MODELING OF EDDY CURRENT LOSSES IN THE IRON CORE OF ELECTRICAL MACHINES BY A FINITE ELEMENT HOMOGENIZATION METHOD

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Abstract - A finite element homogenization method is proposed for the magetodynamic ***h***-conform finite element formulation to compute eddy current losses in electrical steel laminations. The lamination stack is served as a source region carrying predefined current density and magnetic flux density distributions presenting the eddy current losses and skin effects in each lamination. In order to solve this problem, the stacked laminations are converted into continuums with which terms are associated for considering the eddy current loops produced by both parallel and perpendicular fluxes. An accurate model of accuracy is developed via an accurate analytical expression of the eddy currents and makes the method adapted to both low and high frequency effects to capture skin depths of fields along thicknesses of the laminations.

Key words - Eddy current; finite element method; homogenization method; steel laminations; iron cores.

1. Introduction

Iron cores in electrical devices are usually laminated in order to reduce the eddy current losses due to time-varying flux excitations. In order to compute the eddy currents in each lamination, a finite element method (FEM) with a magnetic vector potential formulation has already been applied by many authors in [1]. However, the direct application of the FEM to realistic devices (that consist of multiple steel laminations) is still challenging, and especially requires plenty of time to calculate and simulate eddy currents in each separate lamination (Figure 1), where the currents are first completely ignored, and the Joule losses may be estimated from the results of an eddy current free model. In addition, many years ago, other authors in [2-4], also proposed the homogenization method to directly take these losses into account, but this method has been used for a magnetic vector potential formulation with a time domain.

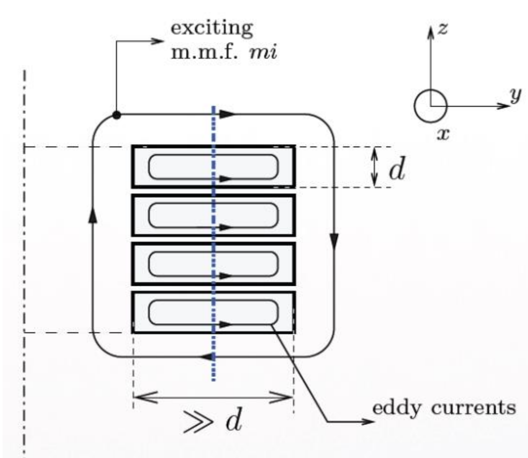

Figure 1. *Model of laminated iron core with the loop of eddy currents*

In this paper, the method is developed for a frequency domain with a magnetodynamic ***h***-conform finite element (FE) formulation. Its extension for accurate consideration of skin depths in the laminations for a wide frequency is also proposed. The method is based on the known analytical formula for eddy current losses. This formulation holds for linear material only and ignores edge effects. Some results are illustrated and compared for test problems.

2. Problem definition

In this definition, the main hypothesis is that the characteristic size of the domain of Ω (with boundary $\partial\Omega = \Gamma = \Gamma_h \cup \Gamma_b$) is much less than the wave-length $\lambda = c/f$ in each medium. The eddy current conducting part of Ω is denoted Ωc and the non-conducting one Ω_c^C, with $\Omega = \Omega c \cup \Omega_c^C$. Stranded inductors belong to Ω_c^C, whereas massive inductors belong to Ωc. Thus, the displacement current density is negligible. Maxwell's equations together with the following constitutive relations can be thus written as [8-9]:

$$\text{curl}\,\boldsymbol{h} = \boldsymbol{j},\quad \text{div}\,\boldsymbol{b} = 0\,,\quad \text{curl}\,\boldsymbol{e} = -\,\partial t\,\boldsymbol{b}, \qquad \text{(1a-b-c)}$$

$$\boldsymbol{h} = \mu{-}1\,\boldsymbol{b},\ \boldsymbol{j} = \sigma\,\boldsymbol{e}, \qquad \text{(2a-b)}$$

where ***h*** is the magnetic field, ***b*** is the magnetic flux density, ***e*** is the electric field, ***j*** is the electric current density, μ is the magnetic permeability, σ is the electric conductivity and ***n*** is the unit normal exterior to Ω. We start by writing a weak form of Faraday's law (1b), i.e.

$$\partial_t(\boldsymbol{b},\boldsymbol{h}')_\Omega + (\boldsymbol{e},\text{curl}\,\boldsymbol{h}')_\Omega + \langle \boldsymbol{n}\times\boldsymbol{e},\boldsymbol{h}'\rangle_\Gamma = 0,\quad \forall\,\boldsymbol{h}' \in F_h^0(\Omega). \qquad (3)$$

The constitutive law (2a-b) is introduced to obtain

$$\partial_t(\mu\boldsymbol{h},\boldsymbol{h}')_\Omega + (\sigma^{-1}\text{curl}\,\boldsymbol{h},\text{curl}\,\boldsymbol{h}')_{\Omega_c} + (\boldsymbol{e},\text{curl}\,\boldsymbol{h}')_{\Omega_c^C} + \langle \boldsymbol{n}\times\boldsymbol{e},\boldsymbol{h}'\rangle_\Gamma = 0,\quad \forall\,\boldsymbol{h}' \in F_h^0(\Omega), \qquad (4)$$

where $F_h^0(\Omega)$ is a curl-conform function space defined in, gauged in Ω_c^C, and containing the basis functions for ***h*** as well as for the test function ***h'*** (at the discrete level, this space is defined by edge FEs; the gauge is based on the tree-co-tree technique); $(\cdot,\cdot)$ and $<\cdot,\cdot>$ respectively denote a volume integral in and a surface integral on of the product of their vector field arguments. The surface integral term accounts for natural BCs, usually zero. The magnetic field ***h*** is expressed as [9]

$$\boldsymbol{h} = \boldsymbol{h}_r + \boldsymbol{h}_s, \qquad (5)$$

where $\boldsymbol{h}_s$ is a source magnetic field defined via an imposed current density $\boldsymbol{j}_s$ in stranded inductors [6-8], and $\boldsymbol{h}_r$ is the associated reaction magnetic field, which is indeed the unknown of our problem. Since

$$\begin{cases} \text{curl}\, \boldsymbol{h_s} = \boldsymbol{j_s} & \text{in } \Omega_s \\ \text{curl}\, \boldsymbol{h} = 0 & \text{in } \Omega_c^C - \Omega_s \end{cases}, \tag{6}$$

one gets

$$\text{curl}\, \boldsymbol{h_r} = 0 \quad \text{in} \quad \Omega_c^C. \tag{7}$$

In the non-conducting regions Ω_c^C, the reaction $\boldsymbol{h_r}$ can be thus defined via a scalar potential ϕ such that $\boldsymbol{h_r} = -\text{grad}\,\phi$. The test field $\boldsymbol{h'_r}$ in the weak form (4) is thus chosen in a subspace of $F_h^0(\Omega)$ for which curl $\boldsymbol{h'_r} = 0$ in Ω_c^C with $\boldsymbol{h} = \boldsymbol{h'_s} + \boldsymbol{h'_r}$.

Thus, the term $(\boldsymbol{e}, \text{curl}\, \boldsymbol{h'})_{\Omega_c^C}$ is omitted and the equation (4) can be rewritten as

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h'})_\Omega + (\sigma^{-1}\boldsymbol{j_s}, \text{curl}\, \boldsymbol{h'})_{\Omega_{ls}} + (\sigma^{-1}\text{curl}\, \boldsymbol{h_r}, \text{curl}\, \boldsymbol{h'})_{\Omega_c} + \langle \boldsymbol{n} \times \boldsymbol{e}, \boldsymbol{h'} \rangle_\Gamma = 0, \tag{8}$$

$\forall\, \boldsymbol{h'} \in F_h^0(\Omega)$, with curl $\boldsymbol{h'_r} = 0$ in Ω_c^C and $\boldsymbol{h} = \boldsymbol{h'_s} + \boldsymbol{h'_r}$. The trace of electric field $\langle \boldsymbol{n} \times \boldsymbol{e}, \boldsymbol{h'} \rangle_\Gamma$ in (8) is defined via homogeneous Neumann boundary condition, i.e. $\boldsymbol{n} \times \boldsymbol{e}|_{\Gamma_e} = 0$ implies $\boldsymbol{n} \cdot \boldsymbol{b}|_{\Gamma_e} = 0$.

3. Homogenization of Laminated Core

Based on the theory presented in Section 2, the *h-conform formulation with homogenized Lamination Stacks* will be proposed in this part. A laminated core region Ω_{ls} is considered as a subset of the source region domain Ω_s (Figure 2). Each lamination has a thickness *d*, an electric conductivity σ and a magnetic permeability μ which can be described by a local coordinate system ($\mathbf{i}_\alpha$, $\mathbf{i}_\beta$, $\mathbf{i}_\lambda$). The directions $\mathbf{i}_\alpha$, and $\mathbf{i}_\beta$ are parallel to the associated lamination, while $\mathbf{i}_\lambda$ is perpendicular to it. The direction $\mathbf{i}_\alpha$ is considered as the *a priori* unknown direction of the magnetic flux density $\boldsymbol{b}_\alpha$ parallel to the associated lamination, while $\mathbf{i}_\lambda$ is perpendicular to it. The direction $\mathbf{i}_\alpha$ is considered as the *a priori* unknown direction of the magnetic flux density $\boldsymbol{b}_\alpha$ parallel to the lamination, and consequently $\mathbf{i}_\beta$ is the main direction of the eddy current loops generated by variations of $\boldsymbol{b}_\alpha$, with associated current density $\boldsymbol{j}_\beta$. In addition, the effect of a varying magnetic flux density perpendicular to the lamination generates a current density denoted $\boldsymbol{j}_{\alpha\beta}$. The current density in one lamination is then expressed as the superposition of eddy current density generated by time-varying flux perpendicular and parallel to the lamination respectively [3-4], i.e.,

$$\boldsymbol{j} = \boldsymbol{j}_{\alpha\beta} + \boldsymbol{j}_\beta, \tag{9}$$

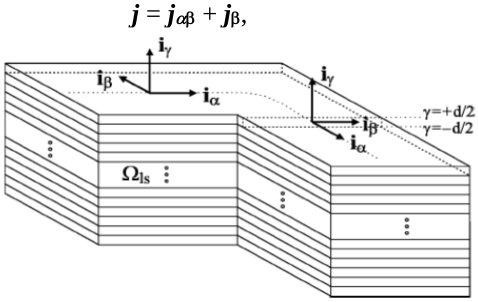

Figure 2. *Laminated iron core with its local coordinate system associated with each lamination [4]*

The current density $\boldsymbol{j}_{\alpha\beta}$ can be considered in the magnetodynamic problem through an anisotropic conductivity with zero components in the direction $\mathbf{i}_\lambda$, while $\boldsymbol{j}_\beta$ should undergo a pre-treatment for avoiding, at the discrete level, the discretization of each lamination separately.

3.1. Eddy current density versus the magnetic flux density

Thanks to the 1-D Faraday equation neglecting fringing fluxes of $\boldsymbol{j}_\beta$ (Figure 3), one gets for one lamination [4].

$$\partial_\lambda \boldsymbol{e}_\beta = \boldsymbol{i}_\lambda \times \partial_t \boldsymbol{b}_\alpha, \tag{10}$$

Figure 3. *Magnetic flux density* $\boldsymbol{b}_\alpha = \mu \boldsymbol{h}_\alpha$ *associated current density* $\boldsymbol{j}_\beta$ *in the cross section of a lamination stack [3]*

where $\boldsymbol{e}_\beta$ is the β component of the electric field. Then, reglecting skin effects for $\boldsymbol{b}_\alpha = \mu \boldsymbol{h}_\alpha$, it gets

$$\partial_{\boldsymbol{\lambda}} \boldsymbol{e}_\beta = \gamma \mu \boldsymbol{i}_\lambda \times \partial_t \boldsymbol{h}_\alpha, \tag{11}$$

where $\boldsymbol{h}_\alpha$ is the so-considered value of the magnetic flux density and γ is the position along the γ direction (equal to zero at the midthickness of the lamination, Figure 2). The Ohm law finally gives

$$\boldsymbol{j}_\beta = \sigma \boldsymbol{e}_\beta = \sigma \gamma \mu \boldsymbol{i}_\lambda \times \partial_t \boldsymbol{h}_\alpha. \tag{12}$$

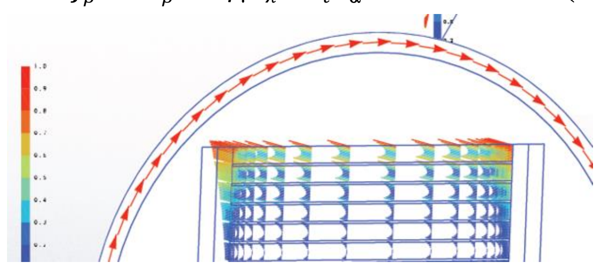

Figure 4. *Distribution of the current density and magnetic field in the coil and laminations*

For high frequency, the skin effects $\boldsymbol{h}_\alpha = \mu^1 \boldsymbol{b}_\alpha$ cannot be neglected, the actual distributions of $\boldsymbol{h}_\alpha$ and $\boldsymbol{j}_\beta$ have to be taken into account. From the Maxwell equations, the components $\boldsymbol{h}_\alpha$ and $\boldsymbol{j}_\beta$ can be defined in one lamination via their analytical expressions [3-4]. One has

$$\boldsymbol{j}_\beta(\gamma) = \boldsymbol{J}\sinh((1+j)\delta^{-1}\gamma, \tag{13}$$

$$\boldsymbol{h}_\alpha(\gamma) = \boldsymbol{H}\cosh((1+j)\delta^{-1}\gamma, \tag{14}$$

where $\boldsymbol{J}$ and $\boldsymbol{H}$ are constants depending on the exterior constrains and δ is the skin depth in the lamination, i.e., $\delta = \sqrt{2/\omega\mu\sigma}$, with the pulsation $\omega = 2\pi f$; f is the frequency. These expressions satisfy the interior constrains $\boldsymbol{j}_\beta(0) = 0$ and $\boldsymbol{h}_\alpha(-d/2) = \boldsymbol{h}_\alpha(d/2)$. From the Ampere law curl $\boldsymbol{h}_\alpha = \boldsymbol{j}_\beta$, it gets $\partial_\lambda \boldsymbol{h}_\alpha = \boldsymbol{j}_\beta$, which implies a relation between $\boldsymbol{J}$ and $\boldsymbol{H}$, i.e.,

$$\mathbf{H} = \mathbf{J}\delta(1-j)/2 \tag{15}$$

Consequently, these remains only one constant $\boldsymbol{J}$ in (13) and (14) for which no expression can generally be obtained *a priori*. The key point is rather to express this constant in terms of the mean magnetic flux density along the thickness of each lamination, which will actually be the field to be considered in the homogenized lamination stack. The magnetic field is defined as [4]

$$\boldsymbol{b}_\alpha = \frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} \boldsymbol{b}_\alpha(\gamma)d\gamma = \boldsymbol{J}j\frac{\delta^2}{d}\sinh((1+j)\delta^{-1}d/2) \qquad (16)$$

From (15), $\boldsymbol{J}$ can be expressed in terms of the magnetic field $\boldsymbol{b}_\alpha$, i.e.,

$$\boldsymbol{J} = -\mu \boldsymbol{b}_\alpha \frac{j\omega d\sigma}{2} / \sinh((1+j)\delta^{-1}d/2). \qquad (17)$$

With (16), (12) and (13) can finally be written in terms of $\boldsymbol{b}_\alpha$.

3.2. Magnetodynamic h-conform formulation with homogenized Lamination Stacks

As presented in Section 3.1, the term associated with the current density $\boldsymbol{j}_\beta$ in the weak formulation (8) can be now written as

$$(\sigma^{-1}\boldsymbol{j}_s, \operatorname{curl} \boldsymbol{h}')_{\Omega_{ls}} = \left(\sigma^{-1}\boldsymbol{j}_\beta(\gamma), \operatorname{curl} \boldsymbol{h}_\alpha(\gamma)\right)_{\Omega_{ls}}, \qquad (18)$$

where $\boldsymbol{j}_\beta(\gamma)$ and $\boldsymbol{h}_\alpha(\gamma)$ are already defined in (13) and (14). By substituting from (12) to (18) into (8), the weak $\boldsymbol{h}$-conform formulation after homogenizing in Ω_{ls} can be defined

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h}')_{\Omega} + (\boldsymbol{J}\sinh((1+j)\delta^{-1}\gamma, \operatorname{curl} \boldsymbol{H}\cosh((1+j)\delta^{-1}\gamma)_{\Omega_{ls}} + (\sigma^{-1}\operatorname{curl} \boldsymbol{h}_r, \operatorname{curl} \boldsymbol{h}')_{\Omega_c} + \langle \boldsymbol{n}\times\boldsymbol{e}, \boldsymbol{h}'\rangle_{\Gamma} = 0, \qquad (19)$$

where $\boldsymbol{J}$ and $\boldsymbol{H}$ are already defined in (15) and (17).

4. Applications

The $\boldsymbol{h}$-conform formulation has been applied to a 2-D stack of rectangular laminations.

The laminations are characterized by a relative permeability $\mu_r = 500$ and conductivity $\sigma = 10^7$ Sm^{-1}. Two values are considered for their thickness: d = 0.3 to 0.6mm. The thickness of the stack is 1.8mm and its width is 10 mm.

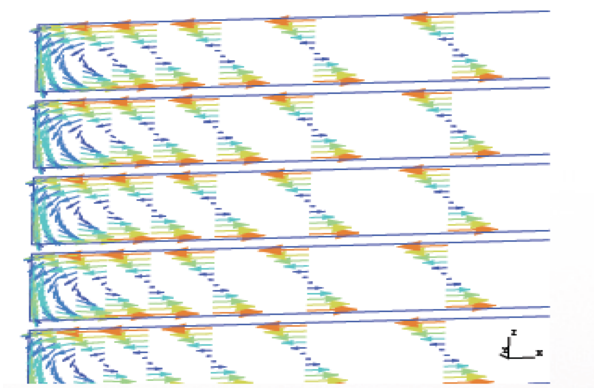

Figure 5. *Distribution of eddy currents in the lamination stack*

It is excited by an electrical current density of 1A, with considered frequencies from 50Hz to 25kHz, with associated skin depth from 1 to 0.016 mm. The distribution of the current density and magnetic field in the coil and laminations is shown in figure 4, for frequency of 50Hz. The loop of eddy currents in each lamination stack is presented in figure 5 with the good insulation. The skin effect at the corners and edges is higher than in the middle of the laminations (f = 500Hz). The real and imaginary part of magnetic flux density along the thickness of the lamination stack with the different frequencies is computed and shown in figure 6. The distribution of the fields depends on the frequency and skin depth. In particular, the field is very high at near edges, and is lower at the middle of the laminations.

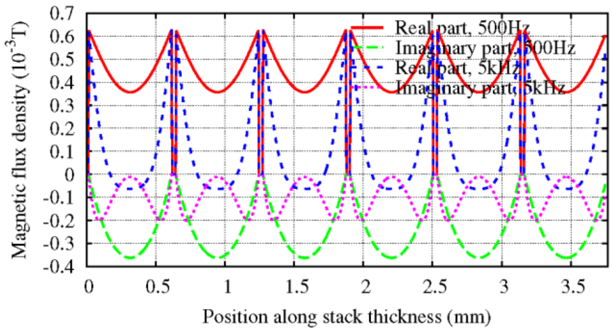

Figure 6. *Magnetic flux density along the thickness of the lamination stack with the different frequencies*

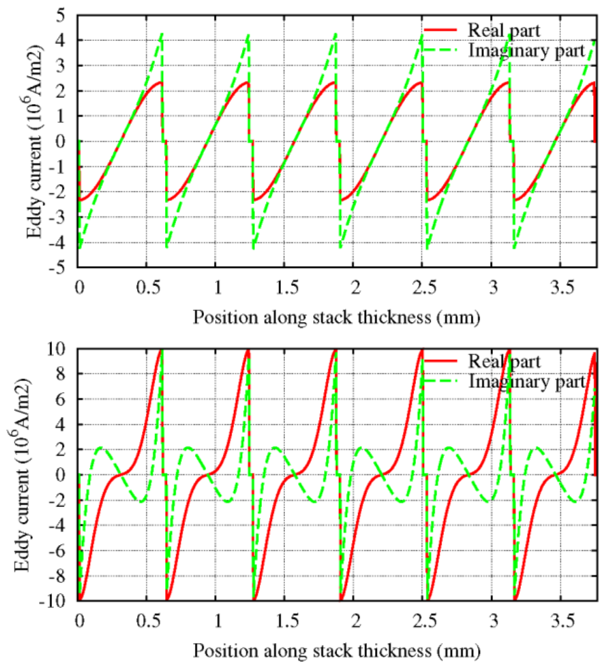

Figure 7. *Current density along the thickness of the lamination stack with the frequencies of 500Hz (top) and 5000Hz (bottom)*

Table 1. *Joule losses in the lamination stack with different frequencies*

No	Frequency *f*	Joule losses P (w)
1	50 Hz	0.25
2	5k Hz	13.3
3	50 kHz	56.8

In the same way, the distribution of eddy currents along the thickness of the lamination stack with frequencies of 50Hz and 5kHz is also pointed out in figure 7. The biggest value is at the corner and edges and is equal to zero at the middle of the laminations. Joule losses in the lamination stack with different frequencies are depicted in Table 1. Significant Joule losses increase with higher frequency.

5. Conclusion

The method for taking the eddy current losses in laminations in a 2-D analysis has been proposed for ***h***- conform finite element formulation. In order to avoid the explicit definition of all laminations, this method allows the laminated region to be converted into a continuum in which the distribution of eddy current losses produced by both parallel and perpendicular fluxes are taken into account thanks to adapted terms of the weak formulation. This helps to mesh and calculate in continuous region instead.

The method is valid for frequencies for which the skin depth in one lamination is greater than its half-thickness and can be applied in both frequency and time domains. The model is based on a precise analytic expression of the eddy currents and makes the method adapted to a wide-frequency range, i.e., for low skin depths in the laminations [4]. This method is limited to the frequency domain, however the analysis of nonlinear lamination stacks can be done through a multi-harmonic approach. The model appears attractive for directly taking into account the eddy current effects which are particularly significant for high frequency components.

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