RELIABILITY ANALYSIS FOR TWO-UNIT SYSTEM COMPOSING OF AN INTERMITTENT OPERATION UNIT

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Abstract - In this paper, a close-form reliability analysis for two-unit system consisting of an intermittent operation (IO) component and a continuous operation (CO) component is developed. While the CO component continuously works, the IO component alternatively works in both operation and standby modes. The deterioration of the CO component could lead to increase working frequency of the IO component, which in result, could speed up the deterioration process of the IO component. Ignoring this deterioration interaction in deterioration modeling and reliability analysis could lead to inaccuracy in lifetime prediction and maintenance planning. Therefore, a deterioration model for the IO component is developed taking into account the deterioration between components. Reliability model of the IO component and the whole system are then derived.

Key words - Reliability analysis; two-unit system; continuous operation component, intermittent operation component

1. Introduction

Reliability analysis based on deterioration modeling has been increased recently [1]. In deterioration modeling, the models based on stochastic processes have obvious superiority in modeling the random fluctuations and achieving failure time distributions, mainly including Wiener process, Gamma process models, and inverse Gaussian process models [2]. The properly models for modeling the deterioration process of the component significantly influence the accuracy of system reliability analysis and lifetime prediction.

For modeling the deterioration process of the components, it is necessary to consider their working characteristics. In practice, components in the system can be classified into two types based on their working characteristics: (1) active and (2) standby components [3]. The active components continuously operate in consistent with the system. Contrarily, the standby components do not work during the normal operation of the system, and being switched to working mode only if its corresponding components failed [4]. Some components, however, alternate between active and standby operation modes. This kind of components is known as intermittent operation components [5]. Multiple speed Gear box for example, which can creates diferent speed levels. For each speed level, only a specific groups of gear is active, while the others are inactive [6]. Modeling the deterioration process of both active and standby components have been greatly investigated [7]. However, there is rarely study on the deterioration modeling for the IO component. To the best of our knowledge, only Zhu et al. proposed a reliability model for an intermittent working system based on Wiener process [8]. Nevertheless, this study only takes the single unit system into account. As far as we are aware, no research has been done on deterioration modeling for two-unit system which consists of an IO component. This creates a demand in modeling the deterioration process for the IO components.

This paper studies a system including two components, one is IO component, and another is CO component. While the CO component operates continuously in consistent with the system, the IO component alternatively works in both operation and standby modes, i.e., it is periodically switched between two operation mode: standby and active modes. This process is regularly repeated. One noticeable characteristic of this type of system is that the deterioration of the CO component can increase the working frequency of the IO component, which in result, could speed up the deterioration process of the IO component. This deterioration rate interaction occurs in a number of systems, air compressor system for example [9]. The air compressor system includes a compressor unit (CU) and an air storage and distribution unit (ASDU) [10]. The CU provide pressurized air into the ASDU until the pressure in the ASDU crosses the upper threshold, its operation mode is then switched to standby mode. By consumption, the pressure in the ASDU declines to below the lower threshold, the CU is then activated to compress air into the storage unit. This procedure is repeated until the system stop working. The deterioration of the ASDU lead to air leakage, which speeds up the pressure declining rate. Therefore, the working frequency of the CU is increased, which in result, speed up the deterioration rate of the compressor unit [11]. This deterioration interaction could adversely impact lifetime of the IO component. Ignoring the deterioration interdependence between the two components in deterioration modeling could lead to inaccurate lifetime prediction, and maintenance planning [12]. Thus, it is important to take into account the deterioration interdependence in the deterioration modeling for the IO.

In summary, this study proposed three main contributions:

- Modeling the deterioration process of the intermittent operation component considering deterioration interactions.

- Developing a reliability analysis for two-component system with an IO component.

- Application of proposed model for reliability analysis of an air compressor system.

2. Motivation and system description

2.1. Motivation example

In this section, we present an engineering example of the air compressor system. Air compressor system presents in almost all industries; companies use compressed air for powering air tools, driving and controlling of automatic machine and production lines, etc. [10]. As introduced in section 1, the system consists of a compressor unit (CU) and the compressed air storage and distribution unit (ASDU). The CU (1) intermittently works to compress air into the ASDU (2); on the other hand, the ASDU continuously works to distribute the compressed air to the end users (3) (Figure 1).

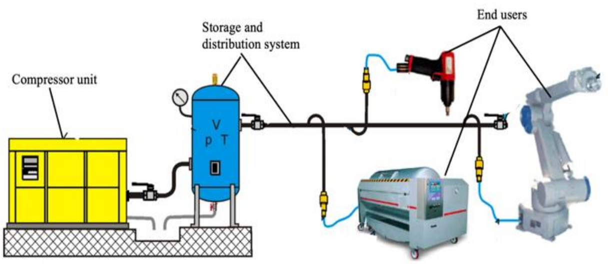

Figure 1. *Illustration of an air compressor system*

The deterioration of the ASDU lead to air leakage. Therefore, the pressure declines faster than it is in normal operation. In that way, working frequency of the CU is increased, which in result, speed up its deterioration rate. In fact, air leakage significantly engages in the energy lost and operation cost of the air compressor system [11].

2.2. System description

Considering a system consisting of two components, while one component continuously works in consistent with the system, another is intermittently worked, i.e., it periodically being switched between working and standby modes. Figure 2 depicts the working properties of the two-unit system with an IO component. The IO component periodically works for a period δ, it then switched to standby mode every amount Δ of time ($\delta < \Delta$). Δ is the working cycle length of the IO.

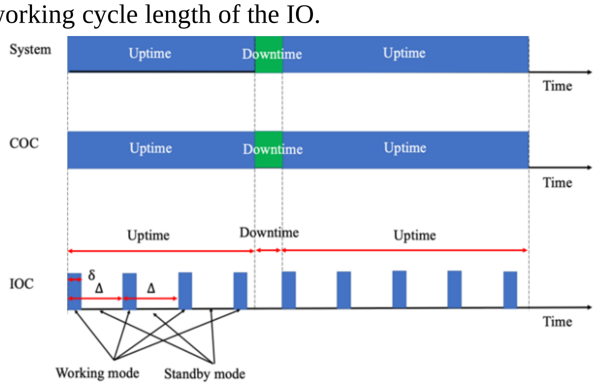

Figure 2. *Working properties of two-unit system with an IO component*

Suppose that each component i ($i = \text{CO}, \text{IO}$) commits to an accumulative continuous deterioration in usage time. This deterioration process can be represented by a scalar variable X_i. Component is known as failed when its deterioration level crosses a failure threshold, denoted as L_i. When a component is stopped working, deterioration level of the component stays the same if there is no maintenance action is conducted.

3. Deterioration modeling

3.1. Deterioration modeling for the CO

A number of models have been proposed and applied for modeling the deterioration process of the components such as Markov chains, Gamma process and Wiener process [2]. Among them, Gamma process is the most popular one. Indeed, gamma process is monotonic increase stochastic process with independent and non-negative increment having a gamma distribution with an identical scale parameter. With these characteristics, gamma process is suitable to describe the deterioration processes caused by accumulative of wear, creep, fatigue, corrosion, etc. [2]. Therefore, in this study, it is assumed that the deterioration of the CO component can be represented by gamma process. In that way, the deterioration level of the CO component at time $t = s + h, s \geq 0, h > 0$ is time period between time t and s, $X_{CO}(t)$, given its deterioration level at time s is $X_{CO}(s) = x_{CO}^s$, is given by:

$$X_{CO}(t) = x_{CO}^s + \Delta X_{CO}(h) \tag{1}$$

Where, $\Delta X_{CO}(h)$ is the addition of the CO deterioration level from time s to time t which is governed by a gamma distribution with shape parameter $\alpha_{co}.h$, and scale parameter β_{co}, its probability distribution function (pdf) is defined as:

$$f_{\alpha_{co}.h,\beta_{co}}(x) = \frac{1}{\Gamma[\alpha_{co}.h]} {\beta_{co}}^{\alpha_{co}.h} . x^{\alpha_{co}.h-1} . e^{-\beta_{co}x} \tag{2}$$

Where, $\Gamma[x] = \int_0^\infty u^{x-1} exp(-u) du, x > 0$ is the Euler gamma function. The scale and shape parameters, α_{co} and β_{co}, can be obtained from historical deterioration data.

The failure time of the CO component is the first time its deterioration level, $X_{CO}(t)$, crosses its failure threshold L_{CO}, and given by:

$$T_{CO}^f = \inf\{t > s > 0 : X_{CO}(t) \geq L_{CO} | x_{CO}^s < L_{CO}\} \tag{3}$$

The cumulative probability function (CDF) of T_{CO}^f is given as:

$$\begin{aligned} F_{\alpha_{co}.h,\beta_{co}}(t) &= \mathbb{P}\left[X_{CO}\left(T_{CO}^f\right) \geq L | T_{CO}^f > s\right] \\ &= \frac{\Gamma[\alpha_{co}.h, \beta_{co}(L_{CO} - x_{CO}^s)]}{\Gamma[\alpha_{co}.h]} \end{aligned} \tag{4}$$

Where, $\Gamma[\alpha, \vartheta] = \int_\vartheta^\infty x^{\alpha-1} . e^{-x} dx$ is the upper incomplete gamma function.

3.2. Deterioration modeling for the IO

The IO component's deterioration level at time $t = s + h, h > 0$, is also described by:

$$X_{IO}(t) = x_{IO}^s + \Delta X_{IO}\left(t_{op}(t, s)\right) \tag{5}$$

Where, $t_{op}(t, s)$ is the working time of the IO component when the system continuosly works in the interval (t, s). Since the IO component intermittently works for a period δ in a working cycle Δ, its cumulative working time is given by:

$$t_{op}(t, s) = N(t). \delta = \frac{t-s}{\Delta} . \delta = \frac{h}{\Delta} . \delta \tag{6}$$

However, when the deterioration level of the CO component becomes large, its performance declines faster. Therefore, working frequency of the IO component also increases, i.e., its working cycle length is declined. It is supposed that the deterioration process of the CO component is discretized into N zones, denoted as 1, 2,.., N, whereas, the CO component deterioration level is in zone $j, j = 1, 2, .., N$, if $X_{j-1} \le X_{CO} < X_j$, $(X_0 = 0, X_{N-1} = L)$. When the CO component deterioration level is in zone j, the IO component works with cycle length Δ_j. Figure 3 illustrates influence of deterioration state of the CO component on the working cycle length of the IO component. When the deterioration level of the CO component is in the first interval, i.e., $X_0 \le X_{CO} < X_1$, the IO component works with cycle length Δ_1. When deterioration level of the CO increases and enter the higher interval, the performance of the CO component declines faster, therefore, the working cycle length of the IO component is declined.

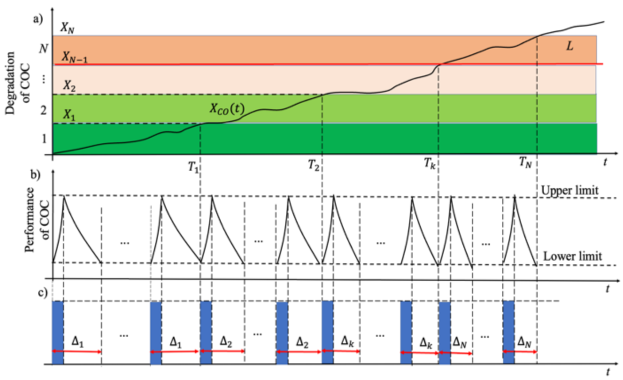

Figure 3. *Impact of (a) the deterioration of the CO component on (b) its performance and (c) the operating cycle of the IO component*

Suppose that at time s, the CO component is in deterioration zone j, $X_{j-1} \le x_{CO}^s < X_j$. Let T_j be the time at which the CO component escapes the deterioration zone j and jump into the deterioration zone $(j+1)$, i.e., T_j is the first pass time of the deterioration process of the CO component to deterioration level X_j. The CDF of T_j is given by:

$$F_{\alpha_{co}.h,\beta_{co}}(t) = \mathbb{P}[T_j \le t | x_{CO}^s < X_j] = \frac{\Gamma[\alpha_{co}.h, \beta_{co}(X_j - x_{CO}^s)]}{\Gamma[\alpha_{co}.h]} \quad (7)$$

The expectation of T_j is given as:

$$\mathbb{E}[T_j] = \int_0^\infty h.f_{\alpha_{co}.h,\beta_{co}}(h).dh = \int_0^\infty h.\frac{F_{\alpha_{co}.h,\beta_{co}}(h)}{dh}.dh \quad (8)$$

Where, $f_{\alpha_{co}.h,\beta_{co}}(h)$ is the pdf of T_j.

In that way, the operation time of the IO component at time $t = s + h$ is given by:

$$t_{op}(t,s) = \begin{cases} \frac{h.\delta}{\Delta_j}, & if\ h \le T_j \\ \left(\frac{T_j}{\Delta_j} + \frac{h - T_j}{\Delta_{j+1}}\right).\delta, & if\ T_j \le h \le T_{j+1} \\ \dots & \\ \left(\frac{T_j}{\Delta_j} + \dots + \frac{T_k - T_{k-1}}{\Delta_k} + \frac{h - T_k}{\Delta_{k+1}}\right).\delta, & if\ h \ge T_k, j < k < N \end{cases} \quad (9)$$

Figure 4 presents a simulation of the deterioration process of the IO component with and without taking into account the deterioration interaction. Knowing that the parameters for the deterioration model are given in Table 1. The historical deterioration of the components can be used for estimating their deterioration data [2]. The results shows that the deterioration interaction importantly impacts on the deterioration process of the IO components.

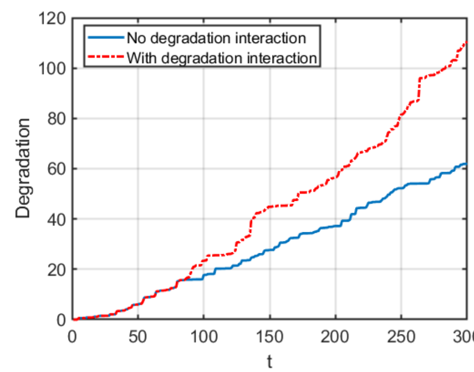

Figure 4. *The deterioration process of the IO component with and without taking into account the deterioration interaction*

Table 1. *Components' deterioration parameters*

Parameters / Component	x^0	α	β	Δ_1	Δ_2	Δ_3	Δ_4	δ	X_1	X_2	X_3	L
1. CO	1.5	0.4	1.0	-	-	-	-	-	30	55	80	80
2. IO	1.0	0.8	1.4	0.1	0.085	0.07	0.055	0.02	-	-	-	50

4. Reliability analysis

For a component subjected to a deterioration process, the reliability of the CO component at time $t = s + h$, $h > 0$, given its deterioration level at time s is x_{CO}^s, is the probability that its deterioration level is smaller than the failure threshold:

$$R_{CO}(t) = P[X_{CO}(t) \le L_{CO}] = 1 - \frac{\Gamma[\alpha_{co}.h, \beta_{co}(L_{CO} - x_{CO}^s)]}{\Gamma[\alpha_{co}.h]} \quad (10)$$

Similarly, the reliability of the of the IO component is given by:

$$R_{IO}(t) = 1 - \frac{\Gamma[\alpha_{IO}.\mathbb{E}[t_{op}(t,s)], \beta_{co}(L_{CO} - x_{CO}^s)]}{\Gamma[\alpha_{co}.\mathbb{E}[t_{op}(t,s)]]} \quad (11)$$

Where, $\mathbb{E}[t_{op}(t,s)]$ is the expectation of the working time of the IO component:

$$\mathbb{E}[t_{op}(t,s] = \begin{cases} \frac{h.\delta}{\Delta_j}, & if\ h \le \mathbb{E}[T_j], \\ \left(\frac{\mathbb{E}[T_j]}{j} + \frac{h - \mathbb{E}[T_j]}{\Delta_{j+1}}\right).\delta, & if\ \mathbb{E}[T_j] \le h \le \mathbb{E}[T_{j+1}] \\ \dots & \\ \left(\frac{\mathbb{E}[T_j]}{\Delta_j} + \dots + \frac{\mathbb{E}[T_k] - \mathbb{E}[T_{k-1}]}{\Delta_k} + \frac{h - \mathbb{E}[T_k]}{\Delta_{k+1}}\right).\delta, & if\ h \ge \mathbb{E}[T_k], j < k < N \end{cases} \quad (12)$$

Where, $\mathbb{E}[T_j]$ is the mathematic expectation of the time left of the deterioration process of the CO to cross the deterioration level X_j, which is given in Eq. (6).

The reliability of the system is given by:

$$R_S(t) = R_{CO}(t).R_{IO}(t) \quad (13)$$

5. Application example

This section aims to show the feasibility and benefit of the developed models. In that way, the developed models are applied for reliability analysis of an air compressor system. As introduced in Figure 2, the system consists of a compressor unit (CU) and an air storage and distribution unit (ASDU). The CU is an IO component, which intermittently works to compress air into the storage and distribution unit. On the other hand, the ASDU is an CO component, which continuously works to distribute the compressed air to the end users. The deterioration of the ASDU lead to air leakage. Therefore, the pressure in the ASDU declines faster than it is when system is in normal operation. In that way, the operating frequency of the CU is increased, which in result, speed up the deterioration rate of the CU. In that way, it is necessary to develop an appreciate deterioration modeling and reliability analysis for the air compressor system.

For the purposed of illustration of the usage and advantages of the developed models, it is supposed that the system's consumption rate is constant, so the changing of the working frequency of the CU (IO component) is due to the deterioration of the ASDU (CO component). The deterioration parameters, that are needed for the proposed models, are assumed to be available (Table 1). Indeed, these parameters can be estimated from system maintenance history.

5.1. Reliability analysis

To demonstrate the practicality of the developed deterioration and reliability models for the IO component, Monte Carlo simulation is adopted to compare the reliability of the IO component by analytic method (obtained by Eq. 11) and simulation method. In simulation, M independent degrading experiments are repeated with the given data. As stated by the law of large number, the statistical results can approximate the analytical results when M is large enough ($M = 2000$). Note that the reliability of the IO component is approximated by the proportion of the experiments that are not failed by time t. The reliability of the IO component acquired by analytic method and simulation one are shown in Figure 5. It shows that the analytic result well matches with the simulation one, which proves the effectiveness of the developed analytic formular for reliability analysis of the IO component.

Figure 6 presents the reliability of the IO component with and without taking into account the deterioration interaction. It shows that the reliability in case of considering deterioration interaction is significantly lower than that one of omitting deterioration interaction. So, it implies that ignoring deterioration interaction can result in underestimating the risk of failure of the IO component.

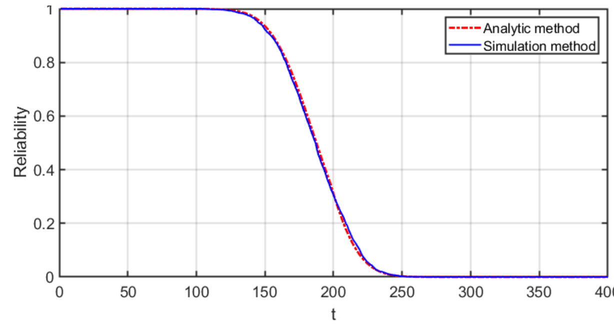

Figure 5. *Reliability comparison of the IO component by analytic and simulation method*

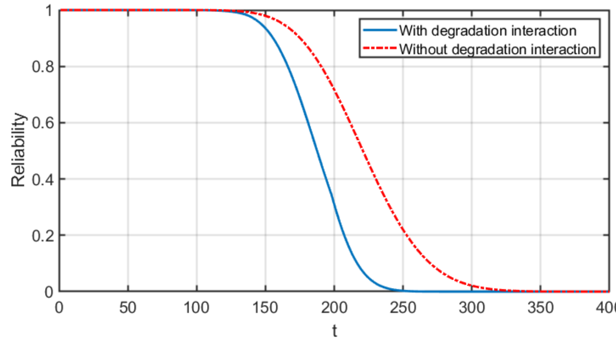

Figure 6. *Reliability of the IO component with and without taking into account deterioration interaction*

In addition, the system reliability in two scenarios with and without taking into account the deterioration interactions are provided in Figure 7. The result shows that deterioration interaction significantly influence the failure risk of the system. Omitting the deterioration interaction can lead to underestimating the failure risk of the system.

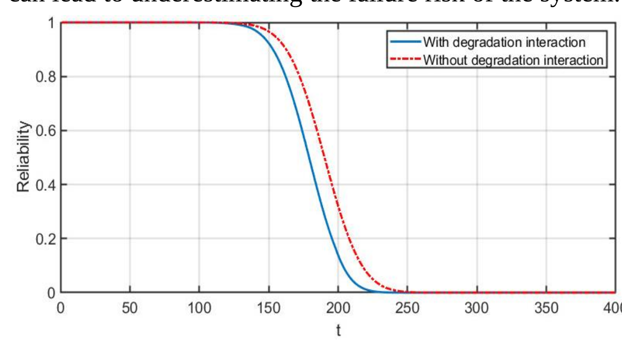

Figure 7. *Reliability of the system with and without considering deterioration interaction*

6. Conclusions and perspectives

In this work, a reliability analysis formula for a systems consisting of an IO component and a CO component is derived. The deterioration of the CO component could speed up the deterioration rate of the IO component. So, the deterioration interaction between the CO and IO components is investigated and the model for describing the deterioration behavior of the IO component is proposed. The formular for reliability analysis of the IO component is then derived. The proposed models have been successfully applied for reliability analysis of an air compressor system. The numerical result shows that (1) the deterioration interaction significantly impact on the IO component's lifetime; (2) considering the interaction in modeling the deterioration process of the IO component is important, and omitting it could have significant impact on estimating the failure risk of the system.

Although this study has made progress in addressing reliability analysis and deterioration modeling issues for system with intermittent operation component, there are still some issues that need to be addressed in more detail in subsequent studies. First, this study only considers two-unit system, studying on multi-component system with intermittent operation components can be considered in future works. Besides, deterioration model based on various other stochastic processes may be further studied in a variety of real-work applications.

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