the overall force acting on the system, $F_M$ denotes the mass’s kinetic energy, $F_C$ denotes the mechanical damping force, $F_K$ denotes the spring’s elastic potential energy, and $F_E$ denotes the internal force brought on by the internal electric process voltage produced by piezoelectric materials. The electric displacement and strain inside the piezoelectric material of the microcantilever in mode 31 or mode 33 are given by component (1) as the following expressions [2]:

$$\delta = \frac{1}{Y_c} \sigma + d_{3i} E$$  

(15)

$$D = d_{3i} \sigma + \varepsilon_{3i} E$$  

(16)

Where $d_{3i}$ and $\varepsilon_{3i}$ are piezoelectric coefficients and dielectric constants in mode 31 or 33. From (15), the stress caused by voltage generation of piezoelectric pieces is as follows [2]:

$$\sigma_e = d_{3i} Y_C E$$  

(17)

From this, the mechanical internal and external forces of an equivalent circuit are defined by the equation [2]:

$$M \ddot{z} + C_m \dot{z} + Kz + \frac{c_1 Y_c d_{3i}}{g_e} \nu = -M \ddot{y}$$  

(18)

Where $\nu$ is the voltage across the load resistance and $c_1$ is the ratio of the binding force to the stress in the
piezoelectric (i.e., $c_1 = F_E/\sigma_e$). The electric surface can be expressed using Kirchoff’s law [2].

$$\frac{c_2 d_3 g e}{\epsilon_3} z - \frac{1}{RC_p} v = \dot{v}$$

(19)

where $c_2$ is the ratio between the stress of the piezoelectric film and the vertical displacement of the block [16] (i.e., $c_2 = z/\delta$). $R$ is the load resistance and $C_p$ is the total capacitance between the electrodes. For a mode 33 receiver with $n$ pairs of IDEs, the total capacitance is the sum of each separate capacitance $C_i$.

Laplace transforms the (18) and (19), output voltage $v$ can be expressed as [2]:

$$v = -j \omega \epsilon_3 g e \frac{c_2 d_3}{\epsilon_3} \dot{y}$$

$$+ \frac{1}{RC_p} \left( 4 \zeta_m \omega_n^2 \left( 4 \left( k_p^2 + 4 \zeta_m \omega_n \frac{4 \epsilon_3}{RC_p} \right) \right) \right)$$

(20)

$k_p^2 = Y_c d_3^2 / \epsilon_3$ is the formula for the electromechanical coupling coefficient, or $k_p$. When the ambient oscillation’s frequency coincides with the piezoelectric beam’s resonance frequency ($n = 1$), (20) may be simplified as follows [2]:

$$v = -j \omega \epsilon_3 g e \frac{c_2 d_3}{\epsilon_3} \dot{y}$$

$$+ \frac{1}{RC_p} \left( 4 \zeta_m \omega_n^2 \left( 4 \left( k_p^2 + 4 \zeta_m \omega_n \frac{4 \epsilon_3}{RC_p} \right) \right) \right)$$

(21)

From there, the output power transmitted to the load will be described by the equation [2]:

$$P = \frac{1}{2} RC_p \omega_n^2 \left( 4 \epsilon_3^2 + k_p^2 \right) + \frac{1}{2} \left( 4 \epsilon_3^2 + k_p^2 \right)$$

$$+ \left( 4 \epsilon_3^2 + k_p^2 \right) \omega_n^2 \right)$$

3. Piezoelectric beam simulation

To obtain the displacement of the end mass and the generated power as a function of excitation frequency, we conducted a simulation of the system. The simulation was performed within a frequency range of 1 to 200 Hz with an acceleration of 0.2 g. The data utilized to simulate a piezoelectric beam is presented in [25].

TABLE 1: Optimized structure parameters of the MEMS device.

<table>
<thead>
<tr>
<th>Description</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>Beam width</td>
<td>5</td>
<td>mm</td>
</tr>
<tr>
<td>Piezoelectric thickness</td>
<td>0.5</td>
<td>mm</td>
</tr>
<tr>
<td>Substrate thickness</td>
<td>0.5</td>
<td>mm</td>
</tr>
<tr>
<td>Mass length</td>
<td>1</td>
<td>mm</td>
</tr>
<tr>
<td>End-mass density</td>
<td>9000</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Piezoelectric material</td>
<td>PZT-5H</td>
<td></td>
</tr>
<tr>
<td>Substrate</td>
<td>Brass</td>
<td></td>
</tr>
</tbody>
</table>

The output power of the MEMS is depicted in Fig. 8 as a function of piezoelectric material type. In the group of all piezoelectric materials shown in Fig. 8, the energy harvested by the MEMS component made of PZT-5H material is the highest, while that of the MEMS made of PZT-5A material is the lowest. Therefore, we have chosen PZT-5H as the piezoelectric material for this study.

3.1. Simulation on NanoHUB

After tweaking the settings, we obtained a peak power of 0.074 $\mu$W and a peak voltage of 1.75 mV at 182 Hz, as seen in Fig. 10 and Fig. 11, respectively.

3.2. Power as a function of load resistance

The output power of the MEMS is depicted in Fig. 12 as a function of load resistance. The output power rises first from 1 to 1000 $\Omega$, then falls as the load resistance rises.
Generated power measured at atmospheric pressure of 10 KPa is shown in Fig. 13. When changing the atmospheric pressure from 1000 Pa to 45 KPa, the harvested energy remains stable and equal to 0.057 µW, as seen in Fig. 14. Generated power measured at a temperature of 350 K is 0.057 µW, as shown in Fig. 15. From Fig. 16, we find that the energy harvested stays constant at 0.057 µW despite the variation in temperature from 300 K to 550 K. These results demonstrate the stability of energy harvesting of the MEMS device when subjected to variations in temperature and environmental pressure.

3.2. Comparison of NanoHUB results and COMSOL results

Generated power measured at atmospheric pressure of 10 KPa is shown in Fig. 13. When changing the atmospheric pressure from 1000 Pa to 45 KPa, the harvested energy remains stable and equal to 0.057 µW, as seen in Fig. 14. Generated power measured at a temperature of 350 K is 0.057 µW, as shown in Fig. 15. From Fig. 16, we find that the energy harvested stays constant at 0.057 µW despite the variation in temperature from 300 K to 550 K. These results demonstrate the stability of energy harvesting of the MEMS device when subjected to variations in temperature and environmental pressure.

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After optimizing the parameters, we achieved a peak voltage of 1.9 mV at 182 Hz, as seen in Fig. 18. This result agrees well with the simulation result of NanoHUB simulation.

4. Conclusion

Through the use of NanoHUB and COMSOL simulation, we examined the effectiveness of a piezoelectric vibration energy harvester in this work. According to the simulation’s results, the MEMS exhibited good performance, including a peak power of 0.074 µW and a peak voltage of 1.9 mV at 182 Hz. Furthermore, the device simulations demonstrated that as the load resistance is raised, the output power first rises before falling. The output voltage of this device can be improved by using the array structure of these devices. We believe that these results could supply power to low-power IoT sensors. This article also demonstrates the stability of energy harvesting of MEMS components when subjected to variations in temperature and environmental pressure.

References

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