SUFFUSION PREDICTION MODEL BASED ON PHYSICAL PARAMETERS FOR GAP-GRADED AND WELL-GRADED SOILS

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Abstract - Suffusion is one of the four types of internal erosion and accounts for 46% of erosion phenomena in earth dams. Suffusion is an erosion of fine particles inside the soil structure that move out of the voids between coarse particles caused by permeability flow. Identification of internal erosion is necessary to mitigate the loss of humans and property as underground erosion occurs. However, determining suffusion susceptibility in situ is complex and takes a lot of time. This study proposed a fine-grained erosion prediction model based on physical parameters related to the grading curve and the dry density. Using the principal component analysis (PCA) tool evaluated the influence of these parameters on suffusion. The proposed predictive model of the erosion resistance index (ERI) supports stakeholders in quickly determining the extent of erosion and then taking appropriate measures to maintain the earth dams.

Key words - Erosion resistance index; well-graded soil; gap-graded soil; prediction model; suffusion.

1. Introduction

There are many reasons for the failures of earth dams [1]. A common cause that induces damage to irrigation construction is internal erosion. This paper focuses on suffusion, a case of internal erosion [2]. According to Wan and Fell [3], suffusion can occur as three conditions are satisfied: fine particle content is sufficiently large, fine particles in the voids of coarse particles that do not subject to effective stress, and the flow rate must be large enough. In addition, suffusion is a complex double process of three activities: detachment, migration, and clogging [4]. Fine particles that do not pass through the constrictions of coarse particles can become trapped in their voids [5]. Loose particles that are moved or filtered during erosion change the soil's hydraulic path pressure, voids, and physical and mechanical properties [6]. Moreover, suffusion may increase the porosity and permeability of the soil layer or soil structures [7]. Fine-grained erosion, although there is little potential risk of soil structure instability or collapse and loss of soil skeleton, can cause long-term damage to earth dams [8]. Thus, fine-grained erosion is one of the common causes of the failure of water structures such as earth dams, dikes, and embankments over time.

Floods that inundate habitats and arable land have severely eroded embankments and dams, resulting in human and economic losses [9-10]. In Vietnam, there are about 7,800 large and small dams, of which a lot of dikes and dams need to be repaired and upgraded [11]. According to statistics, over 2,700 km of dikes protecting densely populated areas are considered grade III above special level. Thus, to ensure the safety of embankments and dams, the prediction of suffusion for the existing structures seems to be an essential issue for managers to ensure economy and sustainability, as well as the future safety of the areas surrounding the dams.

Forecasting suffusion in earthen dams has the function of ensuring safety for plants, animals, and people living around them. Thus, this fine-grained erosion prediction is to optimize maintenance and ensure the integrity of dams. In addition, the length of earth dams is often very long, which leads to a lot of testing to evaluate the erosion level. At the same time, this testing process takes a lot of time. On the other hand, it is complex in the laboratory to describe the degree of suffusion of different soils along the length of the dam [12]. This difficulty can be solved by using a predictive model for optimization that describes the degree of fine-grained erosion.

The internal stability of the soil is one of the crucial factors impacting the overall life of a dam. A few incidents are related to internal instability [6]. The internal stability of the soil is influenced by geometrical conditions (particle pattern, particle distribution, and pore size), hydraulic conditions (gradient and flow rate), and mechanical conditions. Previous research based on the above conditions assessed the potential of soil instability. Istomina defined grain movement or suffusion relative to the uniformity coefficient C_u [13]. Kézdi suggested a criterion about fine content and coarse fraction to assess their self-filtering susceptibility [14]. Kenney and Lau recommended the particle-size curve diagram H-F to determine the internal instability of granular soils [15]. Wan and Fell proposed a method to assess the likelihood of internal instability for well-graded silt-sand-gravel soils depending on two boundary conditions [3]. Li and Fannin suggested a comparative method according to the criteria of Kezdi, and Kenney-Lau [16]. Chang and Zhang recommended extended internal stability criteria using the results of the previous researchers based on the physical understanding of soil microstructure [17].

However, the above criteria only determine the erosion capacity, not the specific erosion level of the soil. So, several recent studies focus on prediction models of erosion susceptibility based on physical parameters [12, 18-19]. Yet, some of these physical parameters are difficult to determine values at the field, such as degree of saturation, percentage of fine particles less than 0.005 mm, plasticity index, methylene, and internal friction angle.

Therefore, this article investigated by statistical analysis to find out parameters highly affecting ERI and,

as well as propose the formula to calculate the predicted erosion susceptibility based on physical parameters that are easily determined to improve efficiency and save costs in the preliminary assessment of the condition of earth dams.

2. Physical parameters

According to Le et al [12], they used three parameters of the soil including dry density (γ_k), internal friction angle (φ), methylene blue value (V_{bs}), and physical parameters relative to particle composition such as Finer Kenny and Lau (KL), gap ratio (G_r), $(H/F)_{min}$, particle content with a diameter smaller than 0.063 mm (P), particle distribution ($d_{90}, d_{60}, d_{50}, d_{20}, d_{15}$, and d_5) to determine the erosion predict model. However, physical parameters related to φ, V_{bs}, and P are hard to confirm in the field or the laboratory. The study proposed eliminating these difficult-to-define parameters and adding others that can be easily determined in the field or the laboratory. Previous research results show that soil instability is related to the uniformity coefficient (C_u) determined based on the distribution curve [13]. At the same time, particle content with a diameter smaller than 0.075 mm (P') affects the stability of the soil [19]. Thus, the study proposed the following parameters, that are input values, to be included in the model to predict the ERI (I_α) being output value:

Parameter I_α: Erosion resistance is related to the ratio between particle size and infiltration length, confining stress, and the visual factor affecting the shape of aquifers on groundwater flow, representing the ability of the soil to resist erosion due to outside impacts. Based on ERI, it is possible to classify the sensitivity to soil erosion, which is determined based on the mass of dry eroded particles and accumulated flow energy [20]. This index is divided into six levels from very easy erosion to very erosion resistant, corresponding to ERI values from 1 to 6.

Parameter γ_k: There is the weight of soil particles in a unit volume of natural soil. Erosion susceptibility is highly dependent on the dry density value. The larger this value is, the more ERI increases [12, 19].

Parameter Finer KL (%): The percent of fine particles KL was determined based on the $(H/F)_{min}$ value of Kenny and Lau's criterion (see Figure 1), which indicates that the particles within the *Finer KL* in the soil skeleton are likely to be eroded [21].

Parameter $(H/F)_{min}$: According to the evaluation criteria for erosion capacity of granular soil [15], which is evaluated by the minimum value of H/F. Particles with a diameter size smaller than D considered (F_{min}) may be eroded from the soil if there are not enough soil particles with sizes from D to 4D considered (H_{min}).

Parameter C_u: [22] Indicating the degree of unevenness of the particle composition. The larger the C_u, the more unequal the soil's size and vice versa. C_u indicates the likelihood that the fine particles pass through the voids formed by the coarse particles in the soil skeleton. The higher this value is, the more susceptible the soil is to erosion [13].

Parameter P' (%): According to Wan and Fell [3], fine particles smaller than 0.075 mm in diameter increase soil erosion resistance. Moreover, fine particles are likely to dominate the stability inside the soil [17].

Parameter G_r: According to Chang and Zhang [17], there is a ratio between the highest and smallest particle sizes ($G_r = D_{max}/d_{min}$) that characterizes the gap in the grading curve (see Figure 2). For continuous grading, the value is taken as 1.0. This parameter has a significant influence on the extent of erosion.

Parameters $d_5, d_{15}, d_{20}, d_{50}, d_{60}, d_{90}$ (mm): [22] The grain diameters have 5%, 15%, 20%, 50%, 60%, and 90%, respectively.
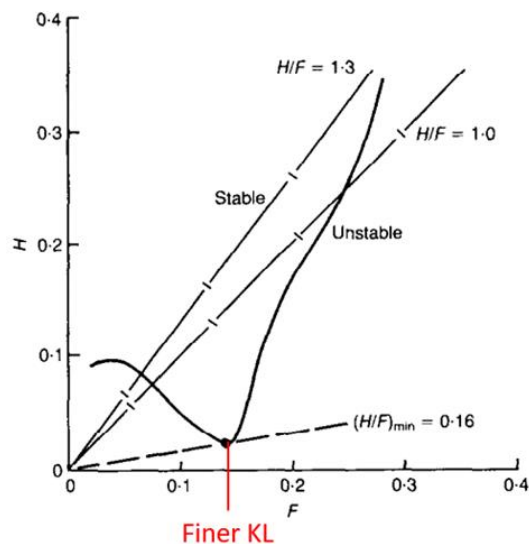

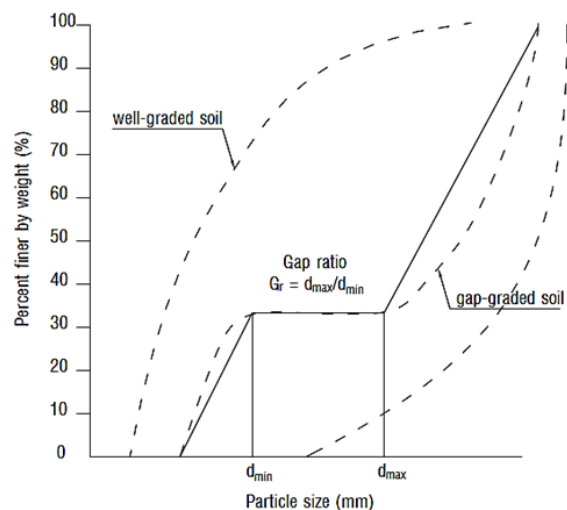*Figure 1. Illustrating the parameter Finer KL [21]*

Figure 2. Illustrating grading curves for both soils [17]

3. Suffusion prediction model

3.1. Data sets

Tables 1 and 2 present datasets on 13 physical parameters and ERI of 16 specimens for gap-graded soils and 11 specimens for well-graded soils from [12, 23-24]. To show accurate results, ERI for the same specimens is averaged.

Table 1. *Data from 16 tests of 14 physical parameters of gap-graded soils [12, 23-24]*

Specimens	ERI (Iα)	Dry density (kN/m³)	Finer KL (%)	min (H/F)	Gr	P' (%)	Cu	d5 (mm)	d15 (mm)	d20 (mm)	d50 (mm)	d60 (mm)	d90 (mm)
1-T-1	3.47	16.43	23.0	0.13	1.60	1.62	14.86	0.14	0.29	0.45	2.97	3.27	3.97
4-T	3.50	16.13	16.5	0.09	2.67	0.47	11.42	0.19	0.45	2.08	3.12	3.35	3.99
5-T	3.89	17.00	25.0	0.00	8.00	5.06	43.75	0.08	0.13	0.15	4.12	4.55	5.86
6-T	3.64	17.00	25.0	0.12	1.60	1.76	15.88	0.13	0.28	0.39	2.92	3.25	3.97
B-q	3.28	17.39	25.0	0.04	2.50	2.18	19.58	0.11	0.21	0.26	2.92	3.25	3.97
B-i	3.25	17.39	25.0	0.04	2.50	2.18	19.58	0.11	0.21	0.26	2.92	3.25	3.97
B-90	3.36	17.39	25.0	0.04	2.50	2.18	19.58	0.11	0.21	0.26	2.92	3.25	3.97
C	2.73	17.39	27.5	0.03	2.50	2.39	20.53	0.11	0.20	0.25	2.86	3.22	3.96
DR-A	4.49	17.87	20.0	0.11	2.40	2.93	14.91	0.09	0.15	0.25	1.56	1.69	2.63
DR-B	3.02	16.00	25.0	0.00	4.80	5.06	26.03	0.08	0.13	0.15	2.41	2.71	4.73
DR-C	2.59	16.00	25.0	0.00	4.80	5.06	35.25	0.08	0.13	0.15	2.99	3.67	5.65
G3-11	2.90	16.00	25.0	0.00	6.00	4.36	30.53	0.08	0.13	0.15	2.92	3.25	3.97
G3-13	3.47	16.00	15.0	0.00	6.00	2.61	25.04	0.10	1.50	2.13	3.15	3.36	3.99
G3-14	3.98	16.00	20.0	0.00	6.00	2.93	29.17	0.09	0.15	0.25	3.05	3.31	3.98
A	4.30	17.39	15.0	0.04	2.50	1.31	14.63	0.15	0.70	2.13	3.15	3.36	3.99
B	3.77	17.39	25.0	0.04	2.50	2.18	19.52	0.11	0.21	0.26	2.92	3.25	3.97

Table 2. *Data from 11 tests of 14 physical parameters of well-graded soils [12, 23-24]*

Specimens	ERI (Iα)	Dry density (kN/m³)	Finer KL (%)	min (H/F)	Gr	P' (%)	Cu	d5 (mm)	d15 (mm)	d20 (mm)	d50 (mm)	d60 (mm)	d90 (mm)
3-T1	4.72	17.00	51.84	0.45	1.00	13.33	22.23	0.01	0.10	0.17	0.54	0.89	3.85
3-T2	3.95	15.50	51.84	0.45	1.00	13.33	22.23	0.01	0.10	0.17	0.54	0.89	3.85
R1	5.13	17.39	15.26	0.59	1.00	1.48	13.17	0.15	0.32	0.63	2.67	3.03	4.48
R2-90	2.83	17.39	25.04	0.20	1.00	3.69	24.50	0.09	0.16	0.26	2.59	3.01	4.48
R2-97	3.41	18.74	25.04	0.20	1.00	3.69	24.50	0.09	0.16	0.26	2.59	3.01	4.48
CH-5	4.71	16.54	60.01	0.41	1.00	4.28	4.25	0.09	0.22	0.26	0.55	0.75	3.63
CH-10	5.49	18.90	40.61	0.44	1.00	0.87	12.92	0.19	0.30	0.37	1.38	3.18	8.35
CD	5.70	19.14	76.46	0.11	1.00	34.13	10.11	0.01	0.03	0.04	0.14	0.18	1.46
R3	4.34	18.66	25.00	0.20	1.00	2.32	18.88	0.11	0.21	0.25	2.59	3.01	4.48
R4	4.21	18.66	20.00	0.24	1.00	1.87	16.94	0.13	0.23	0.33	2.75	3.13	4.53
R5	4.30	18.66	15.00	0.29	1.00	1.41	15.27	0.15	0.31	1.33	2.88	3.24	4.62

3.2. Analysis tool

PCA is a multivariate statistical analysis technique used to decrease the set of dependent variables to a smaller data set of basic variables based on the correlation model of the original variables [25]. In this study, software **R** is used to perform PCA, creating a vector showing the correlation of the parameters. Analyzing the correlation between variables to evaluate the influence of the variable on the predictive model and removing those variables that have a low or no influence on the model. PCA visually represents the influence of the parameters through the correlation circle, by means of vectors with the origin at the center of the circle. The magnitude of the vector represents how much influence that variable has on the model. The direction of the vectors represents the relationship between the variables. When these vectors are farther from the center if vectors are in the same direction and close to each other, the variables are positive correlation; if vectors are opposite, they are negatively correlated; if orthogonal, they are not correlated with each other.

3.3. Results and discussions

The analysis was performed for well-graded and gap-graded soils with the variables being the physical parameters presented in Tables 1 and 2 used for the statistical. The parameters G_r and d_5 should be removed from the model for well-graded soils because their values in all specimens are equal or too small. The erosion prediction model was created by analyzing physical parameters for both soils.

PCA analysis creates new factors from a combination of existing variables having a smaller number of variables but still explains the nature of the old combination. For PCA, it is necessary to select factors with eigenvalues greater than 1.0 [26] and cumulative variances greater than 80% [27]. Scree plots in Figures 3 and 4 show the eigenvalues and cumulative variances of factors for gap-

graded and well-graded soils, respectively. The factors Dim1, Dim2, and Dim3 have eigenvalues greater than 1.0 and the sum of their cumulative variances is greater than 80% (the cumulative variance sum of the three factors in Figure 3 is 81.2%; Figure 4 is 85.1%). Thus, the above three Dims are used to analyze the correlation circle.
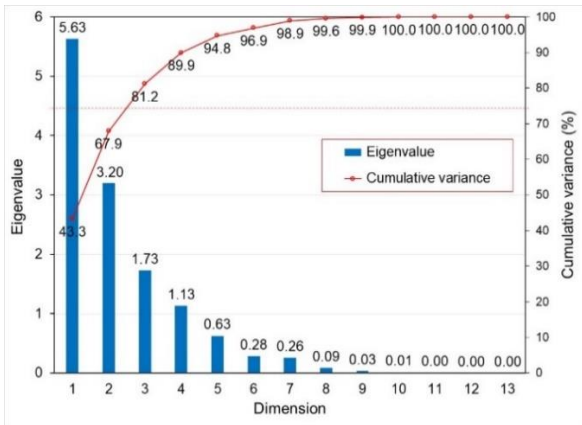

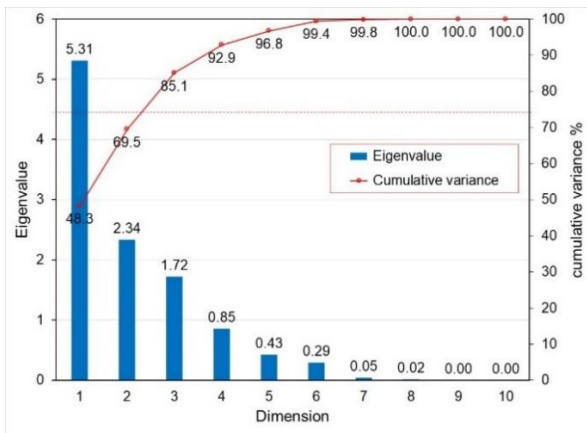*Figure 3. Scree plot – eigenvalues versus cumulative variance in PCA analysis for gap-graded soil*

Figure 4. Scree plot – eigenvalues versus cumulative variance in PCA analysis for well-graded soil

Figures 5 and 6 exhibit that the variables are shown on two graph planes of the factor combination Dim1 - Dim2 and Dim2 - Dim3 for both soil types, the length of the vectors is the degree of influence of the variable on the factor that has been determined in the analysis of the previous factor.

For gap-grade soil, based on plane 1 (Figure 5a) and plane 2 (Figure 5b), vectors d_{20} and d_{15} are in the same direction and close to each other on both planes; i.e., these two parameters are correlated together. In which, d_{15} has a shorter length leading to a lower degree of influence, it is excluded from the model. Vectors G_r and d_{90} are also positively correlated at plane 1. In addition, vectors d_{90} and I_α are orthogonal to each other at plane 2, so d_{90} is excluded from the model. d_5 is positively correlated with I_α on plane 1 but orthogonal to each other on plane 2, so d_5 is excluded from the model. Similarly, d_{50} and d_{60} are eliminated from the model because they are orthogonal to I_α in both planes. Finally, P' is also removed from the model because it is negatively correlated with I_α in plane 1 and orthogonal in plane 2.
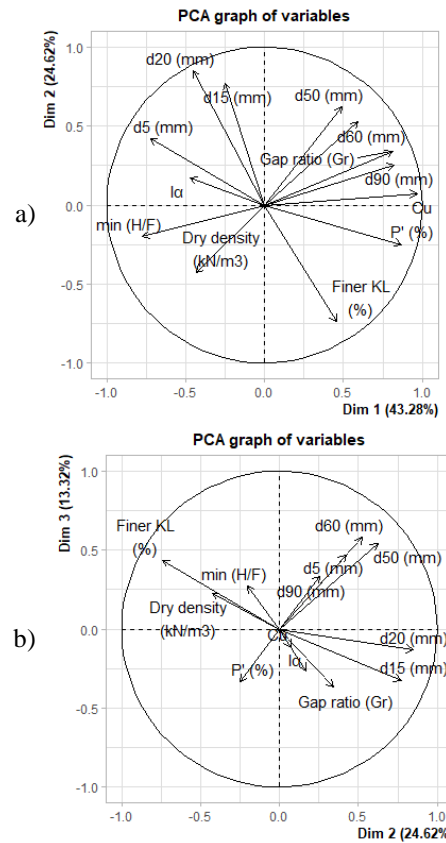

Figure 5. Correlation circles in the PCA of 13 parameters for gap-graded soils about the combination of factors:
a) On plane 1 of Dim1 – Dim2; b) On plane 2 of Dim2 – Dim3
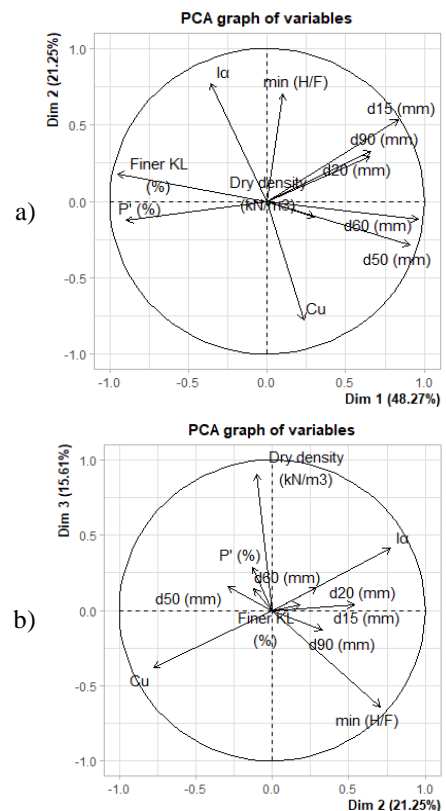

Figure 6. Correlation circles in the PCA of 11 parameters for well-graded soils about the combination of factors:
a) On plane 3 of Dim1 – Dim2; b) On plane 4 of Dim2 – Dim3

For well-graded soil, considering plane 3 (Figure 6a) and plane 4 (Figure 6b), parameters d_{15}, d_{20}, and d_{90} are close together and orthogonal with I_α on plane 3, so these parameters should be eliminated from the model. Discarding the P' because it is orthogonal to I_α on both planes. Vectors d_{50} and d_{60} exhibit positive correlations in both planes, which are orthogonal to vector I_α in plane 4 and should be removed from the model.

It is necessary to reduce the variables of physical parameters by their redundant information. From the results of PCA, six parameters γ_d, Finer KL, $(H/F)_{min}$, C_u, G_r, and d_{20} are used for the erosion prediction model for gap-graded soils while four parameters γ_d, Finer KL, $(H/F)_{min}$, C_u are used for erosion prediction model for well-graded soil. A new correlation between the selected parameters and I_α was determined for both soil types.

For gap-graded soil: (N=16)

$$I_\alpha = -1.73 + 0.51\gamma_d - 0.16 Finer\ KL - 0.01C_u$$
$$+4.72\left(\frac{H}{F}\right)_{min} + 0.15G_r - 0.5d_{20} \quad (R^2 = 0.85)$$
$$(1)$$

For well-graded soil: (N=11)

$$I_\alpha = -7.86 + 0.59\gamma_d + 0.02 Finer\ KL - 0.03C_u$$
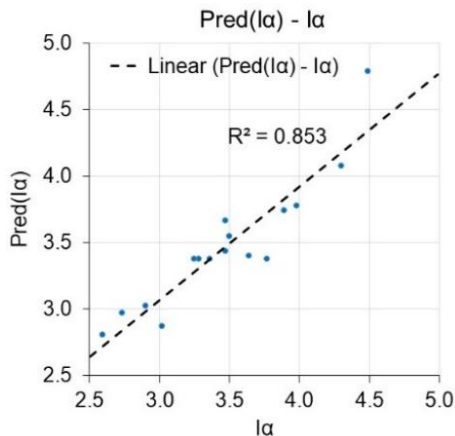$$+4.36(H/F)_{min} \quad (R^2 = 0.91) \quad (2)$$

Figure 7. *Estimated value (Pred(Iα)) versus actual value (Iα) for gap-graded soils*

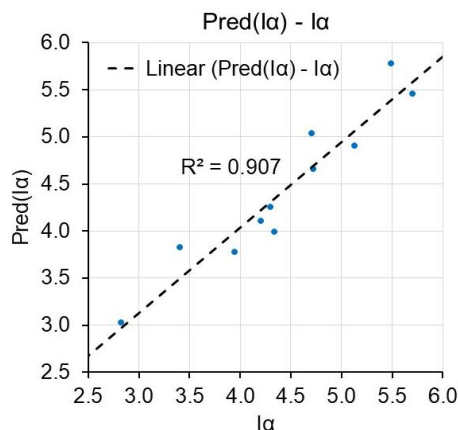**Figure 8.** *Estimated value (Pred(Iα)) versus actual value (Iα) for well-graded soils*

Figures 7 and 8 show the estimated and actual model's estimated and actual ERI values. Multivariate linear regression analysis, building the expression of erosion resistance coefficient I_α through selected physical parameters after PCA analysis. Models 1 and 2 are erosion prediction models for gap-graded and well-graded soils, respectively. The coefficient of determination R2 represents the fitness of the model with the input parameters included in the model. The coefficient of determination between 0.8 and 1.0 indicates that the model is being used at a fairly good level [28]. Therefore, two models are validated for the predictive model.

Table 3. *Effect of parameters to model on gap-graded soil*

Parameters	t	Pr > |t|	Lower bound (95%)	Upper bound (95%)
Dry density	4.01	0.003	0.22	0.79
Finer KL	-3.51	0.007	-0.26	-0.06
(H/F)min	1.94	0.084	-0.78	10.21
Gr	1.11	0.295	-0.16	0.47
Cu	-0.24	0.819	-0.08	0.06
d20	-2.38	0.042	-0.97	-0.02

Table 4. *Effect of parameters to model on well-graded soil*

Parameters	t	Pr > |t|	Lower bound (95%)	Upper bound (95%)
Dry density	4.37	0.005	0.26	0.92
Finer KL	3.79	0.009	0.01	0.04
(H/F)min	4.27	0.005	1.86	6.86
Cu	-1.63	0.155	-0.09	0.02

Results from Tables 3 and 4 show that dry density has the greatest influence on ERI for both gradations ($|t|_{\gamma d \to I\alpha} = 4.01$; $P_r = 0.003$ for gap-graded soil and $|t|_{\gamma d \to I\alpha} = 4.37$; $P_r = 0.005$ for well-graded soil). In addition, Finer KL has the second effect on gap-graded soil ($|t|_{Finer\ KL \to I\alpha} = 3.51$; $P_r = 0.007$) while it has a rather large effect on well-graded soil ($|t|_{Finer\ KL \to I\alpha} = 3.79$; $P_r = 0.009$). Finally, dry density and Finer KL are empirically supported with significance at $t = 2.57$ and $p = 0.01$.

4. Conclusion

This study focused on examining the role of physical parameters affecting suffusion. Thanks to PCA and linear regression, the physical parameters used in the model can reduce. The parameters removed do not affect the erosion prediction results. After PCA-based analysis, the physical parameters that have little influence on the model were eliminated from the 13 proposed physical parameters for both distributions. The six physical parameters γ_d, Finer KL, $(H/F)_{min}$, C_u, G_r, and d_{20} were retained for gap-graded soils. For well-graded soil, removing two parameters G_r and d_{20} out of the six proposed above parameters from the model, keeping four parameters. The results of the study are drawn as follows:

- Dry density has the highest effect on ERI.

- New models to predict ERI are proposed for soils.

Finally, the research results help predict the ERI of earth dams quickly, thereby providing the basis for management agencies to make assessments and decisions on upgrading and repairing dikes and dams. Besides, ensuring safety for the lives of people around the dams.

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