A Method for Fast Synchronization of Chaotic Systems and Its Application to Chaos-based Secure Communication

Tho Nguyen Van, Ngoc Do Thanh Bao*, Ninh Mai Thi An, Rin Nguyen Vy, Dung Le Dinh

Abstract—Chaos theory is one of the fields of research that has many practical applications. An important application of chaos in communication is that it can be used for secure communication. To be able to use the chaotic signal in communication, we need to synchronize the chaotic signal between the receiver and the transmitter. In this paper, a sliding mode controller is proposed for global synchronization between two chaotic systems. The interesting point of this controller is that it can help reduce the synchronization time based on the selection of the appropriate gain parameter. This method has also been applied to a secure communication system with chaos masking. Finally, numerical simulations are given to illustrate the effectiveness of the proposed method.

Index Terms—chaotic, synchronization, secure communication, sliding mode control, Lyapunov stability theory

1. Introduction

CHAOS behavior, an interesting phenomenon in various nonlinear systems, has been discovered by scientists for a long time. In the early 1900s, Henri Poincare published significant findings about small differences in the initial conditions when he researched orbits in the solar system [1]. The first considerable leap in chaos theory was made by Edward Lorenz. Lorenz showed the absence of a period and the divergence of the system with only a very small difference in the initial conditions [2]. His work proposed a simplified mathematical mode including three ordinary differential equations now known as the Lorenz equations. In a chaotic system, the produced signals do not synchronize with any other system. It means two chaotic systems are impossible to synchronize with each other. However, Pecora and Carroll demonstrated that two chaotic systems could be synchronized if they could exchange information correctly [3]. Their work on the synchronization of chaotic systems has attracted a lot of attention in various domains of science and engineering during the last two decades, especially in information technology [4]–[6].

Because of the importance of the synchronization phenomenon in chaotic systems, many synchronization methods have been developed, such as adaptive synchronization [7], pulse synchronization [8], method reverse step design [9], observer-based synchronization [10], [11], and sliding mode control method [12]–[17]. Among these methods, a sliding mode controller with the advantage of a solid response for parameter uncertainty and noise seems attractive for fast synchronization. In [12], Sundarapandian proposed a nonlinear controller to synchronize Lorenz's and Pehlivan's chaotic systems. Rodrigues et al. also proposed a sliding mode control law and a standard observer for the synchronization problem [13]. Besides, the sliding mode controller has been applied to the synchronization of various chaotic systems such as: Rikitake system [14], Four-Scroll Novel Chaotic System [15], forchaotic gyros systems [16], Lu and Bhalekar-Gejji chaotic systems [17].

Motivated by the work on synchronization of chaotic systems proposed by Pecora and Carroll [3] and by the fact that power spectrums of chaotic systems are similar to white noise, the produced signals from chaotic systems can be used for carrying and hiding information over the communication channel. As a result, many studies on secure communication have been published in the literature. In [18], the authors have proposed the chaos mask-based communication scheme. In this scheme, the two chaos generators at the transmitter and receiver will be synchronized with each other. The message is added to the chaotic signal of the transmitter and recovered at the receiver. Later, many models were also published with more robust synchronization methods [19]–[22]. Chaos-based modulation methods have also been proposed [23]–[25]. Chaotic modulation offers a potentially simple solution for wideband communications; it ought to offer better performance under multipath propagation conditions.

In this work, we propose a chaotic synchronization

Nguyen Van Tho and Dung Le Dinh are with The University of Danang - VN-UK Institute for Research and Executive Education.

Ngoc Do Thanh Bao and Ninh Mai Thi An are with Duy Tan University.

Rin Nguyen Vy is with FPT University.

**Corresponding author: Ngoc Do Thanh Bao (E-mail: baongocdt@gmail.com)*

Manuscript received October 30, 2022; revised November 25, 2022; accepted December 23, 2022.
Digital Object Identifier 10.31130/jst-ud.2022.541ICT

method using a sliding controller and apply it to a chaos-based secure communication system. The advantage of this approach is that it is possible to reduce the synchronization time based on the appropriate gain parameter selection of the slide controller. We also conducted a numerical simulation to evaluate the effectiveness of the proposed method and its applicability in the secure communication system.

The rest of the paper is organized as follows. Section 2 describes the Lorenz chaotic system and proposes the chaotic synchronization method based on the stability theory. Section 3 presents simulation results and analyzes the advantages of the synchronous method. Section 4 describes the results of applying the proposed synchronous method to the secure communication model. Conclusions are made in section 5.

2. Lorenz system and Synchronization

2.1. Lorenz system

In this paper, the Lorenz system, a well-known model for systems and its synchronization-based application, is used to exemplify the proposed method.

In general, dynamics of chaotic systems are described by a set of nonlinear differential equations with respect to state variables. The following difference equations describe the Lorenz dynamic continuous 3D system:

$$\begin{aligned} \frac{dx}{dt} &= \sigma(y - x) \\ \frac{dy}{dt} &= x(r - z) - y \\ \frac{dz}{dt} &= xy - bz \end{aligned} \tag{1}$$

where x, y, z is the state variables; $/sigma, r, b$ is the system parameters. Well-known parameter values for Lorenz system showing chaotic behaviors are often used for numerical simulations: $\sigma = 3$, $r = 28$, and $b = 8/3$.

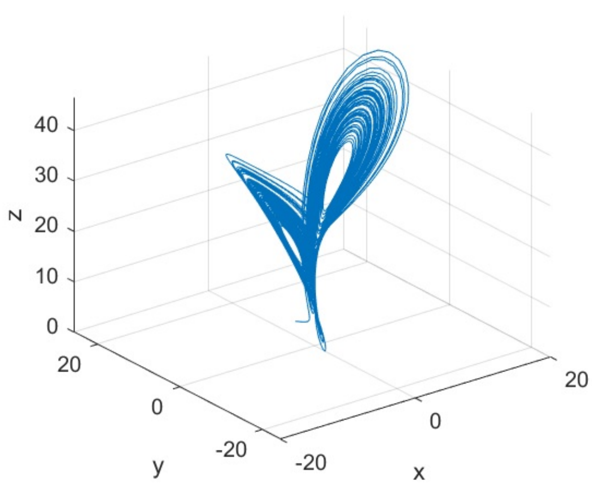


Fig. 1: The orbit of the phase on the Lorenz chaotic system

If the Lorenz system has chaotic behaviors, it operates without cycles. Fig. 1 shows the trajectory of the phase on the Lorenz chaotic system with given parameters. We see that the system is in a non-periodic motion; when the time approaches infinity, the curve in phase space does not go to a fixed point or a periodic trajectory. In addition, this orbital is always in a definite phase space domain and never shifts out of this domain.

The Lorenz system is very sensitive to initiation conditions. Even a very small change in the initial condition can make a massive difference to the system. As can be seen from Fig. 2, two Lorenz systems with slightly different initial conditions will split apart quickly, creating completely different orbits.

Although the operation of a Lorenz system is not cyclical, it is also not a random process. In general, the Lorenz chaotic system is a deterministic system, which could be represented by a set of equations with specific parameters. We can determine the value of the system at a specified time.

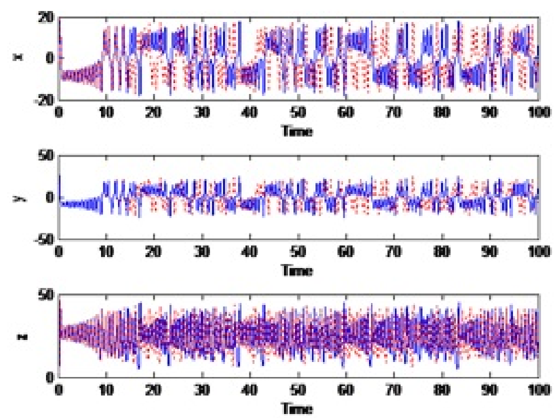


Fig. 2: Variable over time of Lorenz chaotic system with the initial conditions are the very small difference.

Because of its characteristics, the chaotic system has been considered an ideal solution for secure communication systems. The chaotic signal helps mask the information transmitted over the communication channel. Simultaneously, the sensitivity to the system's initial conditions makes it very difficult to estimate a future position from its position in the past.

2.2. Synchronization of Lorenz Systems

Chaos synchronization is a very interesting topic that has been recently studied. Chaos synchronization has the following feature: two identical chaotic systems with different initial conditions will diverge from each other, but they will retain the same pattern of attraction. In order to use in the communication system, the receiver needs a copy of the transmitter's chaotic signal. In other words, we need to synchronize the chaotic signal between the receiver and the transmitter. Synchronization is a requirement of many communication systems; however, the traditional synchronous implementations of these systems are not applicable in a chaotic system.

New methods are therefore required. Consider that the Lorenz system includes one drive system and one response system. The drive system is described by the Lorenz dynamics as follows:

$$\begin{aligned}\frac{dx_1}{dt} &= \sigma(y_1 - x_1)\\ \frac{dy_1}{dt} &= x_1(r - z_1) - y_1\\ \frac{dz_1}{dt} &= x_1y_1 - by_1\end{aligned} \tag{2}$$

The response system is also described by the Lorenz dynamics:

$$\begin{aligned}\frac{dx_2}{dt} &= \sigma(y_2 - x_2) + u_1\\ \frac{dy_2}{dt} &= x_2(r - z_2) - y_2 + u_2\\ \frac{dz_2}{dt} &= x_2y_2 - by_2 + u_3\end{aligned} \tag{3}$$

where $u_i (i = 1, 2, 3)$ is control laws that should be identified to ensure the response system is synchronized with the drive system. The error between two chaos systems is defined as:

$$\begin{aligned}e_1 &= x_2 - x_1\\ e_2 &= y_2 - z_1\\ e_3 &= z_2 - z_1\end{aligned} \tag{4}$$

Take the derivative on both sides of Eq(2), we have system error differential equations as follows:

$$\begin{aligned}\frac{de_1}{dt} &= \sigma(e_2 - e_1) + u_1\\ \frac{de_2}{dt} &= re_1 - e_2) + x_2z_2 - x_1z_1 + u_2\\ \frac{de_2}{dt} &= -be_3 - x_1y_1 + x_2y_2 + u_3\end{aligned} \tag{5}$$

The mission of the control law is to ensure the synchronization between the drive system and the response system so that errors toward zero.

Here, we select Control Act:

$$\begin{aligned}u_1 &= -ke_1 - \sigma e_2\\ u_2 &= -ke_2 - re_1 + x_1z_1 - x_2z_2\\ u_3 &= -ke_3 - x_2y_2 + x_1y_1\end{aligned} \tag{6}$$

where k is customizable gain parameter, $k \geq 0$. From Eq(5) and Eq(6), we have:

$$\begin{aligned}\frac{de_1}{dt} &= (\sigma + k)e_1\\ \frac{de_2}{dt} &= (1 + k)e_2)\\ \frac{de_2}{dt} &= -(b + k)e_3\end{aligned} \tag{7}$$

Eq(7) has a solution:

$$\begin{aligned}e_1 &= e^{-(\sigma+k)t}\\ e_2 &= e^{-(1+k)t}\\ e_3 &= e^{-(b+k)t}\end{aligned} \tag{8}$$

When $t \to \infty$, the e_1,e_2,$e_3 \to 0$.

Select the Lyapunov function to ensure the stability of the Lorenz chaotic system as follow:

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{9}$$

Take the derivative both sides of Eq (9), we have:

$$\frac{dV(e)}{dt} = e_1\frac{de_1}{dt} + e_2\frac{de_2}{dt} + e_3\frac{de_3}{dt} \tag{10}$$

Replace Eq(7) to Eq(10), the result is:

$$\frac{dV(e)}{dt} = -(\sigma + k)e_1^2 - (1 + k)e_2^2 - (b + k)e_3^2 \tag{11}$$

We found that $V(e)$ is a positive function in R_3 and $\frac{dV(e)}{d(t)}$ is a negative function in R_3, i.e., a stable system at equilibrium point $(0, 0, 0)$. According to Lyapunov stable theory, the control law in Eq(6) will ensure stability for Lorenz's chaotic system.

3. Analysys and Simulation

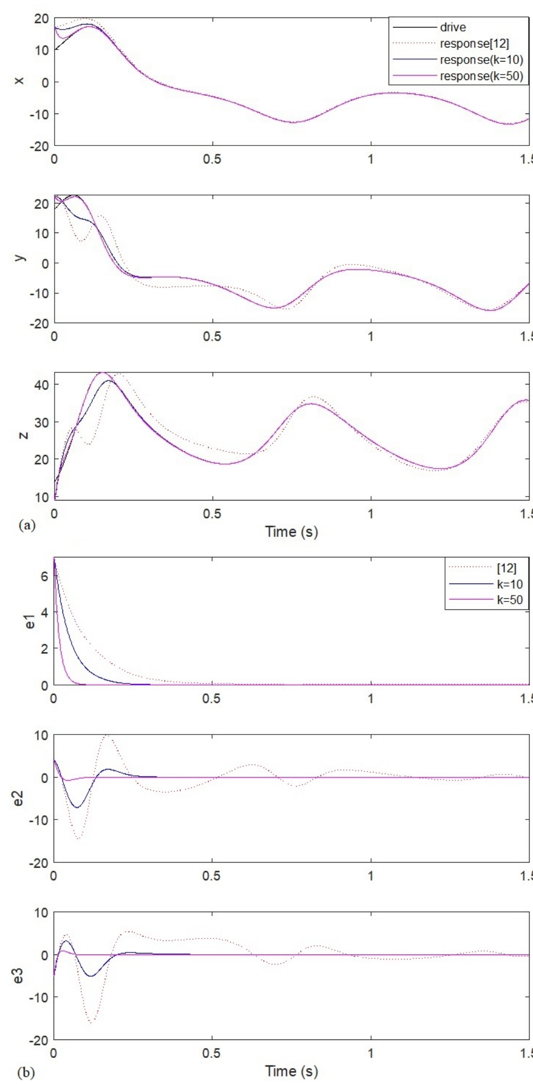


Fig. 3: (a) all states and (b) Estimated for all state.

As can be seen from Eq(6), the proposed control law uses a custom value k as an error amplifier between the drive and response systems. If k is too small, the time required for the two systems to synchronize will be significant. However, when parameter k is too large, the control function oscillates, generating high-frequency square pulses. Therefore, choosing the proper value of k can reduce the time required to synchronize between two systems.

For evaluation, we simulate the proposed synchronous control law with system parameters are $\sigma = 10$, $r = 28$ and $b = 8/3$. The initial conditions of the drive system are $x_1(0) = 10$, $y_1(0) = 18$ and $z_1(0) = 14$; initial condition of the response system are $x_2(0) = 17$, $y_2(0) = 22$ and $z_2(0) = 9$. Simulations were performed with $k = 10$ and $k = 50$, respectively. The simulation results are shown in Fig. 3.

As can be seen from Fig. 3, the response system starts to trace the drive system and finally becomes the same; simultaneously, the synchronization error converges to zero in the case of $k = 10$ and $k = 50$, respectively. However, the time required to synchronize two Lorenz systems in the case of $k = 50$ is $t \geq 0.09s$, much smaller than that of $t \geq 0.44s$ with $k = 10$. .The proposed method is also compared with related work in [12]. By choosing the proper value of k in the proposed method, the synchronization process could be accomplished in a shorter period.

4. Application on Secure Communication

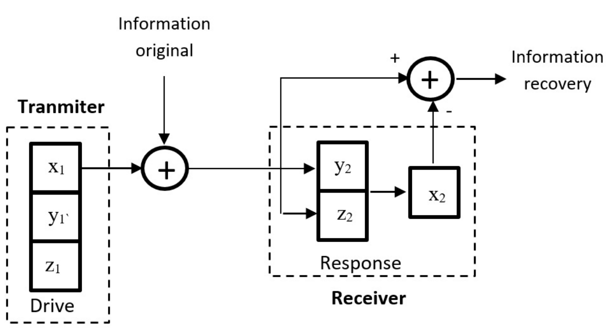


Fig. 4: The chaos-based secure communication system.

In Fig. 4, we display the architecture of secure communication applications based on chaotic systems. At the transmitter, a masking signal produced by the drive system has added the information to create the information-carrying signal. The information-carrying signal is transmitted to the receiver over a communication channel. The masking signal is removed at the receiver, and information is recovered. The mask removal must rely on synchronizing the two chaotic systems at the transmitter and the receiver. We apply the proposed synchronization method for this communication system and use numerical simulations for evaluation.

The simulation results in Fig. 5 show that the receiver can synchronize with the transmitter based on the received signal. The information can successfully be recovered by removing the masking signal afterward.

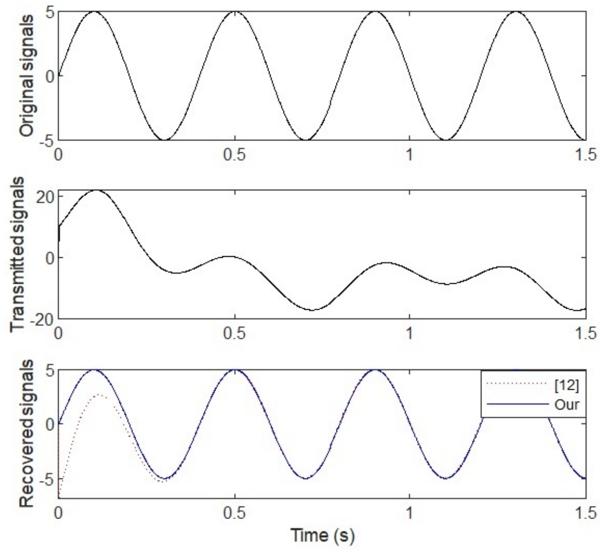


Fig. 5: Simulation chaos-based secure communication.

5. Conclusion

In this paper, a new sliding mode control method has been proposed for the global chaotic synchronization of the Lorenz system. Sliding mode controllers with a customizable gain parameter can reduce synchronization time compared to previous methods. The application of the synchronization method to secure communication systems has been successfully evaluated. Numerical simulations show the effectiveness of the proposed method.

References

[1] D. S. Shafer, “Nonlinear dynamics and chaos: With applications to physics, biology, chemistry, and engineering (steven h. strogatz),” *SIAM Review*, vol. 37, no. 2, pp. 280–281, 1995.

[2] E. N. Lorenz, “Deterministic Nonperiodic Flow,” *Journal of Atmospheric Sciences*, vol. 20, no. 2, pp. 130–148, Mar. 1963.

[3] L. M. Pecora and T. L. Carroll, “Synchronization in chaotic systems,” *Phys. Rev. Lett.*, vol. 64, pp. 821–824, Feb 1990.

[4] M. Brown, “Chaos and nonlinear dynamics: An introduction for scientists and engineers,” *Shock and Vibration*, vol. 3, 01 1996.

[5] R. C. Hilborn, *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers*. Oxford University Press, 09 2000.

[6] M. Kennedy, R. Rovatti, and G. Setti, *Chaotic Electronics in Telecommunications*. CRC Press, 01 2000.

[7] T.-L. Liao and S.-H. Tsai, “Adaptive synchronization of chaotic systems and its application to secure communications,” *Chaos, Solitons & Fractals*, vol. 11, no. 9, pp. 1387–1396, 2000.

[8] L. Zhang and H. Jiang, “Impulsive generalized synchronization for a class of nonlinear discrete chaotic systems,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 4, pp. 2027–2032, 2011.

[9] L.-B. Yang and T. Yang, “Sampled-data feedback control for chen’s chaotic system,” *Acta Physica Sinica -Chinese Edition-*, vol. 49, pp. 1041–1042, 06 2000.

[10] J. Guo, Z. Zhao, F. Shi, R. Wang, and S. Li, “Observer-based synchronization control for coronary artery time-delay chaotic system,” *IEEE Access*, vol. 7, pp. 51 222–51 235, 2019.

[11] J. Li, W. Liu, and Z. Wang, “Observer-based quantized chaotic synchronization,” in *Proceedings of the 29th Chinese Control Conference*, 2010, pp. 4301–4305.

[12] S. Vaidyanathan, "Global chaos synchronization of lorenz and pehlivan chaotic systems by nonlinear control," *International Journal of Advances in Science and Technology*, vol. 2, 01 2011.

[13] V. H. P. Rodrigues and T. R. Oliveira, "Chaos synchronization applied to secure communication via sliding mode control and norm state observers," in *2014 13th International Workshop on Variable Structure Systems (VSS)*, 2014, pp. 1–6.

[14] C.-S. Fang and Y.-Y. Hou, "Sliding mode controller design for chaos synchronization of rikitake chaotic systems," in *2016 IEEE International Conference on Control and Robotics Engineering (ICCRE)*, 2016, pp. 1–4.

[15] S. Vaidyanathan and S. Sampath, "Global chaos synchronization of a four-scroll novel chaotic system via novel sliding mode control," in *2014 International Conference on Science Engineering and Management Research (ICSEMR)*, 2014, pp. 1–6.

[16] L. Yin, Z. Deng, B. Huo, and Y. Xia, "Finite-time synchronization for chaotic gyros systems with terminal sliding mode control," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 6, pp. 1131–1140, 2019.

[17] J. P. Singh, P. P. Singh, and B. K. Roy, "Synchronization of lu and bhalekar-gejji chaotic systems using sliding mode control," in *International Conference on Information Communication and Embedded Systems (ICICES2014)*, 2014, pp. 1–5.

[18] L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz, "Experimental demonstration of secure communications via chaotic synchronization," in *Chua's Circuit*, 1992.

[19] M. Chen, D. Zhou, and Y. Shang, "A sliding mode observer based secure communication scheme," *Chaos, Solitons & Fractals*, vol. 25, no. 3, pp. 573–578, 2005.

[20] F. Zhu, J. Xu, and M. Chen, "The combination of high-gain sliding mode observers used as receivers in secure communication," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 11, pp. 2702–2712, 2012.

[21] V.-N. Giap, S.-C. Huang, and Q. D. Nguyen, "Synchronization of 3d chaotic system based on sliding mode control: Electronic circuit implementation," in *2020 IEEE Eurasia Conference on IOT, Communication and Engineering (ECICE)*, 2020, pp. 156–159.

[22] J. C. L. Chan, T. H. Lee, and C. P. Tan, "Secure communication through a chaotic system and a sliding-mode observer," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 3, pp. 1869–1881, 2022.

[23] G. Kolumban, M. Kennedy, and L. Chua, "The role of synchronization in digital communications using chaos. ii. chaotic modulation and chaotic synchronization," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 45, no. 11, pp. 1129–1140, 1998.

[24] H. Dedieu, M. Kennedy, and M. Hasler, "Chaos shift keying: modulation and demodulation of a chaotic carrier using self-synchronizing chua's circuits," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 40, no. 10, pp. 634–642, 1993.

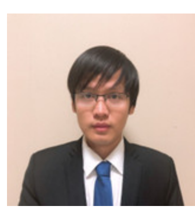
[25] G. Kolumbán, "Basis function description of chaotic modulation schemes," in *International Workshop on Nonlinear Dynamics of Electronic Systems (NDES'00), Catania, Italy*, 2000, pp. 165–169.

Tho Nguyen Van is a lecturer at The University of Danang - VN-UK Institute for Research and Executive Education . His current research interests include Reconfigurable Computing and Architecture, Internet of Things and Chaos-based communication.

Ngoc Do Thanh Bao is a lecturer at Duy Tan University. Her current research interests include Machine Learning Algorithms, Image processings and Chaos-based Secure.

Ninh Mai Thi An is a lecturer at Duy Tan University. Her current research interests include Algorithm theory, Image processings and Natural language processings.

Rin Nguyen Vy is a lecturer at FPT University. His current research interests include Internet of Thing, Networking and Cloud Computing.



Dung Le Dinh was born in Hue, Vietnam, in 1986. He received the B.E. degree from the University of Danang - University of Science and Technology, Da Nang, Vietnam, in 2009, and the M.E. degree from National Chiao-Tung University, Hsinchu, Taiwan, in 2013, and the Ph.D. degree at Nara Institute of Science and Technology, Nara, Japan, in 2019. He is currently working at The University of Danang – VN-UK Institute for Research and Executive Education. His research interests include optical wireless communications and networking, channel coding, and machine learning.