

POINTING ERROR EFFECTS ON PERFORMANCE OF AMPLIFY-AND-FORWARD RELAYING FSO SYSTEMS USING SC-QAM SIGNALS OVER GAMMA-GAMMA ATMOSPHERIC TURBULENCE CHANNELS

Duong Huu Ai¹, Do Trong Tuan², Ha Duyen Trung²

¹VietNam Korea Friendship Information Technology College; aidh@viethanit.edu.vn

²Hanoi University of Science and Technology; trung.haduyen@hust.edu.vn

Abstract - This paper presents the theoretical analysis of the average symbol error rate (ASER) of free space optical (FSO) communication system combined with multiple-input multiple-output (MIMO) relay based on Amplify-and-Forward (AF) technique employing subcarrier quadrature amplitude modulation (SC-QAM) signal over strong atmospheric turbulence, which is modeled by Gamma-Gamma distributions and pointing error. The pointing error effect is studied by taking into account the influence of beamwidth, aperture size and jitter variance on the ASER. The influence of the number of relay stations, link distance and different MIMO/FSO configurations on the system's ASER are also discussed in this paper. The numerical results present the impact of pointing error on the performance of systems and how we use proper values of beamwidth and aperture to improve the performance of such systems.

Key words - AF; atmospheric turbulence; ASER; FSO; QAM; pointing error.

1. Introduction

Free-space optical (FSO) communications have gained significant research attention due to their ability to cater to high bandwidth demand. However, the optical links are vulnerable to adverse channel conditions caused by atmospheric turbulence and pointing error [1]. The turbulence induced scintillation and misalignment-fading is modeled using the Gamma-Gamma distribution, which is suitable for moderate to strong turbulence regimes [2]. To mitigate the impact of turbulence, multi-hop relaying FSO systems have been proposed as a promising measure to extend the transmission links and the turbulence-induced fading. Recently, performance of multi-hop relaying FSO systems over atmospheric turbulence channels has been studied in [3-10].

In this work, the ASER expressions of systems in the Gamma-Gamma atmospheric turbulence channel are analytically obtained by taking into account the influence of pointing errors represented by beam-width, aperture size and jitter variance. The SC-QAM scheme is adopted for the performance analysis. Moreover, the number of relaying stations is included in the statistical model of the combined channel together with atmospheric loss, atmospheric turbulence and pointing error.

The rest of the paper is organized as follows: Section 2 introduces the system description. Section 3 discusses the atmospheric turbulence model of AF MIMO/FSO/SC-QAM systems with pointing error. Section 4 is devoted to ASER derivation of AF MIMO/FSO links. Section 5 presents the numerical results and discussion. The conclusion is reported in Section 6.

2. System description

2.1. AF relaying SISO/FSO system using SC-QAM signals

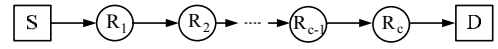


Figure 1. A serial relaying SISO/FSO system

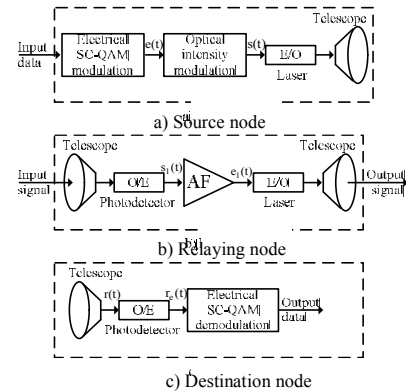


Figure 2. The source node, relaying node and destination node of SISO/FSO systems

We consider an AF relaying FSO system using SC-QAM signals depicted in Figure 1, with single transmitter and single receiver, in which signal from the source node S is transmitted to the destination node D serially through c relay stations $R_1, R_2, \dots, R_{c-1}, R_c$. The schemes are illustrated in Figure 2, QAM symbol is first up-converted to an intermediate frequency f_c . The electrical SC-QAM signal at the output of QAM modulator can be written as

$$e(t) = s_1(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t) \quad (1)$$

where $s_1(t) = \sum_{i=-\infty}^{+\infty} a_i(t)g(t - iT_s)$ and $s_Q(t) = \sum_{j=-\infty}^{+\infty} b_j(t)g(t - jT_s)$ are the in-phase and the quadrature signals, respectively. $a_i(t)$, $b_j(t)$ are the in-phase and the quadrature information amplitudes of the transmitted data symbol, respectively, $g(t)$ is the shaping pulse and T_s denotes the symbol interval. The QAM signal is used to modulate the intensity of a laser of the transmitter, the transmitted signal can be written as

$$s(t) = P_s \{1 + \kappa[s_1(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)]\} \quad (2)$$

where P_s denotes the average transmitted optical power per symbol at each hop and κ is the modulation index. Due to the effects of both atmospheric loss, atmospheric turbulence and the pointing error, the received optical signal at the first relay node can be expressed as

$$s_1(t) = X P_s \{1 + \kappa[s_1(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)]\} \quad (3)$$

where X presents the signal scintillation caused by atmospheric loss, atmospheric turbulence and the pointing error. At each relay node, AF module is used for signal

amplification as shown in Figure 2b. Due to slow turbulence changes, the DC term $\{XP_s\}$ can be filtered out by a bandpass filter. The electrical signal output of AF module at the first relay node therefore can be expressed as

$$e_1(t) = \Re X P_1 P_s \kappa e(t) + v_1(t) \quad (4)$$

where \Re is the PD's responsivity and P_1 is the amplification power of the first relaying node. The receiver noise $v_1(t)$ can be modeled as an additive white Gaussian noise (AWGN) process.

Repeating such manipulations above, the electrical signal output of the PD at the destination node can be derived as follows

$$r_e(t) = P_s e(t) \left[\prod_{i=0}^c \Re^{2i+1} X_{i+1} P_i \right] + \sum_{i=0}^c v_i(t) \quad (5)$$

2.2. AF relaying MIMO/FSO system using SC-QAM signals

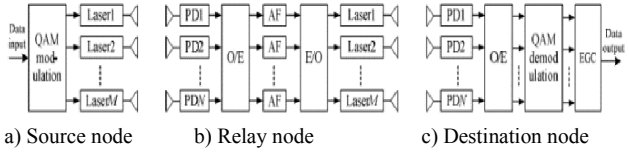


Figure 3. The source node, relay node and destination node of MIMO/FSO systems

In this section, we consider a general AF relaying $M \times N$ MIMO/FSO system using SC-QAM signals with M -lasers pointing toward an N -aperture receiver as depicted. The schemes are illustrated in Figure 3. The MIMO/FSO channel can be modeled by $M \times N$ matrix of the turbulence channel, denoted as $X = [X_{mn}]_{m,n=1}^{M,N}$. The electrical signal at the input of QAM demodulator of the destination node can be expressed as

$$r_e(t) = P_s e(t) \left[\sum_{m=1}^M \sum_{n=1}^N \prod_{i=0}^c (X_{i+1})_{mn} \Re^{2i+1} P_i \right] + \sum_{i=0}^c \left(\sum_{m=1}^M \sum_{n=1}^N v_{mn} \right) (t) \quad (6)$$

where X_{mn} denotes the stationary random process for the turbulence channel from the m^{th} laser to the n^{th} PD. In this system model, the instantaneous electrical SNR can be expressed as a finite sum of sub-channels as

$$\gamma = \left(\sum_{m=1}^M \sum_{n=1}^N \prod_{i=0}^c (\sqrt{\gamma_{imn}}) \right)^2 \quad (7)$$

where γ_{imn} is the random variable defined as the instantaneous electrical SNR component of the sub-channel from the m^{th} laser to the n^{th} PD, it can be expressed as

$$\gamma = \frac{\left(\frac{1}{MN} \kappa \Re^{2i+1} P_s \prod_{i=1}^c X_{i+1} P_i \right)^2}{N_0} = \bar{\gamma} \left(\prod_{i=0}^c X_{i+1} \right)^2 \quad (8)$$

where $\bar{\gamma}$ is defined as the average electrical SNR.

3. Atmospheric turbulence models with pointing error

In Eqs. (7) and (8), X is the channel state. X models the optical intensity fluctuations resulting from atmospheric loss X_l , atmospheric turbulence fading X_a and pointing

error X_p . They can be described as

$$X = X_l X_a X_p \quad (9)$$

3.1. Atmospheric loss

Atmospheric loss X_l is a deterministic component and no randomness, thus acting as a fixed scaling factor over a long time period. It is modeled in [4] as

$$X_l = e^{-\sigma L} \quad (10)$$

where σ denotes a wavelength and weather dependent attenuation coefficient, and L is the link distance.

3.2. Gamma-gamma atmospheric turbulence

For moderate to strong turbulence, the most widely accepted model is the gamma-gamma distribution, which has been validated through studies [4, 5]. The pdf of the irradiance intensity of gamma-gamma channel is given as

$$f_{X_a}(X_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} X_a^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta}X_a) \quad (11)$$

where $\Gamma(\alpha)$, $\Gamma(\beta)$ is the Gamma function and $K_{\alpha-\beta}(\cdot)$ denotes the modified Bessel function of the second kind. The positive parameter α represents the effective number of large-scale cells of the scattering process, and the positive parameter β represents the effective number of small-scale cells of the scattering process in the atmospheric.

$$\alpha = \left[\exp \left(\frac{0.49\sigma_2^2}{(1 + 0.18d^2 + 0.56\sigma_2^{12/5})^{7/6}} \right) - 1 \right]^{-1} \quad (12)$$

$$\beta = \left[\exp \left(\frac{0.51\sigma_2^2 (1 + 0.69\sigma_2^{12/5})^{-5/6}}{1 + 0.9d^2 + 0.62\sigma_2^{12/5}} \right) - 1 \right]^{-1} \quad (13)$$

In Eqs. (12) and (13), $d = \sqrt{kD^2/4L}$ where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, L is the link distance, and D is the receiver aperture diameter, and σ_2 is the Rytov variance, it expressed by [4]

$$\sigma_2^2 = 0.492 C_n^2 k^{7/6} L^{11/6} \quad (14)$$

where C_n^2 is the refractive-index structure parameter.

Through c relay stations, there are $(c+1)$ turbulence channels, The pdf of X^{c+1} is as follows:

$$f_{X_{mn}}(X_{mn}^{c+1}) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{(c+1)\Gamma(\alpha)\Gamma(\beta)} X_{mn}^{\frac{\alpha+\beta-2}{2}-c} \times K_{\alpha-\beta}(2\sqrt{\alpha\beta}X_{mn}) \quad (15)$$

3.3. Pointing error model

A statistical pointing error model is developed in [7, 8], the pdf of X_p is given as [7]

$$f_{X_p}(X_p) = \frac{\xi^2}{A_0^{\xi^2}} X_p^{\xi^2-1}, \quad 0 \leq X_p \leq A_0 \quad (16)$$

where $A_0 = [\text{erf}(v)]^2$ is the fraction of the collected power at radial distance 0, v is given by $v = \sqrt{\pi}r/(\sqrt{2}\omega_z)$ with r and ω_z respectively denote the aperture radius and the beam

waist at the distance z and $\xi = \omega_{zeq} / 2\sigma_s$, where the equivalent beam radius can be calculated by [7]

$$\omega_{zeq} = \omega_z (\sqrt{\pi} \operatorname{erf}(v) / 2v \times \exp(-v^2))^{1/2} \quad (17)$$

where, $\omega_z = \omega_0 \left[1 + \varepsilon (\lambda L / \pi \omega_0^2)^2 \right]^{1/2}$ with ω_0 is the transmitter beam waist radius at $z = 0$, $\varepsilon = (1 + 2\omega_0^2) / \rho_0^2$ and $\rho_0 = (0.55 C_n^2 k^2 L)^{-3/5}$ is the coherence length.

3.4. Combined channel model

We derive a complete statistical model of the channel considering the combined effect of atmospheric turbulence, atmospheric loss and pointing error. The unconditional pdf of the channel state is obtained [8, eq. (17)].

$$f_X(X) = \int f_{X|X_a}(X|X_a) f_{X_a}(X_a) dX_a \quad (18)$$

where $f_{X|X_a}(X|X_a)$ denotes the conditional probability given a turbulence state, and it can be expressed by [8].

$$f_{X|X_a}(X|X_a) = \frac{1}{X_a X_l} f_{X_p} \left(\frac{X}{X_a X_l} \right) \quad (19)$$

As a result, we can derive the unconditional pdf. The pdf can be expressed by

$$f_X(X) = \frac{2\xi^2(\alpha\beta)^{(\alpha+\beta)/2}}{(c+1)(A_0 X_l)^\xi \Gamma(\alpha)\Gamma(\beta)} X^{\xi-1} \times \int_{X/X_l A_0}^{\infty} X_a^{\frac{(\alpha+\beta)}{2}-1-\xi^2-c} K_{\alpha-\beta}(2\sqrt{\alpha\beta X_a}) dX_a \quad (20)$$

The modified Bessel function of the second kind $K_{\alpha-\beta}(\cdot)$ can be expressed as a special case of the Meijer G-function, given by the following relationship [11]

$$f_X(X) = \frac{\xi^2(\alpha\beta)^{c+1}}{(c+1)(A_0 X_l)\Gamma(\alpha)\Gamma(\beta)} \left(\alpha\beta \frac{X}{A_0 X_l} \right)^{\frac{\alpha+\beta}{2}-1-c} \times G_{1,3}^{3,0} \left[\alpha\beta \frac{X}{A_0 X_l} \left| \begin{matrix} 1-\frac{\alpha+\beta}{2}+\xi^2+c \\ -\frac{\alpha+\beta}{2}+\xi^2+c, \frac{\alpha-\beta}{2}, \frac{\beta-\alpha}{2} \end{matrix} \right. \right] \quad (21)$$

Eq. (21) can be further simplified using [12, eq. (9.31.5)] as

$$f_X(X) = \frac{\xi^2(\alpha\beta)^{c+1}}{(c+1)(A_0 X_l)\Gamma(\alpha)\Gamma(\beta)} \times G_{1,3}^{3,0} \left[\alpha\beta \frac{X}{A_0 X_l} \left| \begin{matrix} \xi^2 \\ \xi^2-1, \alpha-1-c, \beta-1-c \end{matrix} \right. \right] \quad (22)$$

4. Aser calculation

We can derive the average symbol error rate of AF relaying MIMO/FSO/SC-QAM, which can be generally expressed as

$$P_{se} = \int_0^\infty P_e(\gamma) f_\gamma(\gamma) d\gamma \quad (23)$$

where $P_e(\gamma)$ is the conditional error probability (CEP), $\Gamma = \{\Gamma_{nm}, n=1, \dots, N, m=1, \dots, M\}$ is the matrix of the MIMO/FSO channels. For using SC-QAM modulation, the conditional error probability presented as

$$P_e(\gamma) = 1 - \left[1 - 2q(M_I)Q(A_I\sqrt{\gamma}) \right] \left[1 - 2q(M_Q)Q(A_Q\sqrt{\gamma}) \right] \quad (24)$$

where $q(x) = 1 - x^{-1}$, $Q(x)$ is the Gaussian Q -function, $Q(x) = 0.5 \operatorname{erfc}(x/\sqrt{2})$, $A_I = \left(6 / \left[(M_I^2 - 1) + r^2 (M_Q^2 - 1) \right] \right)^{1/2}$, $A_Q = \left(6r^2 / \left[(M_I^2 - 1) + r^2 (M_Q^2 - 1) \right] \right)^{1/2}$, in which $r = d_o/d_i$ as the quadrature to in-phase decision distance ratio, M_I and M_Q are in-phase and quadrature signal amplitudes, respectively. Eq. (24) can further be written as follows

$$P_e(\gamma) = 2q(M_I)Q(A_I\sqrt{\gamma}) + 2q(M_Q)Q(A_Q\sqrt{\gamma}) - 4q(M_I)q(M_Q)Q(A_I\sqrt{\gamma})Q(A_Q\sqrt{\gamma}) \quad (25)$$

Assuming that MIMO sub-channels' turbulence processes are uncorrelated, independent and identically distributed, according to Eq. (8), Eq. (22) and formula contact between probability density function, we obtain the pdfs of AF relaying MIMO/FSO systems over gamma-gamma channel as

$$f_{\Gamma_{mn}} \left(\gamma \frac{c+1}{2} \right) = \frac{\xi^2(\alpha\beta)^{c+1}}{(c+1)(A_0 X_l)\Gamma(\alpha)\Gamma(\beta)} \frac{1}{2\gamma} \times G_{1,3}^{3,0} \left[\alpha\beta \frac{(\gamma/\gamma)^{0.5}}{A_0 X_l} \left| \begin{matrix} \xi^2 \\ \xi^2-1, \alpha-1-c, \beta-1-c \end{matrix} \right. \right] \quad (26)$$

Substituting Eq. (25) and Eq. (26) into Eq. (23), the ASER of the systems can be obtained as

$$P_{se}(\gamma) = 2q(M_I) \int_0^\infty Q(A_I\sqrt{\gamma}) f(\gamma) d\gamma + 2q(M_Q) \int_0^\infty Q(A_Q\sqrt{\gamma}) f(\gamma) d\gamma - 4q(M_I)q(M_Q) \int_0^\infty Q(A_I\sqrt{\gamma})Q(A_Q\sqrt{\gamma}) f(\gamma) d\gamma \quad (27)$$

5. Numerical results and discussion

Using previous derived expressions, Eq. (26) and Eq. (27), we present numerical results for ASER analysis of the AF relaying MIMO/FSO systems. The systems's ASER can be estimated via multi-dimensional numerical integration with the help of the Matlab™ software. Relevant parameters considered in our analysis are provided in Table 1.

In Figure 4, the system's ASER is presented as a function of transmitter beam waist radius under several of number relay stations. In these figures it is clearly depicted that for a given condition including specific values of number relay stations, aperture radius and average SNR, the minimum of ASER can be reached to a specific value of ω_0 . This value is called the optimal transmitter beam waist radius. Apparently, the more the value of transmitter beam waist radius comes close to the optimal one, the lower the value of system's ASER is.

Table 1. Sysem parametters and constants

Parameter	Symbol	Value
Laser Wavelength	λ	1550 nm
Photodetector responsivity	\Re	1 A/W
Modulation Index	κ	1
Total noise variance	N_0	10^{-7} A/Hz

In-phase/Quadrature signal amplitudes	M_I/M_Q	8/4
Index of refraction structure	C_n^2	$3 \times 10^{-14} \text{m}^{-2/3}$

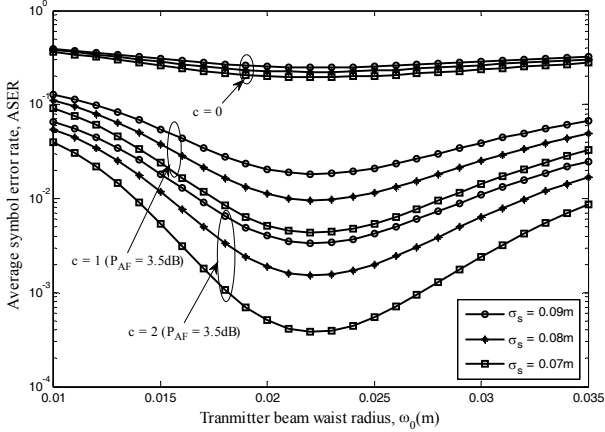


Figure 4. ASER performance versus transmitter beam waist radius ω_0 for various values of σ_s with the aperture radius $r = 0.065 \text{m}$, the average SNR = 23 dB and $L = 1000 \text{m}$.

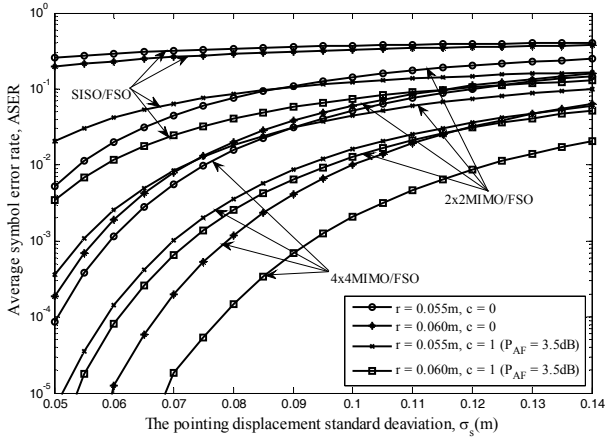


Figure 5. ASER performance against σ_s for various values of aperture radius with transmitter beam waist radius $\omega_0 = 0.022 \text{m}$, the average SNR = 23 dB and $L = 1000 \text{m}$.

Figure 5 illustrates the ASER performance against the pointing error displacement standard deviation of the AF relaying MIMO/FSO systems with different MIMO configuration of 2×2 and 4×4 MIMO/FSO systems. As it is clearly shown, the system's ASER is improved significantly with the increase of number of lasers, receivers and relay stations. In addition, the pointing error effects impact more severely on the system's performance since higher values of ASER are gained. The impact of the aperture radius and the transmitter beam waist radius on the system's performance is more significant in low σ_s regions than in high σ_s regions.

Figure 6 illustrates the ASER performance against the aperture radius under various relay stations of the AF relaying MIMO/FSO systems. As a result, the system's ASER significantly decreases when the values of aperture radius and number relay stations increase. It is found that, in low-value region when aperture radius increases, system's ASER does not change much. However, when aperture radius exceeds the threshold value, ASER

plummets when aperture radius increases.

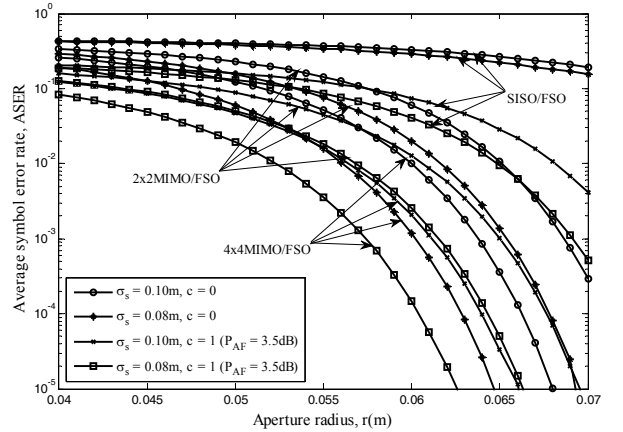


Figure 6. ASER performance against the aperture radius r for various values of σ_s with the beam waist radius $\omega_0 = 0.022 \text{m}$, the average SNR = 23 dB and $L = 1000 \text{m}$.

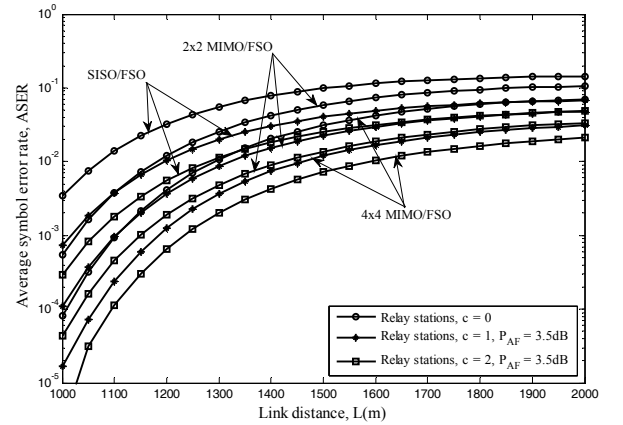


Figure 7. ASER performance versus link distance with of SISO, 2×2 and 4×4 MIMO/FSO systems. $\sigma_s = 0.08 \text{m}$, beam waist radius $\omega_0 = 0.022 \text{m}$, $r = 0.055 \text{m}$ and $L = 1000 \text{m}$.

Figure 7 depicts the ASER performance as function of the transmission link distance L for various number of relay stations with different MIMO/FSO configurations. It can be seen from the figure that the ASER increases when the transmission link distance is longer. In addition, when using relay stations combined with MIMO/FSO systems, ASER will get better performance. Figure 8 illustrates the ASER as a function of the average electrical SNR for different number relay stations and different MIMO configurations. The ASER decreases with the increase of the SNR, number of relay stations and MIMO/FSO. It can be found that simulation results show a closed agreement with analytical results.

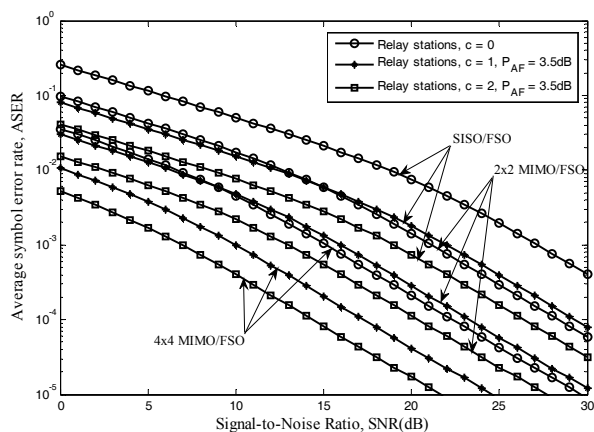


Figure 8. ASER performance versus average SNR of SISO, 2×2 and 4×4 MIMO/FSO system. $\sigma_s = 0.08$ m, beam waist radius $\omega_0 = 0.022$ m, $r = 0.055$ m and $L = 1000$ m.

6. Conclusion

In this paper, we have theoretically analyzed the ASER of AF relaying MIMO/FSO communication systems employing SC-QAM signals over Gamma-Gamma atmospheric turbulence and pointing error. The results present the theoretical expressions for ASER performance of SISO and MIMO systems by taking into account the number of AF relay stations, MIMO configurations and the pointing error effect. The numerical results show the impact of pointing error on the system's performance. By analyzing ASER performance, we can conclude that using proper values of aperture radius, transmitter beam waist radius, partially surmounted pointing error and number of relay stations combined with MIMO/FSO configurations could greatly benefit the performance of such systems.

REFERENCES

- [1] X. Zhu and J. M. Kahn, "Free-space optical communication through atmospheric turbulence channels," *IEEE Trans. Commun.*, vol. 55, no. 8, pp. 1293–1300, Aug. 2002.
- [2] M. A. Al-Habash, L. C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," *Opt. Eng.*, vol. 40, no. 8, pp. 1554–1562, 2001.
- [3] Mona Aggarwal, Parul Garg and Parul Puri, "Exact Capacity of Amplify and-Forward Relayed Optical Wireless Communication Systems," *IEEE Photonics Technology Letters*, VOL. 27, NO. 8, April 15, 2015.
- [4] Majumdar, A. K., Ricklin, J. C., *Free-space laser communications: principles and advances*, New York Springer (2008)
- [5] Andrews, L., Phillips, R., Hopen, C., *Laser Beam Scintillation with Applications*, Bellingham, WA: SPIE Press (2001).
- [6] Safari, M., Uysal, M., "Relay-Assisted Free-Space Optical Communication," *IEEE Trans. Wireless Communication*. (2008) 5441-5449.
- [7] Amon, S., "Effects of atmospheric turbulence and building sway on optical wireless communication systems," *Opt. Lett.* 28 (2003) 129–131
- [8] Trung, H. D., Tuan, D. T., Anh, T. P., "Pointing error effects on performance of free-space optical communication systems using SC-QAM signals over atmospheric turbulence channels," *AEU-Int. J. of Elec. and Commun.* 68 (2014) 869-876
- [9] Kostas P. Peppas, "A New Formula for the Average Bit Error Probability of Dual-Hop Amplify-and-Forward Relaying Systems over Generalized Shadowed Fading Channels," *IEEE Wireless Communications Letters*, VOL. 1, NO. 2, APRIL 2012.
- [10] Kostas P. Peppas, Argyris N. Stassinakis, Hector E. Nistazakis, and George S. Tombras, "Capacity Analysis of Dual Amplify-and-Forward Relayed Free-Space Optical Communication Systems Over Turbulence Channels With Pointing Errors," *J. OPT. COMMUN. NETW./VOL.* 5, NO. 9/SEPTEMBER 2013.
- [11] Harilaos G. Sandalidis, Theodoros A. Tsiftsis, Member, and George K. Karagiannidis, Senior, "Optical Wireless Communications With Heterodyne Detection Over Turbulence Channels With Pointing Errors," *JOURNAL OF LIGHTWAVE TECHNOLOGY*, VOL. 27, NO. 20, OCTOBER 15, 2009.
- [12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. New York: Academic, 2008.

(The Board of Editors received the paper on 05/03/2016, its review was completed on 25/03/2016)