CONTROL AND DETERMINATION OF THE KINEMATIC MODEL POSITION OF PLANAR PARALLEL MECHANISM

Vo Dinh Tung[1]*, Kheylo Surgey[2], Rakhilin Konstantin[3]

[1]HUTECH University, Hochiminh, Vietnam
[2]The Kosygin State University, Russia
[3]Bauman Moscow State Technical University, Russia

*Corresponding author: vd.tung@hutech.edu.vn

(Received: November 18, 2022; Revised: January 28, 2023; Accepted: January 31, 2023)

Abstract - The mechanisms of parallel structure are the subject of many publications due to specific functional properties of these objects, such as load capacity and accuracy This paper offers a solution for the determination of the position and planar parallel two degrees mechanism control. Two problems are solved: the problem of the position of the mechanism and the problem of transition by the executive link of the mechanism through a special position and calculated the motor forces to pass through a special position. There was a proposed and tested control algorithm that offered an example of controlling. The solution for the accuracy position determination for a planar parallel mechanism with two degrees of freedom was offered.

Key words - Parallel mechanism; control; inverse problems of dynamics; accuracy; singularity; special position

1. Introduction

Currently, in machine-building production, installations for welding, cutting, and processing sheet materials (steels, non-ferrous alloys, non-metallic materials) are widely used. There are various process flow diagrams: Material transfer plants, tool transfer plants, and hybrid plants. The main differences between these types are related to the movement of the cut material and the cutting head. High-accuracy mechanisms are required to move the XY table with fixed material and to move the cutting head. One of these types of mechanisms is mechanisms of a parallel structure, which are widely used as executive bodies of machines, equipment, automatic machines, and machine tools [1–7]. Installations with these mechanisms allow the accurate cutting of flat objects with high productivity and reliability.

The important functional part of an industrial manipulator is its control system. The main tasks of the control system are:

- Implementation of the movement of the executive body along a given trajectory determined by equations.

- Ensuring the given laws of coordinates change in time.

The executive system must meet technical requirements and be stable, transient processes must meet the specified quality indicators. There has been significant development of the controlling drives - techniques, methods, and algorithms for their design have been developed. Actuators are servo systems and have feedback for output coordinates, speed, acceleration, or force. Control problems are solved using the methods of the theory of automatic control. In this case, only the mechanisms of robots of a sequential structure were considered. Along with this, difficulties are caused by the problems of controlling the mechanisms of a parallel structure with changing moments of inertia, the nonlinearity of the equations of constraints, the interaction between the degrees of freedom, and a variable gear ratio between the input (generalized) and output (absolute) coordinates of the output link (working body).

The control algorithm based on minimizing the position, velocity, and acceleration errors, uses the solution to the inverse problem of dynamics. This algorithm is associated with the calculation of forces for a given motion. The method of inverse problems of dynamics allows you to build control algorithms based on mathematical models of controlled processes.

However, in the process of operation, the links of the mechanisms of the parallel structure may fall into special positions within the working area. The degree of freedom is lost in them, or there is a loss of control. In addition, the load capacity of parallel manipulators is reduced in the vicinity of special positions. To get out of the special positions, additional actuators can be provided, which are activated when approaching the special positions. After exiting the area of special positions, additional drives are disabled, and the main drives are enabled again. Thus, the problem of developing a control algorithm for a mechanism to get it out of a special situation is of practical interest. One of the drawbacks of the parallel structure mechanisms is the presence of singularity. Often, when synthesizing mechanisms at the design stage, they try to avoid the singularity. However, the simple mechanism considered in the article has the property of singularity and it is necessary to provide solutions that allow the mechanism to be taken out of the singularity point more efficiently than high-accuracy technological operations.

2. Kinematic problem decision

The considered flat mechanism has two degrees of freedom. This mechanism is provided as a device for welding and cutting, where the beam is deflected by a system of two mirrors (Figure 1) [8 – 12].

In the mechanism, the input links *BD* and *CE* (length L1) are connected to the drives installed on the base, and two other links *AB* and *AE* (length L2), hinged together, move the chain coupled to the laser. These links move the *AF* kinematic chain, which is coupled with a laser mounted on the base. This kinematic chain includes a translational pair. The optical axis of the laser is located along the axis of the rotational pair *F*. The beam is deflected by two mirrors located at points *A*, and *F*. Thus, the considered mechanism consists of three kinematic chains.

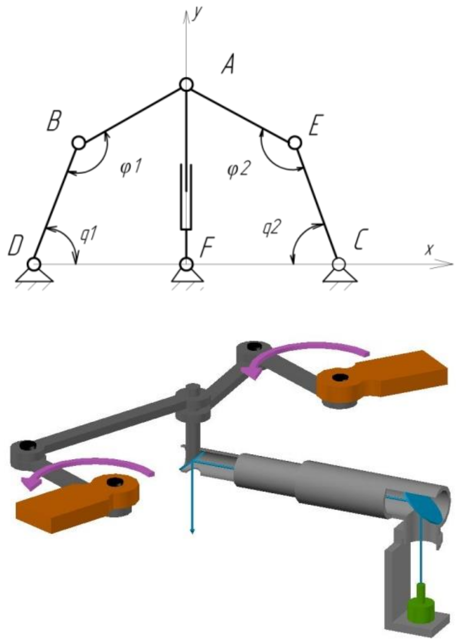

Figure 1. *Scheme of a mechanism with two degrees of freedom for laser welding and cutting*

The control of parallel mechanisms is complex. In this case, there is a nonlinearity of the equations of constraints, the mutual influence between the degrees of freedom, and the inconstancy of the gear ratios between the input (generalized) and output (absolute) coordinates of the output link (working body). Any movement of the output link requires a coordinated movement of the drives. In addition, it is necessary to ensure that the motion error is minimized according to the prescribed law.

This mechanism is widely used in various tap mechanisms such as laser installations, relative manipulation devices, medical robotics. This explains such an interest in the study of control, dynamic and kinematic positioning accuracy of robotic devices using this mechanism. The positioning error is determined by several geometric errors: deviations from the nominal dimensions of the links, gaps in the hinges, loosening in fasteners, temperature fluctuations, and dynamic errors (inaccuracy of the control system, robot vibrations caused by adjacent equipment, residual vibrations). These objectives have not previously been addressed in the comprehensive assessment.

The mechanism must overcome a possible special position - a singularity, in which controllability is lost, and the forces in the drives have an unacceptably large value. Exceeding the specified maximum by the generalized force is considered a dynamic criterion of proximity to the singularity. In this case, control must be transferred to the auxiliary drives, and the main motors must be turned off. After overcoming the singularity, the main drives are switched on again.

3. Manipulator control decision

In robotics, there are different approaches to solving the control problem, while in most publications only mechanisms of robots of a sequential structure were considered [13]. When controlling the mechanism under consideration, we use the compensation control algorithm [14 – 19].

The control problem is formulated in such a way that it is required to find the moments in the drives at which the actuator moves according to a given law, which minimizes the error:

$$\Delta_i = q_i(t) - q_{pi}(t), \qquad \dot{\Delta}_i = \dot{q}_i(t) - \dot{q}_{pi}(t),$$

$$\ddot{\Delta}_i = \ddot{q}_i(t) - \ddot{q}_{pi}(t), \qquad i = 1, 2.$$

Where, $q_i(t)$, $\dot{q}_i(t)$, $\ddot{q}_i(t)$: Real values of coordinates, velocities, and accelerations of the input link (generalized coordinates), $q_{pi}(t)$, $\dot{q}_{pi}(t)$, $\ddot{q}_{pi}(t)$: Specified values of coordinates, velocities, and accelerations of the input link.

To assess the speed of attenuation and the magnitude of the deviation, we use the quadratic integral estimate of the transient process:

$$J_i = \int_{t_0}^{T} (\Delta_i^2 + k_{1i} \cdot \dot{\Delta}_i^2 + k_{2i} \cdot \ddot{\Delta}_i^2) dt$$

J - To assess the speed of damping and the magnitude of the deviation, we use the quadratic integral estimate of the transient process

Where k_{1i}, k_{2i}: are constant coefficients, the value of this integral must take a minimum value.

The minimum functionality is implemented under the condition:

$$\ddot{\Delta}_i + \gamma_{1i} \cdot \dot{\Delta}_i + \gamma_{0i} \cdot \Delta_i = 0 \qquad (1)$$

where, γ_{1i}, γ_{0i} are constant coefficients.

To ensure control, it is necessary to fulfill the condition that errors in the transient process are a solution to equation (1). To define the constant coefficients in equation (1), we represent this expression in the form corresponding to the vibrational link:

$$\tau_i^2 \ddot{\Delta}_i + 2\zeta_i \tau_i \cdot \dot{\Delta}_i + \Delta_i = 0$$

$$\tau_i^2 = \frac{1}{\gamma_{0i}}, \quad 2\zeta_i \tau_i = \frac{\gamma_{1i}}{\gamma_{0i}},$$

where: τ_i time constants, ζ_i damping coefficient of natural vibrations.

It is known that the most preferable is the mode with the damping coefficient of the natural oscillations $\zeta = \frac{\sqrt{2}}{2}$. In this case, the overshoot value $\sigma \approx 5\%$, the duration of the transition process is $t \approx \frac{3 \cdot \tau}{\xi}$ s, and constant γ_0, γ_1 are equal: $\gamma_0 = \frac{1}{\tau^2}$, $\gamma_1 = \frac{\sqrt{2}}{\tau}$ [11].

Using the principle of possible displacements, we can write the ratios:

$$\begin{cases} m\ddot{x}\dfrac{\partial x}{\partial q_1}\delta q_1 + m\ddot{y}\dfrac{\partial y}{\partial q_1}\delta q_1 + J\ddot{q}_1\delta q_1 + M_1\delta q_1 = 0 \\ m\ddot{x}\dfrac{\partial x}{\partial q_2}\delta q_2 + m\ddot{y}\dfrac{\partial y}{\partial q_2}\delta q_2 + J\ddot{q}_2\delta q_2 + M_2\delta q_2 = 0 \end{cases} \quad (2)$$

Where J: a moment of inertia of the input links, m: output link weight, $\dfrac{\partial x}{\partial q_i}, \dfrac{\partial y}{\partial q_i}$: variable coefficients, M_1, M_2: moments in drives.

In the calculations, the moments of inertia of links AB and AE are not considered, because their mass is not commensurate with the mass of the output link. Therefore, in the calculations, the masses of the AB and AE links can be neglected.

The constraint equations can be represented by a system of equations in general form:

$$\begin{cases} F_1 = L_2^2 - \left(x_c + L_1 \cdot \mathbf{cos}(q_2) - x\right)^2 - \left(y_c + L_1 \cdot \mathbf{sin}(q_2) - y\right)^2 = 0 \\ F_2 = L_2^2 - \left(x_D + L_1 \cdot \mathbf{cos}(q_1) - x\right)^2 - \left(y_D + L_1 \cdot \mathbf{sin}(q_1) - y\right)^2 = 0 \end{cases} \quad (3)$$

Where: (x_C, y_C), (x_D, y_D): Coordinates of points D, C, in which the drives are located. L_1, L_2: The length of the links. q_1, q_2: Generalized coordinates (angles of rotation in drives). (x, y): Output link coordinates.

Special provisions can be determined by studying the properties of matrixes composed of constraint equations. We write the equation of velocities in the form:

$$(\mathbf{A})\cdot(\mathbf{V}) = -(\mathbf{B})\cdot(\dot{\mathbf{q}}) \quad (4)$$

Where: **V**: output link speed, $\dot{\mathbf{q}}$: speed of input links.

$$\mathbf{A} = \begin{pmatrix} \dfrac{\partial F_1}{\partial x} & \dfrac{\partial F_1}{\partial y} \\ \dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_2}{\partial y} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \dfrac{\partial F_1}{\partial q_1} & \dfrac{\partial F_1}{\partial q_2} \\ \dfrac{\partial F_2}{\partial q_1} & \dfrac{\partial F_2}{\partial q_2} \end{pmatrix}, \quad \dot{\mathbf{q}} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}$$

In special positions, the determinant of one of the matrixes (**A**) or (**B**) is equal to zero (Figure. 2). Consider a special position in which the links of the mechanism line up and the determinant of the matrix (**A)** is equal to zero.

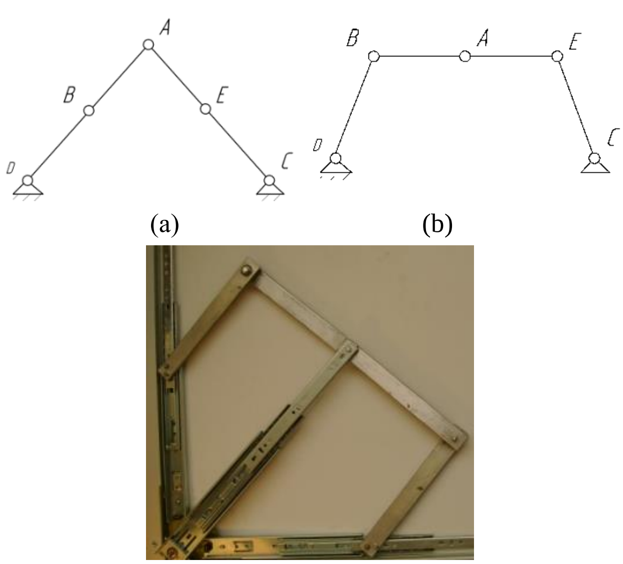

Figure 2. *(a, b): Special positions of the mechanism*

The relationship between the distances of the OC drives and the lengths of the links affects the appearance of singularities.

For numerical simulation, we set the damping coefficient. $\zeta = \dfrac{\sqrt{2}}{2}$ and transient time; t =0,05 s. In this case, the time constant will be equal to $\tau \approx 0{,}011$s, and the feedback coefficients will be γ_0 = 7200, γ_1 = 120. In this case, the mass of the output link is considered.

Based on the presented algorithm, you can find a generalized acceleration:

$$\ddot{q}_i = \ddot{q}_{pi} + \gamma_{1i}\cdot(\dot{q}_i - \dot{q}_{pi}) + \gamma_{0i}\cdot(q_i - q_{pi}) \text{, i= 1,2} \quad (5)$$

Moments *Mi* can be found by substituting accelerations $\ddot{q}_i$ from equations (5) to equations of motion (2):

$$M_i = J_i\left(\ddot{q}_{pi} + \gamma_{i1}(\dot{q}_i - \dot{q}_{pi}) + \gamma_{i0}(q_i - q_{pi})\right) + m\ddot{y}\frac{\partial y}{\partial q_i} + m\ddot{x}\frac{\partial x}{\partial q_i}$$

Acceleration $\ddot{x}, \ddot{y}$ of the output link is determined from the equations obtained by differentiating the velocity equation (4)

Let us set the law of motion in which the output link crosses the point of the special position, y = 0, 7.sin(t); x=0; the mass of the output link is m = 7 kg, the lengths of the links are $L1$ = $L2$ = 0.5 m, the coordinates of points D, C are respectively (-0.4 m; 0), (0.4 m; 0).

As a result of the numerical simulation of the movement of the mechanism according to a given law, the values of the moments in the drives are obtained.

When approaching the special position, there is a sharp increase in the torque (Nm) in the actuator (Figure. 3). To reduce the load in the main drives, it is necessary to provide for a redistribution of the load considering the additional drive.

We place additional drives at points B, E. In this case, the position problem can be written as follows:

$$\begin{cases} F_1^* = \left(L_1^2 + L_2^2 - 2L_1L_2\cos\phi_1\right) - \left(x - x_D\right)^2 - \left(y - y_D\right)^2 \\ F_2^* = \left(L_1^2 + L_2^2 - 2L_1L_2\cos\phi_2\right) - \left(x - x_c\right)^2 - \left(y - y_c\right)^2 \end{cases} \quad (6)$$

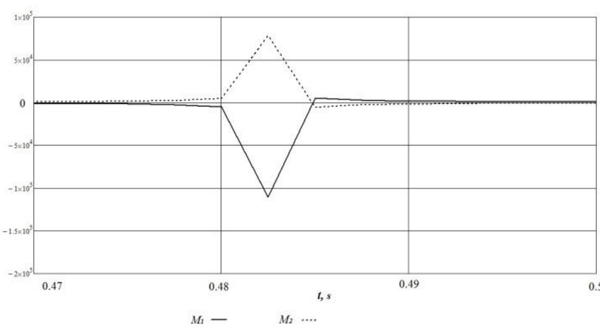

Figure 3. *Graph of changes in the torques (Nm) of the main drives*

Figure 3 shows that from 0.48 to 0.49 seconds when crossing the singularity, peak loads occur in the drives.

Let us express φ_1, φ_2 from equations (6) of the position problem:

$$\phi_1 = 2\mathbf{arccos}\left(\frac{L_1^2 + L_2^2 - y^2 - (x - x_D)^2}{2 \cdot L_1 \cdot L_2}\right)$$

$$\phi_2 = 2\mathbf{arccos}\left(\frac{L_1^2 + L_2^2 - y^2 - (x - x_c)^2}{2 \cdot L_1 \cdot L_2}\right)$$

Using the principle of possible movements (turning actuators on corners $\delta\varphi_1, \delta\varphi_2$) Let's compose the equations of motion:

$$m\ddot{x}\frac{\partial x}{\partial \phi_1}\delta\phi_1 + m\ddot{y}\frac{\partial y}{\partial \phi_1}\delta\phi_1 - J\ddot{q}_1\frac{\partial q_1}{\partial \phi_1}\delta\phi_1 + M_1^*\delta\phi_1 = 0$$

$$m\ddot{x}\frac{\partial x}{\partial \phi_2}\delta\phi_2 + m\ddot{y}\frac{\partial y}{\partial \phi_2}\delta\phi_2 - J\ddot{q}_2\frac{\partial q_2}{\partial \phi_2}\delta\phi_2 + M_2^*\delta\phi_2 = 0$$

Where: m: output link weight;

$\frac{\partial x}{\partial \phi_i}$: variable coefficients;

M_1^*, M_2^*: moments in additional drives, Nm.

We write the equation of velocities in the form:

$$(\mathbf{A}^*) \cdot (\mathbf{V}) = -(\mathbf{B}^*) \cdot (\omega)$$

Where, $\mathbf{V}_i$: output link speed; $\boldsymbol{\omega}_i$: speed of input links.

$$(\mathbf{A}^*) = \begin{pmatrix} \frac{\partial F_1^*}{\partial x} & \frac{\partial F_1^*}{\partial y} \\ \frac{\partial F_2^*}{\partial x} & \frac{\partial F_2^*}{\partial y} \end{pmatrix}, (\mathbf{V}) = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}, \ (\mathbf{B}^*) = \begin{pmatrix} \frac{\partial F_1^*}{\partial \phi_1} & \frac{\partial F_1^*}{\partial \phi_2} \\ \frac{\partial F_2^*}{\partial \phi_1} & \frac{\partial F_2^*}{\partial \phi_2} \end{pmatrix}, (\boldsymbol{\omega}) = \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix}$$

Variable coefficients $\frac{\partial x}{\partial \phi_i}$, $\frac{\partial y}{\partial \phi_i}$ are determined from the velocity equation.

In the example under consideration, when approaching a special position at the time *t* = 0.48 sec, there is a sharp increase in the torque in the main drive and at *t* = 0.49 sec there is a sharp decrease in torque to the permissible value. During this time interval, the main drive is turned off and the additional one is turned on. The torque values in the auxiliary drive are shown in Figure. 4.

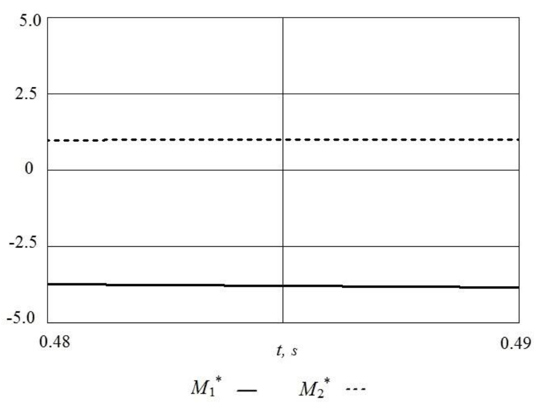

Figure 4. *Torque curve of the auxiliary drive*

Thus, additional drives M1*, and M2* are switched on when the output link is in a special position (singularity). To get out of the special position, a sharp increase in force occurs in the main drives. To get out of the special position, it is enough to create small forces in additional drives M1 *≈ 1Nm, M2 *≈ 3Nm (Figure 4), which eliminates the overloading of forces in the main drives.

4. Positioning accuracy determination

One of the characteristics of assessing the quality of functioning of robotic systems is the positioning accuracy of the working body. The task of ensuring accuracy should be solved at the design stage of machinery and equipment [18, 20, 21].

Positioning can appear because of:

- Systematic errors (inaccuracy of the control system, deviations from the nominal size of the links);

- Random component errors (gaps in hinges, loosening of fastenings, temperature fluctuations, robot vibrations caused by adjacent equipment, residual vibrations, fatigue deformations of links).

Systematic errors can be partially compensated for. Reducing random errors should be considered in design and operation.

The article shows the calculation of the systematic component -and ?? the inaccuracy of manufacturing the links of the mechanism.

Solving the problem of finding the deviation of the output link from the design scheme when the size of the links of the mechanism deviates, we have a system of two linearly independent equations. The system of equations solution below allows us to determine the positioning error.

In general, the position problem is specified in the form of implicit functions:

$$F_1 = (x, y, L_1, L_2, q_1, q_2)$$

$$F_2 = (x, y, L_1, L_2, q_1, q_2)$$

The full differential of the function can be written as:

$$\frac{\partial F_1}{\partial x}\delta x + \frac{\partial F_1}{\partial y}\delta y + \frac{\partial F_1}{\partial L_1}\delta L_1 + \frac{\partial F_1}{\partial L_2}\delta L_2 + \frac{\partial F_1}{\partial q_1}\delta q_1 + \frac{\partial F_1}{\partial q_2}\delta q_2 = 0$$

$$\frac{\partial F_2}{\partial x}\delta x + \frac{\partial F_2}{\partial y}\delta y + \frac{\partial F_2}{\partial L_1}\delta L_1 + \frac{\partial F_2}{\partial L_2}\delta L_2 + \frac{\partial F_2}{\partial q_1}\delta q_1 + \frac{\partial F_2}{\partial q_2}\delta q_2 = 0$$

accept drive increments equal to zero $\delta q_1 = \delta q_2 = 0$. The implicit function equations can be written as follows:

$$\frac{\partial F_1}{\partial x}\delta x + \frac{\partial F_1}{\partial y}\delta y = -\left(\frac{\partial F_1}{\partial \theta_{12}}\delta L_1 + \frac{\partial F_1}{\partial \theta_{11}}\delta L_2\right)$$

$$\frac{\partial F_2}{\partial x}\delta x + \frac{\partial F_2}{\partial y}\delta y = -\left(\frac{\partial F_2}{\partial \theta_{12}}\delta L_1 + \frac{\partial F_2}{\partial \theta_{11}}\delta L_2\right)$$

From the obtained equations it is possible to obtain the values of the positioning deviation of the executive body δx, δy:

$$\delta x = \frac{\frac{\partial F_1}{\partial y}\cdot\frac{\partial F_2}{\partial \theta_{12}}\cdot\delta L_1 - \frac{\partial F_1}{\partial \theta_{12}}\cdot\frac{\partial F_1}{\partial y}\cdot\delta L_1 + \frac{\partial F_1}{\partial y}\cdot\frac{\partial F_2}{\partial \theta_{11}}\cdot\delta L_2 - \frac{\partial F_1}{\partial \theta_{11}}\cdot\frac{\partial F_2}{\partial y}\cdot\delta L_2}{\frac{\partial F_1}{\partial x}\cdot\frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial y}\cdot\frac{\partial F_2}{\partial x}}$$

$$\delta y = \frac{\frac{\partial F_1}{\partial x}\cdot\frac{\partial F_2}{\partial \theta_{12}}\cdot\delta L_1 - \frac{\partial F_1}{\partial \theta_{12}}\cdot\frac{\partial F_1}{\partial x}\cdot\delta L_1 + \frac{\partial F_1}{\partial x}\cdot\frac{\partial F_2}{\partial \theta_{11}}\cdot\delta L_2 - \frac{\partial F_1}{\partial \theta_{11}}\cdot\frac{\partial F_2}{\partial x}\cdot\delta L_2}{\frac{\partial F_1}{\partial x}\cdot\frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial y}\cdot\frac{\partial F_2}{\partial x}}$$

Calculation justification of accuracy is one of the main tasks arising in the design of mechanisms. The presented solution to the problem of determining the deviation of the robot's actuator takes and considers inaccuracies in the manufacture of the mechanism, in particular the lengths of the links.

The obtained dependencies make it possible to determine the deviations of the output link using the accuracy theory. This considers the non-linear nature, since the mechanisms of the parallel structure are multi-links, with the mutual influence of the drives. This complicates the solution to the problem associated with assessing the accuracy of robots functioning.

The deviation of the output link can be determined at any point in the working area, and constructive solutions for its compensation can be proposed. This approach to determining the positioning error makes it possible to calculate the output link deviations for similar parallel structure mechanisms.

5. Conclusion

The paper shows that when the mechanism approaches the special position, there is a sharp increase in the load in the main drive. In this case, turning off two main drives and switching on two additional ones is necessary. After the mechanism moves through a special position, the additional drives should be turned off and switched to the main ones. This solution avoids main drive motor overloads and eliminates loss of control. The permissible torque in the drives can be used as one of the singularity criteria.

The presented estimate of the kinematic accuracy of the mechanism allows compensation for the positioning error at the synthesis stage.

REFERENCES

[1] J. P. Merlet, *Parallel robots*, Kluwer Academic Publishers, 2000.

[2] X. Kong and C. Gosselin, *Type Synthesis of Parallel Mechanisms*, Springer, 2007.

[3] L.-W. Tsai, *Robot analysis: the mechanics of serial and parallel manipulators*, John Wiley & Sons, 1999.

[4] S.V. Amirinezhad and P. Donelan, “Kinematic constraint maps and c-space singularities for planar mechanisms with prismatic joints”, *In International Symposium on Advances in Robot Kinematics*, Springer, 2018, pp. 212–220.

[5] H. M. Daniali, P. Zsombor-Murray, and J. Angeles, “ingularity singularity analysis of planar parallel manipulators”, *Mechanism and Machine Theory*, vol. 30, no. 5, pp. 665–678, 1995.

[6] G. Yang, W. Chen, and I. M. Chen, “A geometrical method for the singularity analysis of 3-RRR planar parallel robots with different actuation schemes”, In*: Intelligent Robots and Systems, IEEE/RSJ International Conference*, vol. 3, pp. 2055–2060, 2002.

[7] D. Zlatanov, R. G. Fenton, and B. Benhabib, “Identification and classification of the singular configurations of mechanisms, *Mechanism and Machine Theory*, vol. 33, no. 6, pp. 743–760, 1998.

[8] H. Dong, S. Xu, C. Xu, and D. Wan, “A Probability Way on Kinematic Positioning Error Manipulator Caused by Joint Clearance”, *Advances in Mechanism and Machine Science, Mechanisms and Machine Science* vol. 73, pp. 2459- 2467, 2019.

[9] Patent RU 2060135, *Laser Cutting Machine*, 01.12.1992.

[10] A. Martín-Parra, D. Rodríguez-Rosa, L. Corral-Gómez, J. Rosado-Linares, F. J. Castillo-García, and E. Ottaviano, “Gateway Points on Scara Parallel Robots. Ultrafast Pick and Place Operations”, *Advances in Mechanism and Machine Science, Mechanisms and Machine Science* vol. 73, pp. 589-598, 2019.

[11] M. Acevedo, “Conditions for Dynamic Balancing of Planar Parallel Manipulators Using Natural Coordinates and their Application”, *The 14th IFToMM World Congress*, Taipei, Taiwan, 2015, pp. 419-427.

[12] X.-J. Liu, J. Wang, and G. Pritschow, “Kinematics, singularity and workspace of planar 5R symmetrical parallel mechanisms, *Mech and Mach Theory*, vol. 41, no. 2, pp. 145-169, 2006.

[13] G. Alici, “An inverse position analysis of five-bar planar parallel manipulators” *Robotica*, vol. 20, no. 2, pp. 195-201, 2002.

[14] R. Paul, *Modelling, trajectory calculation and serving of a computer-controlled arm*, M. Nauka, 1976.

[15] V. A. Glazunov, M. G. Esina, and R. E. Bykov, “Control of parallel manipulator while intersecting singular zones”, *Machinery Manufacture and Reliability*, no. 2, pp. 78-84, 2004.

[16] J. Geng and V. Arakelian, “Design of Partially Balanced Planar 5R Symmetrical Parallel Manipulators via an Optimal Motion Planning”, *Advances in Mechanism and Machine Science, Mechanisms and Machine Science*, vol. 73, pp. 2211-2222, 2019.

[17] F. Bourbonnais, P. Bigras, and I. A. Bonev, “Minimum-time trajectory planning and control of a pick-and-place five-bar parallel robot”, *IEEE/ASME Transactions on Mechatronics*, vol. 20, no. 2, pp. 740-749, 2015.

[18] M. Acevedo and J. M. Reyes, “Dynamic balancing conditions of planar parallel manipulators”, *Proceedings of the IFToMM Symposium on Mechanisms Design for Robotics*, 2015, pp. 55-64.

[19] V. Glazunov and S. Kheylo, “Dynamics and control of planar, translational, and spherical parallel manipulators", *In: Dynamic balancing of mechanisms and synthesizing of parallel robots*, Springer, 2016, pp. 365–403.

[20] V. A. Glazunov, S. V. Kheylo, and A. V. Tsarkov, “The Control Complex Robotic System on Parallel Mechanism”, *In: Smart Electromechanical Systems, Studies in Systems, Decision and Control*, Springer, 2019, pp. 137–146.

[21] S. Kheylo and V. Glazunov, “Kinematics, Dynamics, Control and Accuracy of Spherical Parallel Robot*”, Advances on Theory and Practice of Robots and Manipulators, In: Proceedings of ROMANY 2014 XX CISM-IFToMM Symposium on Theory and Practice of Robots and Manipulators.* Springer Cham Heidelberg New York Dordrecht London, 2014, pp. 133-140.