AN IMPROVED METHOD FOR LATERAL SURFACE DEVELOPMENT OF A CIRCULAR TRUNCATED CONE USING VERTICAL PROJECTION AND ITS APPLICATIONS

CẢI TIẾN PHƯƠNG PHÁP KHAI TRIỂN MẶT NÓN CỤT TRÒN XOAY SỬ DỤNG HÌNH CHIẾU ĐỨNG VÀ CÁC ỨNG DỤNG

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Abstract - In the field of mechanical engineering, particularly within the automotive, shipbuilding the utilization of metal sheets to create intersecting componentsis of great importance. However, the precise development of the circular surfaces that intersect with the circular truncated conical surface poses a critical challenge. It is essential to establish the intersection and accurately represent it on engineering drawings. This paper aims to overcome the limitations of existing approximate methods for the 2D development of circular truncated conical surfaces. This research offers a comprehensive solution for the accurate development of circular truncated cones, thereby enhancing precision and efficiency in engineering designs within industries such as automotive, shipbuilding, and pipeline engineering. By providing a quick and versatile development method based solely on vertical projection, contribute to the advancement of practical applications in these industries.

Key words - Circular truncated conical tubes; vertical projection; intersection problems; engineering drawings

Tóm tắt - Trong lĩnh vực kỹ thuật cơ khí, đặc biệt là trong công nghiệp ô tô và đóng tàu, việc sử dụng tấm kim loại để tạo ra các chi tiết giao nhau có vai trò quan trọng. Tuy nhiên, việc khai triển chính xác các bề mặt tròn xoay giao với mặt nón cụt tròn xoay đặt ra một thách thức lớn. Do đó, việc thiết lập và biểu diễn chính xác sự giao nhau này trên bản vẽ kỹ thuật là rất cần thiết. Bài báo này nhằm loại bỏ các hạn chế của các phương pháp gần đúng hiện có cho việc khai triển 2D của các mặt nón cụt tròn xoay. Nghiên cứu này đề xuất phương pháp khai triển chính xác mặt nón cụt tròn xoay, nâng cao độ chính xác và hiệu suất trong thiết kế kỹ thuật, đặc biệt là trong các ngành công nghiệp như ô tô, đóng tàu và kỹ thuật đường ống dẫn. Phương pháp đề xuất dựa trên hình chiếu đứng, cung cấp một cách tiếp cận nhanh chóng và linh hoạt, góp phần vào sự phát triển của các ứng dụng thực tế trong những ngành công nghiệp này.

Từ khóa - Ống hình nón cụt tròn xoay; hình chiếu đứng; bài toán giao tuyến; bản vẽ kỹ thuật

1. Introduction

In the general mechanical engineering industry, particularly in the automotive, shipbuilding, and pipeline sectors, various components are manufactured using metal sheets. Prior to the production of these parts, it is essential to accurately depict the development of their lateral surfaces on engineering drawings. This serves as a crucial step in the manufacturing process, as it provides a blueprint for subsequent operations such as bending, shaping, or welding, which are necessary to achieve the desired surface configuration. In practical terms, when working with engineering drawings, it is quite common to encounter intersecting circular tubes, specifically circular truncated conical tubes. These tubes, with their unique shape, present a challenge when it comes to accurately developing their lateral surfaces. Therefore, it becomes imperative to devise an effective method for precisely developing the lateral surface of a circular truncated cone. This is particularly important in the fabrication of pipe surfaces, as any inaccuracies can result in faulty connections or compromised structural integrity.

In the past, there have been several publications addressing the development of lateral surfaces for 3D circular truncated cones. These publications have introduced various methods, including one based on orthogonal projections. However, it is important to note that these methods are approximate and have limitations [1]. Specifically, the method based on orthogonal projections is only applicable when the bases of the truncated cones are two circles. Furthermore, this method does not offer a general solution for cases where the truncated conical surface intersects with other circular surfaces. To further explore this topic, additional information can be found in references [1], [2], and [3]. These references provide valuable insights and knowledge related to the lateral surface development of circular truncated cones.

The primary objective of this study is to conduct in-depth research and propose an efficient and comprehensive development method for circular truncated cones. The proposed method focuses exclusively on vertical projection as the basis for the development process. This approach ensures a rapid and accurate determination of the lateral surface of the cone. Unlike existing methods, our approach is designed to accommodate bases with arbitrary curves, allowing for greater flexibility in real-world applications. Furthermore, our method considers scenarios where the vertex of the cone lies outside the drawing, ensuring its applicability in a wide range of practical situations. In addition to addressing the development challenges of circular truncated cones, our research also investigates the complexities associated with the intersection of the truncated conical surface with other revolution surfaces commonly encountered in industrial engineering drawings. This analysis contributes to the practical implementation of the proposed method across various industries.

2. Methods and problems in the study

2.1. Methods

The study primarily concentrates on the development of the circular truncated conical surface, specifically when the vertex of the cone is located outside the drawing and the cone axis is parallel to the vertical projection plane. In order to accurately develop this truncated cone, we take into account the following cases:

a. Circular truncated cone with two circular bases (Figure 1a).

The procedure for developing the lateral surface of a circular truncated cone with two circular bases, as follows:

- Let D represent the diameter of the larger base circle of the given truncated cone, serving as a reference for subsequent calculations.

- To facilitate the development process, an auxiliary circular cone is constructed, similar to the given cone. The vertex of the auxiliary cone is denoted as point S, and the axis t' is parallel to the axis t of the given cone. It is recommended to select the diameter d of the base of the auxiliary cone within the circle plane of diameter D, ensuring that the ratio D/d = k is typically an integer (k = 2, 3, 4 and so on). The auxiliary cone can be positioned either inside or outside the given truncated cone. This choice enables convenient and accurate calculations during the development process. The construction of the auxiliary cone can be performed either as shown in Figure 1a or outside the truncated cone.

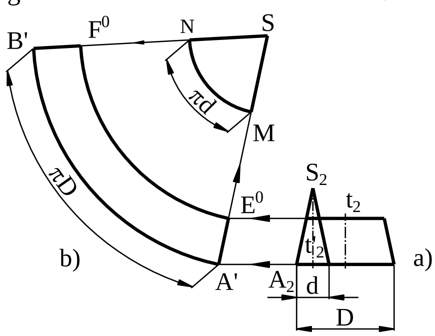


Figure 1. *Development of a circular truncated cone with circular bases*

- Draw a segment SA' that is parallel to S_2A_2 and has a length equal to k times the length of the contour generation line of the vertical projection of the auxiliary cone: SA'// = k.S_2A_2 with $A_2A' \perp t_2$ (Figure 1b).

- On segment SA', establish SM = S_2A_2 = 1/k.SA'.

- Draw an arc MN = πd and center it at point S. The sector SMN represents the development of the auxiliary cone.

- As the given truncated cone is similar to the auxiliary cone with a ratio of k, it becomes apparent that the two arcs A'B' and the arc E^0F^0, intercepted by the angle MSN and centered at S, depict the development of the two base circles of the circular truncated cone. Furthermore, the segment A'E^0 corresponds to the development of the generation line of the truncated cone (refer to Figure 1b).

b. Circular truncated cone with one base as an arbitrary curve

Similar to case a), in this scenario, the smaller base of the truncated cone takes the form of a curved surface denoted as h, which possesses a symmetrical plane parallel to the vertical projection plane (refer to Figure 2a).

The procedure for developing the lateral surface of this circular truncated cone is as follows:

- Let D represent the diameter of the larger base circle of the given truncated cone.

- In this case, an auxiliary circular cone is constructed that is similar to the given cone. The vertex of the auxiliary cone is denoted as point S, and the axis t' is parallel to the axis t of the given cone. To ensure a convenient and accurate development process, it is recommended to select the diameter d of the base of the auxiliary cone in the circle plane of diameter, such that the ratio $\frac{D}{d} = k$, where k is an integer (as illustrated in Figure 2a).

- The auxiliary cone, with vertex S and a large base circle of diameter D, is developed using the same procedure as demonstrated in Figure 1b. By following this procedure, we obtain the resulting fans SMN and SA'B', as depicted in Figure 2b.

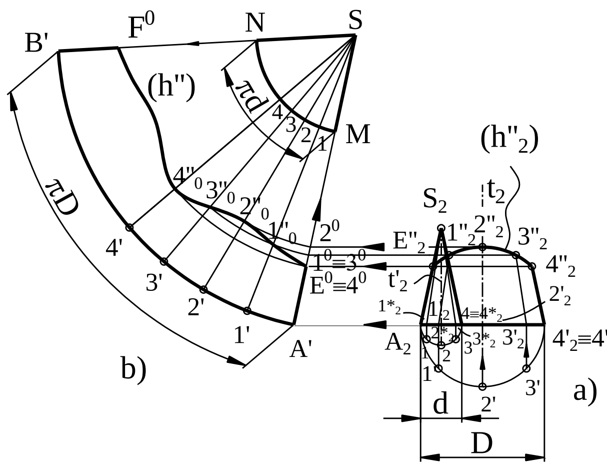


Figure 2. *Developing a circular truncated cone with arbitrary curved small base*

- Since the symmetrical plane of the truncated cone is parallel to the vertical projection plane, the development of the curve (h) requires dividing the halves of the circles with diameters d and D into equal parts. These equal divisions are represented by points 1, 2, 3, 4, etc., for the smaller base, and points 1', 2', 3', 4', etc., for the larger base, as shown in Figure 2a. Subsequently, the two semicircular arcs MN and A'B' are also divided into equal segments using the corresponding division points, as illustrated in Figure 2b.

- Draw radial lines Si' that divide the fan into equal parts, where i' = 1', 2', 3', 4', etc belong to the half of the sector A'B' (Figure 2b).

- Since the two cones are similar, their corresponding sets of generation lines must be parallel. Therefore, we can construct the vertical projection of the dividing generation lines of the truncated cone: $1'_21''_2$ // $S_21^*_2$, $2'_22''_2$ // $S_22^*_2$, $3'_23''_2$ // $S_23^*_2$, $4'_24''_2$ // $S_24^*_2$, etc., as illustrated in Figure 2a. Among these lines, points 1_2, 2_2, 3_2, 4_2, etc., belong to the curve (h_2).

- To determine the development of the endpoints 1, 2, 3, 4, etc., which belong to the curve (h), we connect them to circles on the surface of the truncated cone. These circles lie in planes that are perpendicular to the t-axis of the cone.

- Draw fan-shaped arcs centered at S, which represent the development of the corresponding circles of the cone. These arcs should pass through the endpoints $1^0 \equiv 3^0$, 2^0, $E^0 \equiv 4^0$, etc., belonging to SA' with $1^0 1''_2$ // $2^0 2''_2$ // $3^0 3''_2$ // $4^0 4''_2$ // $A'A_2$.

- Proceed by drawing the intersection points: E_0, 1_0, 2_0, 3_0, 4_0, etc.. These points are obtained by intersecting the newly constructed fan arcs with the corresponding divided fan radii. These intersection points will serve as the established development points.

- Finally, connect the points E_0, 1_0, 2_0, 3_0, 4_0, etc., with a smooth curve.

- The second half of the development can be constructed using either the same method or by exploiting the symmetry about the axis S4'. This approach yields the curved line (h_0) as the development of the curved line (h) of the smaller base of the truncated cone.

c. Circular truncated cone with arbitrary curved bases

In this scenario, the given truncated cone has its axis, denoted as t, parallel to the vertical projection plane. The small base is in the form of an ellipse, represented as (h), while the large base comprises two elliptical arcs, (e) and (g), connected by the endpoints G. The vertex of the cone lies outside the drawing, and the symmetrical plane of the cone runs parallel to the plane of vertical projection (as depicted in Figure 3a).

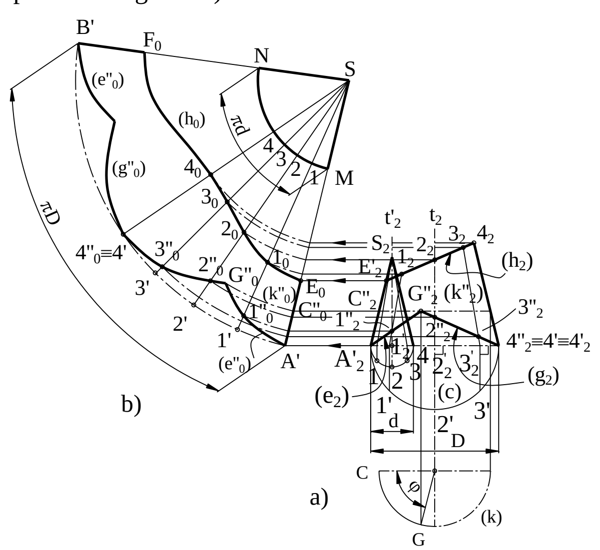


Figure 3. *Developing a circular truncated cone with both bases being arbitrary curves*

The procedure for developing the lateral surface of this circular truncated cone is as follows:

- Let D represent the diameter of the auxiliary circle (c) that passes through point A'. This point corresponds to the endpoint of the contour generation line in the vertical projection of the truncated cone and is located in a plane perpendicular to the t-axis of the cone, as shown in Figure 3a.

- We construct an auxiliary circular cone that is similar to the given cone. The vertex of the auxiliary cone is denoted as point S, and the axis t' is parallel to the axis t of the given cone. It is recommended to select the diameter d of the base of the auxiliary cone in the circle plane of diameter D in such a way that the ratio D/d = k, where k is an integer.

- The auxiliary cone, with vertex S, and the auxiliary circle (c) with diameter D of the truncated cone are developed using a similar procedure as shown in Figure 1b. This development yields the fans SMN and SA'B' as illustrated in Figure 3b.

- Since the symmetrical plane of the truncated cone is parallel to the vertical projection plane, developing the elliptic curves (h), (e), and (g) as the bases of the truncated cone involves dividing half of the circles with diameters d and D into equal parts. We mark these divisions as points 1, 2, 3, 4, etc., for the smaller base, and points 1', 2', 3', 4', etc., for the larger base, as shown in Figure 3a. Furthermore, the two semicircular arcs MN and A'B' are divided into equal parts using the corresponding division points, as depicted in Figure 3b.

- Draw the dividing fan radii Si', where i' = 1', 2', 3', 4', etc..

- Since the two cones are similar, their corresponding pairs of generation lines are parallel. Consequently, we can construct the vertical projection of the dividing generation lines for the truncated cone, such as $1''_2 1_2$, $2''_2 2_2$, $3''_2 3_2$, $4''_2 4_2$, etc., as shown in Figure 3a. The points 1_2, 2_2, 3_2, 4_2, etc., belong to the curve (h_2), while $1''_2$ belongs to (e_2), and $2''_2$, $3''_2$, $4''_2$, etc., belong to (g_2).

- Let G = (e) ∩ (g), and attach point G to the circle (k) of the truncated cone. In this way, we can easily determine the length of the fan arc $C''_0 G''_0$ in Figure 3b, which represents the development of the arc CG ∈ (k).

- Similar to case b), the endpoints on the dividing generation lines belonging to the semi-elliptic curves (h), (e), and (g) are developed into a set of points: (E_0, 1_0, 2_0, 3_0, 4_0) for curve (h); (A', $1''_0$, G''_0) for curve (e); and (G''_0, $2''_0$, $3''_0$, $4''_0$) for curve (g). The arc length C_0G_0 is equal to the arc length CG of the circle (k).

- Draw smooth curves connecting the set of points: (E_0, 1_0, 2_0, 3_0, 4_0); (A', $1''_0$, G''_0); and (G''_0, $2''_0$, $3''_0$, $4''_0$).

- The second half of the development can be constructed using the same method or by utilizing the symmetry about the axis S4'. This approach yields the development of (h_0), (c_0), and (g_0), which represent the bases of the truncated cone, as depicted in Figure 3b.

2.2. Challenges in developing intersecting circular truncated conical tubes in conjunction with rotating tubes

- **Problem 1:**

Given the vertical projection of a circular cylindrical tube with a radius R, axis $k \perp P_3$, and a circular truncated conical tube with axis $t \perp P_1$, where the surfaces share a common plane of symmetry parallel to P_2 (as depicted in Figure 4a). Construct the intersection of these two surfaces and develop them.

Construct the auxiliary spherical surface with center O = t ∩ k. This spherical surface is inscribed in the cylindrical surface along the circle (ω), and intersects the conical surface at the circle (g). The intersection point 3 = (ω) ∩ (g), as described in references [1], [4], [5]. The two given surfaces intersect to form a fourth-degree curve. Due to the common plane of symmetry parallel to P_2, the vertical projection of the intersection results in a hyperbolic arc denoted as $1^0{}_2 3^0{}_2 6^0{}_2$. Point $3^0{}_2$ serves as the vertex of this arc, with the line $O_2 3^0{}_2$ acting as its axis. Refer to Figure 4a for a visual representation.

Let D represent the diameter of the larger base circle of the given truncated cone, as illustrated in Figure 4a. We construct an auxiliary circular cone that is similar to the given cone. The vertex of the auxiliary cone is denoted as point S, and the axis t' is parallel to the axis t of the given cone. It is recommended to select the diameter d of the base of the auxiliary cone in the circle plane of diameter D in such a way that the ratio $\frac{D}{d} = 3$, as shown in Figure 4a.

We divide half of the circles with diameters d and D into six equal parts, marking the division points as 1, 2, 3,... 6 and 1', 2', 3',... 6', respectively, as depicted in Figure 4a.

Construct the generating lines of the auxiliary cone that pass through the dividing points. As the two cones are similar, their corresponding pairs of generation lines are parallel. Therefore, we can construct the vertical projection of the dividing generation lines for the truncated cone, namely $1'_2 1^0{}_2$, $2'_2 2^0{}_2$, $3'_2 3^0{}_2$, $4'_2 4^0{}_2$, $5'_2 5^0{}_2$, and $6'_2 6^0{}_2$ (refer to Figure 4a). Among these lines, the points $1^0{}_2$, $2^0{}_2$, $3^0{}_2$, and $4^0{}_2$ belong to the vertical projection of the hyperbola formed by the intersection.

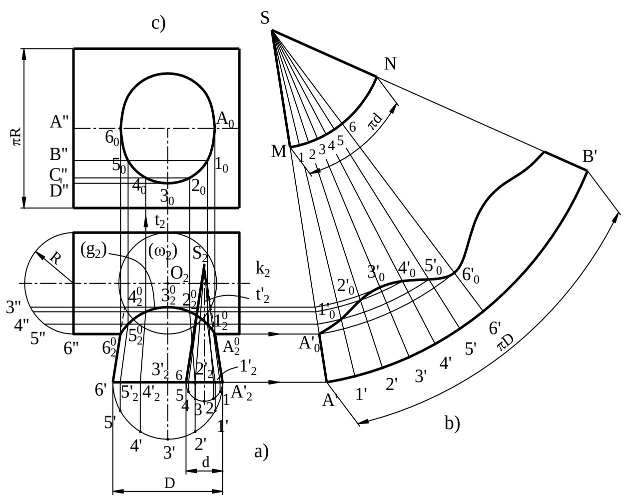


Figure 4. *Developing the intersection between circular cylindrical and truncated conical tubes*

The auxiliary cone, along with the large base circle having diameter D, and the vertical projection of the hyperbolic curve of the intersection, are developed according to the illustration in Figure 2b. The resulting outcome is presented in Figure 4b.

The lower half of the cylinder is developed in parallel with the contour generation line of its vertical projection. The segments A"B", B"C", and C"D" correspond to the lengths of arcs 6"5", 5"4", and 4"3", respectively. As a result, development points belonging to the semi-intersection are determined: A_0, 1_0, 2_0, 3_0, 4_0, 5_0, and 6_0, as depicted in Figure 4c.

Draw a smooth curve connecting the set of points: A_0, 1_0, 2_0, 3_0, 4_0, 5_0, and 6_0.

The second half of the development can be achieved by employing the same method or leveraging the symmetry about the axis $A_0 6_0$. This leads to the successful development of the cylinder, as demonstrated in Figure 4c.

- **Problem 2**

Given the vertical projection of a circular cylindrical tube with a radius of R and an axis t perpendicular to P_1, along with two branches of circular truncated conical tubes with axes k and k' symmetrically forming a Y shape and parallel to P_2, as depicted in Figure 5a, the objective is to construct the intersection of the adjacent surfaces and develop them.

It is observed that the circular truncated cones and the cylinder are circumscribed about the sphere with center O, where O = t ∩ k. Consequently, they intersect in the arcs of elliptic curves (e), (e'), and (g) through the points of intersection of the tangent curves. Since the common symmetry plane of the surfaces is parallel to the vertical projection plane, the vertical projections of these intersecting elliptical arcs are reduced to segments (e_2), (e'_2), and (g_2), as detailed in references [1-5].

To develop the cylindrical surface, we divide the semicircle of its base into six equal parts by marking points 1, 2, 3… 7. Next, we draw the vertical projection of the dividing generation lines of the cylinder, such as $1_2 1'_2$, $2_2 2'_2$, $3_2 3'_2$… $7_2 7'_2$. These points, namely $1'_2$, $2'_2$, $3'_2$… $7'_2$, belong to the vertical projection of the intersecting ellipses (e_2) and (e'_2).

To develop the lateral surface of the cylinder parallel to its vertical projection generation line, we can construct the development of the base circle of the given cylinder as the segment PQ, where PQ = 2πR. Since the semicircle of the cylinder's base is divided into six equal parts, we can also divide the semicircle PQ into six equal parts using the dividing points 1, 2, 3… 7, respectively.

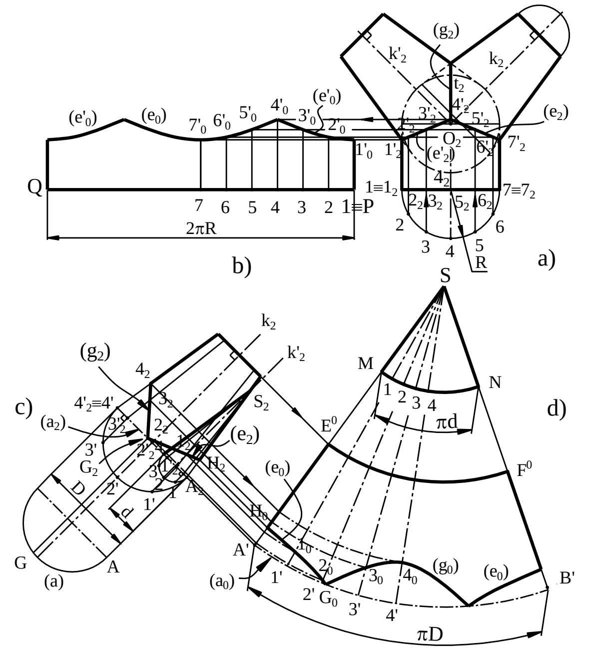


Figure 5. *Developing the intersection between a circular truncated conical tube and a cylindrical tube*

To develop the endpoints 1', 2', 3', 4', 5', 6', 7' on the dividing generation lines, which belong to the intersecting ellipses (e) and (e'), we can attach them to the horizontal lines that are perpendicular to the t_2 axis of the cylinder. These lines will intersect with the vertical lines passing through the corresponding dividing points on PQ. As a result, we will obtain the intersections $1'_0$, $2'_0$, $3'_0$, $4'_0$, $5'_0$, $6'_0$, $7'_0$, which represent the developed points to be constructed (Figure 5b). This process is detailed in references [6-7].

Draw smooth curves passing through the set of points: ($1'_0$, $2'_0$, $3'_0$, $4'_0$) and ($4'_0$, $5'_0$, $6'_0$, $7'_0$). By utilizing the symmetry about the axis $77'_0$, we can obtain the curves (e_0) and (e'_0) (Figure 5b).

To clearly develop the truncated cone, we separate it from the cylinder (Figure 5c).

Let $G = (e) \cap (g) \rightarrow G_2 = (e_2) \cap (g_2)$. Let D represent the diameter of the auxiliary circle on the truncated cone in the plane perpendicular to the k axis and passing through point G (Figure 5c). We construct an auxiliary circular cone similar to the given cone, with point S as the vertex and the axis t' parallel to t. It is advisable to select the diameter d of the base of the cone in the circle plane of diameter D so that the ratio $\frac{D}{d} = 3$ (Figure 5a).

The procedure for developing the auxiliary cone, the circle with diameter D, and the small base of the truncated cone follows a similar approach as demonstrated in Figure 1a. In this process, we ensure that SA' // $3.S_2A_2$, and SM = S_2A_2. As a result, various shapes are formed, including the fan shapes SMN, SE^0F^0, and SA'B' as depicted in Figure 5d. To develop the two intersecting ellipses (e) and (g), we divide each of the half circles with diameters d and D into four equal parts, employing the dividing points 1, 2, 3, and 4 for one circle, and 1', 2', 3', and 4' for the other circle. Similarly, we divide the semicircular arcs MN and A'B' into four equal parts, using their respective division points (as shown in Figure 5d). Subsequently, we draw the fan radii Si' through the corresponding dividing points i' = (1', 2',... 4'). Since the two cones are similar, their corresponding pairs of generation lines must be parallel. Therefore, we can construct the vertical projection of the auxiliary dividing generation lines for the truncated cone: $1'_21_2$, $2'_22_2$, $3'_23_2$, $4'_24_2$ (as depicted in Figure 5c). Among these lines, the points 1_2 and $2_2 \in (e_2)$, while the points 3_2 and $4_2 \in (g_2)$.

To develop the points 1_2 and $2_2 \in (e_2)$, as well as the points 3_2 and $4_2 \in (g_2)$, we attach them to the circles of the truncated cone located in planes perpendicular to the k-axis of the cone. These circles are further developed into fan-shaped arcs with center S. These arcs intersect the corresponding dividing fan radii at points H_0, 1_0, 2_0, 3_0, 4_0. It is important to note that the arc length $A'G_0$ is equal to the arc length AG (as shown in Figure 5d).

Draw smooth curves passing through the points H_0, 1_0, 2_0, G_0, as well as through the points G_0, 3_0, 4_0. Utilizing the symmetry about the axis S4', we can derive the curves (e_0) and (g_0), which are part of the development of the truncated cone (as illustrated in Figure 5d).

- **Problem 3**

Given the vertical projection of a circular truncated conical tube with the axis t ⊥ P1, and a sphere with center O (as shown in Figure 6a), where their common symmetry plane is parallel to P_2 and the vertex of the truncated cone is positioned outside the drawing. We are required to construct the intersection of these two surfaces and develop them. The intersection between the circular truncated connical surface and the sphere surface results in a 4th-degree rapids curve. Due to the parallelism between the common symmetry plane and P_2, the vertical projection of the intersection forms a parabolic arc. This concept is detailed in references [1], [3-5], [10-11]. To determine the points along this parabolic arc, we can employ the following procedure:

To determine the vertical projection of the intersection, begin by drawing the line OI ⊥ t and construct an auxiliary sphere with center I, which is inscribed in the truncated conical surface along the circle (ω). This spherical surface intersects the given spherical surface, forming a circle (g). Locate point $A_2 = (\omega_2) \cap (g_2)$. Point A_2 serves as the vertex of the vertical projection parabola of the intersection. The line x, passing through point A_2 and perpendicular to the axis t_2, represents the axis of this parabola. Detailed information regarding this procedure can be found in references [8-9]. The resulting vertical projection corresponds to the parabolic arc $1_2A_28_2$.

We proceed by constructing an auxiliary circular cone that is similar to the given cone. This auxiliary cone has its vertex at point S and its axis, denoted as t', is parallel to the original axis t. To ensure proper proportions, it is recommended to choose the diameter d of the base of the cone in such a way that the ratio between the diameter D of the cone's base in the circle plane, ensuring that the ratio $\frac{D}{d} = k$ is typically an integer, where k is assigned to be 3.

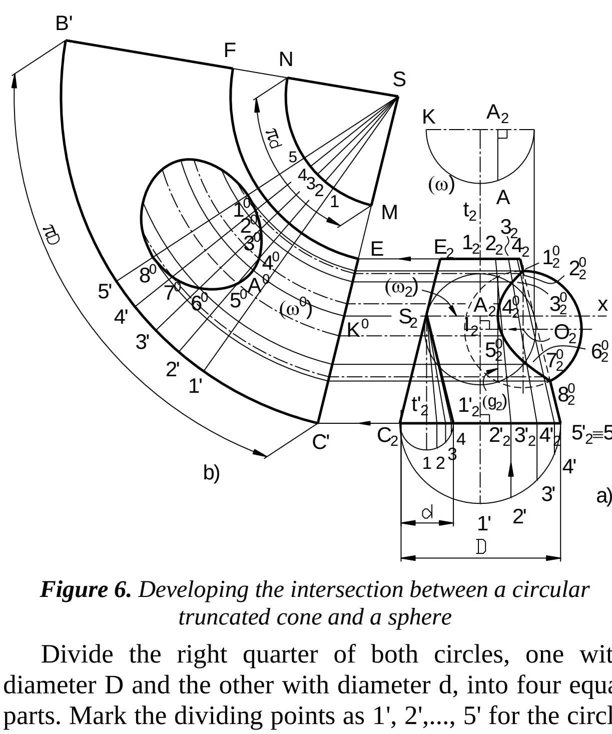


Figure 6. *Developing the intersection between a circular truncated cone and a sphere*

Divide the right quarter of both circles, one with diameter D and the other with diameter d, into four equal parts. Mark the dividing points as 1', 2',..., 5' for the circle with diameter D, and as 1, 2,..., 5' for the circle with diameter d. Next, draw the dividing generation lines of the auxiliary cone passing through these dividing points. As the two cones are similar, their corresponding pairs of generation lines are parallel. Hence, we can proceed to construct the vertical projection of the dividing generation lines for the truncated cone: $1'_2 1_2$, $2'_2 2_2$, $3'_2 3_2$, $4'_2 4_2$ (Figure 6a). These lines intersect the vertical projection parabola at points 1^0_2, 2^0_2, 3^0_2, 4^0_2, 5^0_2, 6^0_2, 7^0_2, 8^0_2. The auxiliary cone with vertex S and the two base circles of the truncated cone are developed using a similar method, as illustrated in (Figure 1a).

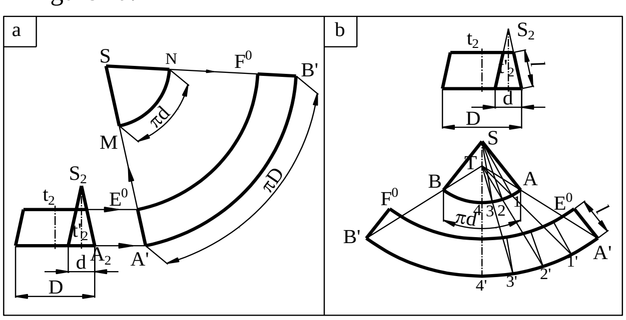
To develop the points belonging to the intersection in the vertical projection: 1^0_2, 2^0_2, 3^0_2, 4^0_2, 5^0_2, 6^0_2, 7^0_2, 8^0_2, we connect them to the circles on the conical surface in the plane perpendicular to the t-axis. These circles are then developed into fan arcs with center S. The fan arcs intersect the corresponding dividing fan radii Si' (i' = 1', 2', 3', 4', 5') at points 1^0, 2^0, 3^0, 4^0, 5^0, 6^0, 7^0, 8^0, respectively. These points represent the development points that need to be constructed (Figure 6b).

The point A_2 corresponds to the vertex of the developed vertical projection parabola, represented as A^0. It is important to emphasize that the length of the arc K^0A^0 is equal to the length of the arc KA.

Draw smooth curves connecting the set of points 1^0, 2^0, 3^0, 4^0, 5^0, 6^0, 7^0, 8^0. By taking advantage of the symmetry about the axis S5', we can achieve the complete development of the intersection, as depicted in Figure 6b.

3. Results and discussion

The study results have introduced a comprehensive and accurate method for developing the lateral surface of a circular truncated cone using only its vertical projection. This method is applicable to cones with bases of arbitrary shapes and positions, taking into account that the cone's vertex is positioned outside the drawing, as demonstrated in Figure 7a.



a) Research results *b) Based on reference [1]*

Figure 7. *Compare the study results and reference [1]*

The development of the circular truncated conical surface is accomplished using the approximation method, as outlined in reference [1] and depicted in Figure 7b.

The study results demonstrate that the base circles of the truncated cone accurately develop into the arcs A'B' and E^0F^0. According to reference [1], the base circles of the truncated cone develop into approximate curves A'B' and E^0F^0. Based on reference [9], the development of the circular truncated conical surface is achieved using a 3D model.

4. Conclusion

The study results presented in this study offer significant implications for the efficient development of intricate components of circular truncated conical tubes that intersect with other circular tubes, such as conical, cylindrical, and spherical shapes. This development method not only holds great value in the field of mechanical engineering but also has wide-ranging applications in the pipeline industry. Its quick and reliable approach enables accurate construction and visualization of these complex structures, ultimately enhancing the overall design and manufacturing processes.

Furthermore, the results of this research can be effectively incorporated into educational materials and curricula, particularly in the field of mechanical engineering. By integrating these findings into engineering programs, students can enhance their proficiency in designing and developing surfaces commonly encountered in technical drawings. This integration will bridge the gap between theoretical knowledge and practical design expertise, empowering students with the necessary skills and capabilities to work with surfaces extensively utilized in various engineering applications.

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