INTERNAL FORCE ANALYSIS OF BEAMS ON ELASTIC FOUNDATION USING DIFFERENTIAL EVOLUTIONARY OPTIMIZATION ALGORITHM BASED ON HYBRID MUTATION TECHNIQUE

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Abstract - The paper presents the Finite element method (FEM) that calculates the internal force and the displacement of beams on elastic foundation in the case of the uncertainty input parameters described in terms of the number intervals. Using the interval function optimization algorithm combined with the FEM to determine the internal force values of the span reinforced concrete beam structure. This study applies the hybrid differential evolutionary optimization algorithm combined with FEM interval functions to determine the required internal force results of reinforced concrete beams placed on an elastic foundation. The calculation process is programmed using Maple.17 software to determine the displacement output and the resulting internal force in the beam. To check the correctness of the program calculated on Maple.17, the input declared dataset corresponds to the central values of the interval numbers and is recalculated by SAP2000v21 software, then evaluates the results error of internal force on the beam under consideration.

Key words - Interval numbers; Interval function; Finite element method; Displacement Output; Hybrid differential evolutionary.

1. Introduction

Types of beams placed on an elastic foundation such as foundation beams placed on the ground, pontoon bridges, ferries lying on the water surface, railway sleepers spread on rocks, are a type of indeterminate static problem that is especially popular in practice. economic. Currently, the calculation of beam structure on elastic foundation to determine internal force placed on an elastic foundation such as foundation beams placed on the ground, pontoon bridges, conveying ferries lying on water, railway sleepers spread on rock, are a particularly common type of statically indeterminate problem in reality. Currently, the calculation of beam structure on elastic foundation determining internal force [1–4], deformation, stress or displacement of beam is a rather complicated problem. It has many different computational opinions. And often use many different local elastic deformation background models such as Pasternak model, Rivkin's model, Philonhenco-Bolodis model, V.D. Vlaxov model, Khentini model, and Winkler model. In which, the popular model in practice when calculating beams placed on elastic foundation is the Winkler model. Today's engineering problems are well suited to these types of simple models. The model assumes that the reaction of the soil at any point is proportional to the settlement of the soil at that point. Therefore, the foundation is conceived as an infinitely independent system of springs, the reaction of the soil at each point is linearly proportional to the elastic settlement at that point, through the constant elastic coefficient k of the foundation. that for each point of each background type. According to the Winkler base model, consider the ground reaction per unit area P, and Y is the settlement at a point under investigation, expressed by the following relationship:

$$p(x) = K_z \times y(x) \quad (1)$$

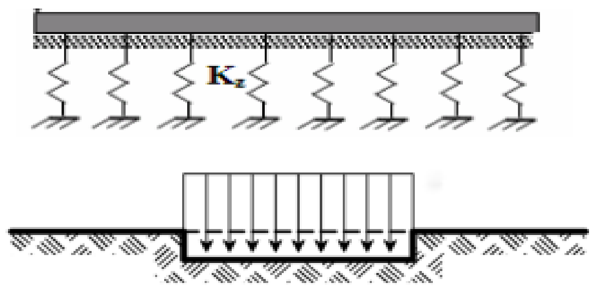

Figure 1. *Beam on elastic foundation – Winkler ground model*

Some previous case studies such as the study of Zimmos P. Mourwlatost and Michael G. Parsons [1]. The article analyzes beams on a continuous elastic foundation using the displacement finite element method. A consistent and complete three-dimensional model to explain the effects of both the Filonenko-Borodich and Pastemak platform models. A variational principle is given by the slope field due only to bend and the displacement field being approximated by independent quantities subject to change. Some examples illustrate element accuracy, the importance of shear, axial and shear-axial interactions in relation to the elastic continuum. In [2], The author has analyzed the static structure of beams on an elastic foundation using the computer code according to the finite-element method. Euler-Bernoulli beam on a two-parameter elastic foundation (abbreviated as EBBEF2p), is written in the Matlab program package to deal with many static load problems, related to unidirectional beams supported on elastic foundations. For example, the problem discussed was solved by the theoretical base code, the computer program.

A new method in the problem of beam bending on an elastic foundation in article [5] the results are obtained with closed-form analysis, the basic equations by the differential formula based on the minimum value of the total potential function. With a reasonable approach, the author has proposed to solve the balanced equation and use load-varying boundary conditions based on singular functions.

The finite difference method was used in the study [6] on the linear elastic behavior of non-prismatic beams on the Winler foundation. The study solved the differential equations governing for different configurations of the non-prismatic cross-section and load cases with different supports. Another example solves the problem of the Winler hypothesis model by using the ANSYS software package [3] to study the behavior of beams on an elastic foundation under the effects of static and dynamic loads. The author has analyzed beams on elastic foundations subjected to point loads in transverse direction through different methods. The proposed solution [4] is often encountered in practice, presenting a finite difference method to solve the deformation problem of a beam resting on an elastic foundation with a variable modulus of ground reaction, load bearing and geometry. Using an Excel workbook to calculate beam deflections gives numerical and graphical outputs with long beams of arbitrary load and constant cross-section. In article [7], the study has analyzed beam and beam-column deflection based on elastic strain differential equation, which is an approximation method. The author uses the central difference method of the finite difference method with Euler-Bernoulli beams and beam-columns bearing on an elastic, nonlinear foundation with discrete rigid or elastic supports. Use the Laplace variable method to verify the results of the above methods. The result is a quadratic central finite difference diagram during numerical analysis with five nonlinear behaviors of equally spaced springs.

The research in [8] is a review on previous work on linear elastic behavior of beams resting on uniform and non-uniform Winkler foundation. The 3D plate, beam and solid elements in beam and spring elements for the Winler foundation model were previously built based on the finite-element method. The results obtained based on the different methods are discussed and compared, checking the accuracy between those solutions. In study [9] the beam shear deformation analysis on the elastic foundation is based on the unified beam finite element. The beam and foundation matrix stiffness are obtained by introducing an analytical shear-rotation coefficient, allowing the shear and flexural curvatures to be separated. The shape functions for bending and shear strain are third and quadratic polynomials, respectively. And a major drawback of previous finite-element models is that the element result is free from shear locking. Therefore, the results using this element are considered to be in agreement with the classical beam theories of combined bending and shear strain.

However, most of the previous studies only considered the determinism of the input parameters, the output results were also determined values. This can show that when calculating and determining structural output according to deterministic parameters, it is impossible to comprehensively evaluate when taking into account the effects of uncertain factors formed of random, fuzzy and interval. Furthermore, in the process of surveying, designing, constructing and using construction works, there are many uncertain quantities affecting the structure. And in fact, the engineering field is significantly characterized by uncertainty that always exists both within the structure and from external factors such as lack of information, the result of people and equipment, due to use or maintenance. In fact, through surveys, the stiffness of the frame nodes of beams and columns can receive intermediate values in the range [0,1] depending on the stiffness of the beams, columns and the connection structure between them. Or when calculating the structure, some special loads such as wind, earthquake, section characteristics, elastic modulus E, drag coefficient of the structure model, or the coefficient of the foundation are considered as uncertain factors. Therefore, it is necessary to analyze the texture state with uncertain input parameters in the form of intervals. The process of analyzing and determining the internal force and bearing capacity of the structure when considering the uncertainty is carried out according to the calculations of the uncertainty model.

In this study, the author uses the range of input parameter variables based on available references, the input data of the problem has uncertainty in the form of intervals. Simultaneously, the calculation and determination of the structural internal forces are performed according to the arithmetic interval operations [10] and the interval optimization algorithm [11, 12]. Specifically, the study determines the required internal force results of concrete beams placed on an elastic foundation, applying the hybrid mutant differential evolutionary (HMDE) optimization algorithm [13] combined with the finite-element method (FEM) to compute the interval function. From there, the author determines the range of displacement and internal force results in the beams programmed by the author on Maple.17. Finally, studying the correctness of the program made on Maple.17, the author evaluated the results of the internal force error of the beam under consideration using the input dataset corresponding to the weighted value centers of the interval numbers and recalculated using SAP2000 V.21 software.

2. Research method

Beam structure with the uncertain input data formed as interval numbers then, the output will be interval numeric. Considering the output as a function that depends on the input variables of the interval, the analysis to determine the output should be performed according to interval arithmetic optimization operations to determine the lower and upper bound values of the result [3, 6].

$$y_j = f_j\left(x_1, x_2, \ldots, x_n\right) \rightarrow \min$$

Constraint condition $a_j \le x_j \le b_j$ (2)

$y_j = f_j(x_1, x_2, \ldots x_n) \rightarrow \max$,

Constraint condition $a_j \le x_j \le b_j$ (3)

Solve problems (2) and (3) to get the minimum and maximum value of the output. Below, the author briefly presents the algorithm to optimize the function containing the interval variable to determine the output in the form of interval number.

2.1. HMDE optimization algorithm

The Differential Evolutionary Algorithm (DEA), developed by Storn and Price [5], is an evolutionary algorithm for solving optimization problems. The general idea of the algorithm is that from a randomly generated population of individuals, new individuals will be generated and selectively compete with the old ones. During this selection process, good individuals will be passed on to the next generations; otherwise, inferior individuals will perish. Here, the instances will be evaluated through an objective function f(x) defined by a particular optimization problem. This process is similar to the process of natural selection described in Darwin's theory of evolution. DEA is an evolved version of the genetic algorithm with the steps of crossover and mutation clearly described by mathematical formulas. Experimentally, DEA is said to have the ability to find the optimal solution very well through mining and exploiting the search space. The DE algorithm is depicted in Figure 2.

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1: Define the parameters of the algorithm: Number of design variables (D),
number of individuals (P), maximum number of iterations (G)
2: Initialize instances of the first population according to (4)
3: For g = 1: G
4: Assess the population and identify the best individual xbest
5: For i = 1: P
6: Identify the instance mother xi
7: Generate 3 random positive integers r1, r2, r3
8: Determination of mutation coefficient F = N (0.5, 0.22) and the probability of
hybridization Cr = 0.8
9: Create mutant vectors that follow (5) or (6)
10: Create vector ci by (7)
11: IF f (ci) <f (xi) THEN xi = ci
12: IF f (ci) <f (xbest) THEN xbest = ci
13: End For
14: End For
15: Return xbest
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Figure 2. *Differential evolution algorithm*

Determine the parameters of the algorithm: The parameters of the algorithm include the number of design variables (D), the number of instances (P), and the maximum number of generations (G). Usually, the number of instances P = 4.D÷8.D, the maximum number of generations G is usually set so that the algorithm converges. The algorithm terminates when the maximum number of generations condition is satisfied.

Initialization phase: An individual is represented by a vector where the number of components of the principal vector is equal to the number of design variables D. Thus, a population will be represented by a matrix PxD. Instances of the first population are randomly initialized as follows:

$$x_{ij} = LB_j + rand\left(0;1\right).\left(UB_j - LB_j\right) \quad (4)$$

Where, LB_j and UB_j are the minimum and maximum values of the design *variable j, j = 1,2,…,D. rand(0;1)* is a random initialized real number in the interval [0, 1].

Mutation: Each vector x in the current generation g is called a 'parent vector'. For each 'parent vector', a 'mutant vector' $d_{i,g}$ can be generated in many ways, two ways of creating a 'mutation vector' or chosen are the mutated type *DE/rand/1* and the *DE/best/1* mutation according to:

$$\text{DE/rand/1:} \quad d_{i,g} = x_{r1,g} + F.\left(x_{r2,g} - x_{r3,g}\right) \quad (5)$$

$$\text{DE/best/1:} \quad d_{i,g} = x_{\text{best},g} + F.\left(x_{r1,g} - x_{r2,g}\right) \quad (6)$$

Where: *r1*, *r2*, and *r3* are 3 randomly generated integers in the range [1; N]; These 3 integers are generated so that they do not coincide with the I of the 'parent vector'. F is the amplitude of mutations generated according to the normal distribution N(0.5, 0.22). x_{best} is the best individual in the population. g is the symbol for the current generation.

The mutation process according to (5) tends to exploit the search space, making it difficult for the algorithm to fall into the local optimal region, but the convergence process will be slow. The mutation process according to (6) tends to exploit the found xbest value, this method has the advantage of helping the algorithm to converge quickly, but it is easy to fall into the local optimal region when the search problem is complicated.

Crossover: Diversify the existing population by exchanging components of the target vector and the mutation vector. During this phase, a new vector is created and named the test vector. The test vector is also known as the offspring. The test vector can be formed as follows:

$$c_{j,i,g} = \begin{cases} d_{j,i,g}, \text{if rand}_j \le C_r \text{ or } j = rnb\left(i\right) \\ x_{j,i,g}, \text{if rand}_j > C_r \text{ or } j \ne rnb\left(i\right) \end{cases} \quad (7)$$

Where, *rand$_j$* is a randomly generated real number of [0,1]. C_r is the probability of the hybrid being chosen = 0.8.*rnb(i)* is a randomly chosen positive integer in the interval [1, P].

Selection: The individuals 'child vector' $c_{i,g}$ and 'mother vector' $x_{i,g}$ are compared with each other. The instance with a correspondingly worse objective function value will be discarded:

$$x_{i,g+1} = \begin{cases} c_{i,g} \text{if } f\left(c_{i,g}\right) \le f\left(x_{i,g}\right) \\ x_{i,g} \text{if } f\left(c_{i,g}\right) > f\left(x_{i,g}\right) \end{cases} \quad (8)$$

In order to improve the optimization ability of DE algorithm, Hoang's research [13] proposed a new DE optimization method "mixed mutant differential evolutionary optimization - HMDE". HMDE has the same basic steps (population initialization, hybridization, selection) as conventional DE methods, however, in the mutation step of individuals, Hoang [13] proposes a new mutation equation, this new equation is a combination of equations (3) and (4). The new method helps to accelerate the convergence of the algorithm, while avoiding the search process from falling into a locally optimal solution. The mixed mutation equation is described as follows:

$$d_{i,g} = \gamma.x_{best,g} + \left(1-\gamma\right)x_{r1,g} + F.\left(x_{r2,g} - x_{r2,g}\right) \quad (9)$$

Where, $\gamma = 1 - \exp(\frac{-g}{100})$ is the coefficient that determines the influence of the xbest vector on the mutation process. It is easy to see that when g changes from 1 → G_{max} (the maximum number of generations or the maximum number of iterations of the algorithm), then γ changes from 0 → 1. When the evolution is nearing its end, the participation of vector- The more xbest vectors, the faster the convergence of the algorithm.

2.2. FEM for calculating beams on elastic foundation according to Winkler model

2.2.1. Beam element stiffness matrix on elastic foundation

Considering the Winkler model with beams on an elastic foundation with beams placed on springs, the stiffness k is completely independent as shown in Figure 3. The stiffness matrix of the flexural beam plus the stiffness matrix of the foundation acting on the beam will determine the stiffness matrix of the beam on an elastic foundation by the finite-element method [1–3, 9].

Displacement at the node of a beam element with two ends i and j:

$$q = \left\{q_1, q_2, q_3, q_4\right\}^T$$

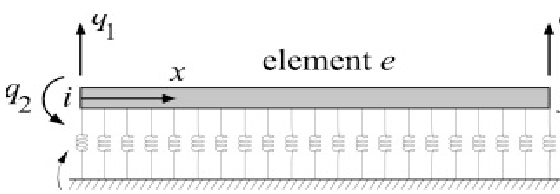

Figure 3. *Element diagram of beam on elastic foundation*

The flexural beam element stiffness matrix has the form (10).

$$\left[k\right]_{eb} = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{2EI}{l} & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \quad (10)$$

The beam element stiffness matrix under the influence of the foundation has the form (11)

$$\left[k\right]_{ef} = \frac{k}{420}\begin{bmatrix} 156l & 22l^2 & 54l & -13l^2 \\ 22l^2 & 4l^3 & 13l^2 & -3l^2 \\ 54l & 13l^2 & 156l & -22l \\ -13l^2 & -3l^2 & -22l^2 & 4l^3 \end{bmatrix} \quad (11)$$

Where: E (kN/m^2), I (m^4), l (m) are the elastic modulus of the material, the moment of inertia of the bar section, the length of the bar, respectively.

k is the stiffness coefficient of the spring converted from the Winkler coefficient K; $k = K \times b$.

K(T/m^3), b (m) is looked up according to the table in [1,10], and the width of the bar placed on the ground, respectively.

So, the beam stiffness matrix on an elastic foundation has the form: $\left[k\right]_e = \left[k\right]_{eb} + \left[k\right]_{ef}$ (12)

Effect of structural stiffness taking into account the uncertainty of the material characteristic $\breve{E}$ and the foundation coefficient $\breve{k}$

Apply the principle of virtual work:

$$\left[\breve{k}\right].\left\{\breve{q}\right\} = \left\{\breve{f}\right\} \quad (13)$$

Determination of displacement and internal force of beam structure is the final result based on finite element method combined with interval optimization algorithm.

2.2.2. Structural analysis steps by the interval Finite Element method

From the basic equation of the structural system according to the Finite Element method with the interval parameter (13), after removing boundary conditions), we rewrite the equation as follows:

$$\left\{\breve{q}\right\} = \left[\breve{k}\right]^{-1}.\left\{\breve{f}\right\} \quad (14)$$

Expanding equation (14) we have:

$$\left\{\breve{q}\right\} = \begin{Bmatrix} \breve{q}_1 \\ \breve{q}_2 \\ \dots \\ \breve{q}_n \end{Bmatrix} = \left[\breve{k}\right]^{-1}\left\{\breve{f}\right\} = \begin{bmatrix} \breve{k}_{11} & \breve{k}_{12} & \dots & \breve{k}_{1n} \\ \breve{k}_{21} & \breve{k}_{22} & \dots & \breve{k}_{2n} \\ \dots & \dots & \dots & \dots \\ \breve{k}_{n1} & \breve{k}_{n2} & \dots & \breve{k}_{nn} \end{bmatrix}^{-1} \times \begin{Bmatrix} \breve{f}_1 \\ \breve{f}_2 \\ \dots \\ \breve{f}_n \end{Bmatrix} \quad (15)$$

Where $[\breve{\delta}] = [\breve{k}]^{-1}$, $[\breve{\delta}]$ calculated directly by software Maple.17 with the determinant of $[\breve{k}]$ is non-zero.

Consider the ith equation of the system of equations

$$\breve{q}_i = \breve{\delta}_{i1}\breve{f} + \breve{\delta}_{i2}\breve{f}_2 + \dots + \breve{\delta}_{in}\breve{f}_n \quad (16)$$

In equation (18), the left hand side is the i-th interval displacement component to be found, which is determined from the interval parameters $\breve{\delta}_{ij}$ and $\breve{f}_i$ (i,j = 1,2,…,n), We consider equation (18) as an interval function that determines the output variable in terms of the input variables and by optimizing the interval to find the max and min values of $\breve{q}_i$. Doing it for all equations of system (17) will determine all the components of the displacement of the structure. Determine the internal force and stress of the structure after determining the displacement of the nodes. In Figure 4, the sequence diagram of the structural analysis steps is shown:

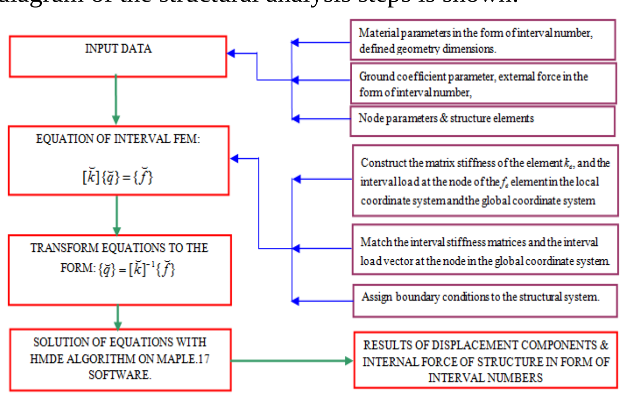

Figure 4. *Block diagram of structural analysis steps*

2.3. Applied problems and input data

2.3.1. Problem

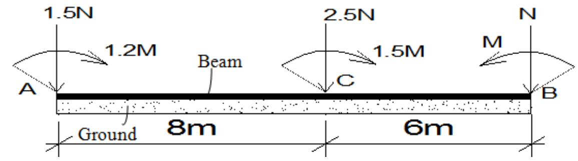

Figure 5. *Structure diagram of foundation beam under load*

Consider an actual construction in Vietnam with a two-span foundation beam structure bearing the load as shown in Figure 5. The foundation beam material uses concrete of strength grade B25, the cross-sectional dimension of the beam is $b \times h = 700 \times 1500\left(mm^2\right)$, placed on medium grain sand background. The problem requires calculating the internal force of the beam structure with input parameters in the form of intervals.

Assuming to be ±5% deviation from the mean E=3100kN/cm^2 and the external load has a deviation of 15% from the mean N=1800 kN and M=270 kNm:

$$\breve{E} = \left[E^L; E^U\right] = \left[2945;3225\right] kN / cm^2$$

$$\breve{N} = \left[N^L; N^U\right] = \left[1530;2070\right] kN$$

$$\breve{M} = \left[M^L; M^U\right] = \left[21250;28750\right] kN.cm$$

The medium-grained sand base has the soil foundation coefficient: $\breve{K} = \left[K^L; K^U\right] = \left[30000;50000\right] kN / m^3$

2.3.2. Sequence of calculation

The number of elements and the number of displacements of the structural nodes.

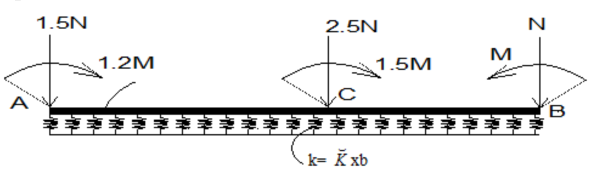

Figure 6. *Calculation model of beams on elastic foundation*

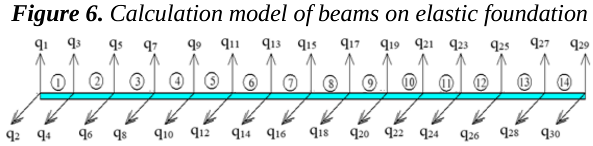

Figure 7. *Diagram of division of sub-elements*

The beam is divided into 14 sub-elements and is numbered and displaced according to local coordinates as shown in Figure 7, based on the beam calculation model in Figure 6.

Table 1. *The number of elements and displacements of the nodes of the beam*

Elements	Displacement of element's node in local coordinate			
	1	**2**	**3**	**4**
	Displacement of element's node in global coordinate			
1	1	2	3	4
2	3	4	5	6
3	5	6	7	8
4	7	8	9	10
5	9	10	11	12
6	11	12	13	14
7	13	14	15	16
8	15	16	17	18
9	17	18	19	20
10	19	20	21	22
11	21	22	23	24
12	23	24	25	26
13	25	26	27	28
14	27	28	29	30

The equations for the finite element method with interval parameters:

$$\left[\breve{k}tt\right]_{30x30} \cdot \left\{\breve{q}\right\}_{30x1} = \left\{\breve{f}\right\}_{30x1} \quad (17)$$

Convert equation (24) to the form:

$$\left\{\breve{q}\right\}_{30x1} = \left[\breve{k}tt\right]^{-1}_{30x30} \cdot \left\{\breve{f}\right\}_{30x1} \quad (18)$$

Based on the block diagram 4, the author analyzes, calculates and simulates on Mable.17, integrates the HMDE algorithm into solving the system of equations with interval parameters. Calculation code program named FEM.BEF to determine internal moment force, shear force at girder structural node elements.

3. Calculation results and discussion

Through the data, the problem condition posed of the beam structure placed on the elastic foundation. Applying calculation programs FEM.BEF and software Sap2000v21 to conduct coding, output internal force results including moment, shear force of beam structure.

From there, compare and evaluate the deviation of the calculated results of the above methods, shown in Table 2.

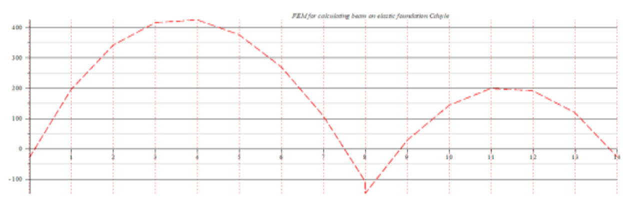

Figure 8. *Moment graph in FEM.BEF*

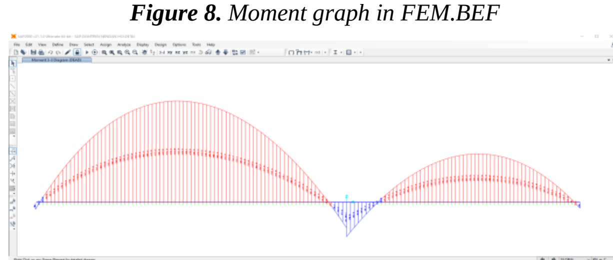

Figure 9. *Moment graph in Sap.2000*

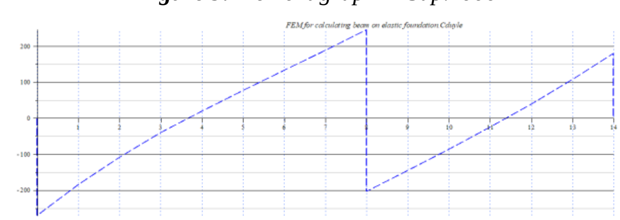

Figure 10. *Shear graph in FEM.BEF*

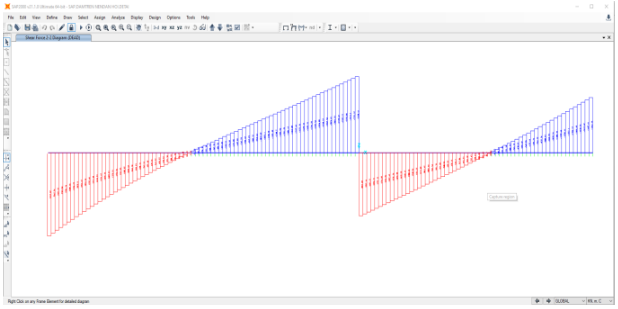

Figure 11. *Shear graph in Sap.2000*

Looking at the results of Table 2 and Table 3, it shows that the values bearing the sign (-) are conventionally used for the upper fiber tension of the beam. Conversely, the sign (+) is the lower fiber tension of the beam. In addition, shear force values bearing the (-) sign representing the conventional sign of negative magnitude are given on the negative y-axis drawing.

Through Table 2, the data show that the error rate between the two methods giving results is less than 1% for the moment component and less than 5% for the shear force component, which is considered as an insignificantly small percentage., acceptable in the calculation process, check. At the same time, the results allow to confirm the high reliability of the program FEM.BEF on Maple.17 software. Therefore, the determination of internal forces in the form of intervals with elastic modulus $\breve{E}$, external load $\breve{N}$, $\breve{M}$ and Winler's foundation coefficient $\breve{K}$, with reinforced concrete beams on elastic foundation is reliable, shown in Table 3.

Table 2. *The calculated result value according to the methods of moment, shear force (FEM.BEF and SAP2000v21)*

Node order number	**Value of internal force in beam**					
	Moment (kN.m)		**Shear (kN)**		**Error (%)**	
	FEM.BEF.	**SAP2000v21**	**FEM.BEF.**	**SAP2000v21**	**Moment**	**Shear**
0	300.00	300.00	-2700.00	-2653.86	0.00	1.708
1	-1961.30	-1942.99	-1840.12	-1874.54	0.933	1.87
2	-3413.81	-3381.21	-1080.62	-1109.87	0.954	2.706
3	-4150.93	-4111.07	-406.11	-409.81	0.96	0.911
4(Middle of span 1)	-4247.14	-4265.39	205.13	212.28	0.429	3.451
5	-3753.74	-3722.73	777.15	742.79	0.826	4.421
6	-2698.22	-2680.29	1332.91	1295.94	0.664	2.773
7	-1087.01	-1083.35	1890.90	1854.05	0.336	1.948
8 (Middle node)	1087.01(left) -1462.06 (right)	1076.98(left) -1451.98(right)	2461.98(left) -2038.01(right)	2430.16(left) -2011.03(right)	0.922 0.649	1.292 1.324
9	-285.49	-287.94	-1453.13	-1478.99	0.858	1.779
10	-1439.34	-1437.42	-856.40	-881.36	0.133	2.914
11(Middle of span 1)	-1990.74	-1991.55	-262.87	-270.43	0.046	3.112
12	-1916.22	-1906.04	397.30	401.37	0.531	1.024
13	-1183.51	-1176.55	1075.29	1039.24	0.588	3.352
14	250.00	250.00	1800.00	1762.35	0.00	2.091

Table 3. *Interval internal force value of beam*

Node order number	**Interval Moment** $\breve{M}$ **(kNm)**	**Inteval Shear** $\breve{Q}$ **(kN)**
0	[255.000; 339.000]	[-3105.000; -2294.99]
1	[-2345.915; -1605.731]	[-2188.770; -1558.101]
2	[-4016.262; -2907.368]	[-1347.740; -909.945]
3	[-486.160; -3614.068]	[-571.383; -336.786]
4 (Middle of span 1)	[-4966.284; -3769.816]	[100.530; 260.8123]
5	[-4388.819; -3405.839]	[612.857; 914.778]
6	[-3161.185; -2540.025]	[1113.969; 1548.960]
7	[-1284.096; -1178,723]	[1613.847; 2185.621]
8 (Middle node)	[682.600; 1226.834] [1007.182; 1645.632]	[2118.791; 2837.668] [-2337.332; -1706.209]
9	[-742.779; -128.905]	[1240.441; 1603.111]
10	[-1760.635; -1289.479]	[733.918; 906.733]
11 (Middle of span 1)	[-2382.310; -1714.045]	[212.618; 198.527]
12	[-2287.043; -1605.716]	[-449.930; -388.029]
13	[-1438.240; -946.394]	[-1231.161; -947.20]
14	[212.500; 282.500]	[-2070.000; -1530.00]

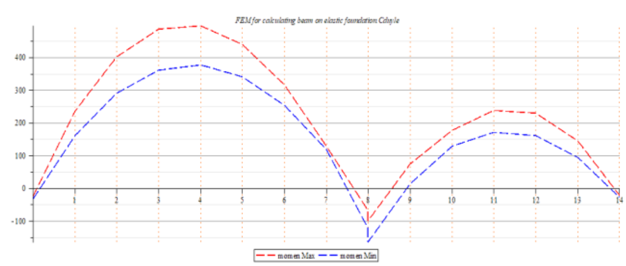

Figure 12. *Interval moment graph in FEM.BEF*

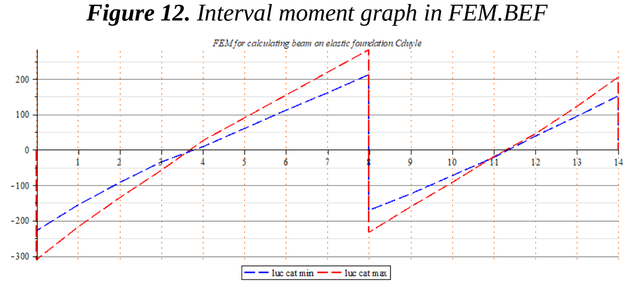

Figure 13. *Interval shear graph in FEM.BEF*

In fact, the uncertainty parameters greatly affect the calculation results, when taking into account the uncertainty of the input parameters, it is possible to predict the output range of the structure to serve as the basis for state estimation. structural safety. When the input parameters have a numerical uncertainty in the interval, the output also has an uncertainty about the interval. The outputs are in the form of interval numbers allowing a more realistic reflection of the actual deviations of the input parameters affecting the output of the structure.

4. Conclusion

The study has applied the hybrid differential evolution algorithms combined with interval finite-element theory. Based on the FEM.BEF coding program programmed in the Maple.17 language. From there, determine the internal force state in the case of irregularity in the number of intervals and calculate the output of the beam structure on the elastic foundation.

The result of internal forces and displacements in the form of a number of intervals is determined by the interval optimization method, on the basis of applying the mixed mutation differential evolutionary algorithm that gives the results suitable to the requirements of the problem. The correctness of FEM.BEF is verified with the calculation results in SAP.2000.V21 software, allowing to confirm the reliability of the calculation program.

From the research results achieved, the results of internal force calculation bring many advantages and advantages to test the bearing capacity as well as evaluate the reliability of the bearing strength of beams and some other factors.

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