PRESCRIBED PERFORMANCE-BASED ADAPTIVE SLIDING MODE CONTROL FOR A STRUCTURE UNDER EARTHQUAKE EXCITATION

ĐIỀU KHIỂN TRƯỢT THÍCH NGHI DỰA TRÊN HIỆU SUẤT QUY ĐỊNH CHO KẾT CẤU DƯỚI SỰ KÍCH THÍCH ĐỘNG ĐẤT

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Abstract - In this research paper, an adaptive sliding controller designed to enhance seismic response mitigation in structural systems with prescribed performance is presented. The performance function converts the error to a converging error within a predefined neighborhood. Furthermore, the adaptive algorithm is also designed to ensure the controller operates effectively under uncertain system parameters. The serial–parallel estimation model was adopted for parameter vector and error model. The Lyapunov function is chosen to ensure the convergence stability of the control strategy. Through comprehensive theoretical analysis and simulation studies, the efficiency of the performance function has been simulated for a one-degree-of-freedom structure. Simulation results demonstrated the effectiveness of the control algorithm in reducing the seismic response.

Key words - Prescribed performance; seismic response reduction; active isolator; adaptive control.

Tóm tắt - Trong nghiên cứu này, một bộ điều khiển trượt thích nghi được thiết kế để tăng cường chống lại tác động của động đất lên kết cấu dựa trên hiệu suất quy định được trình bày. Hàm hiệu suất chuyển đổi sai lệch thành sai lệch hội tụ trong vùng lân cận được xác định trước. Bộ điều khiển hoạt động hiệu quả với các thông số hệ thống không chắc chắn. Mô hình ước lượng song song nối tiếp được áp dụng cho vectơ tham số và mô hình sai số. Hàm Lyapunov được chọn để đảm bảo tính ổn định hội tụ của thuật toán điều khiển.Thông qua phân tích lý thuyết toàn diện, hiệu quả của bộ điều khiển dựa trên hiệu suất qui định đã được sử dụng để mô phỏng cho kết cấu cách ly tích cực một bậc tự do. Kết quả mô phỏng đã chứng minh tính hiệu quả của thuật toán điều khiển trong việc giảm tác động của động đất.

Từ khóa – Hiệu suất qui định; giảm tác động động đất; cách chấn tích cực; điều khiển thích nghi

1. Introduction

An important challenge in structural engineering research is to find an effective and reliable method to protect structures and their materials from dangerous external influences, such as strong winds or earthquakes [1-3]. Active isolator is one of the techniques to prevent damage caused by earthquakes. This method isolates and reduces earthquakes before they impact infrastructure and civil works. The isolator works to minimize the impact of earthquakes on a building using controls. The goal is to isolate all or part of the structure from earthquakes to ensure safety. The isolator can generate external forces to absorb earthquake energy. Sensors monitor and detect earthquakes and send response signals to the controller. The controller uses a control algorithm to determine the required control force on the structure. A suitable active control algorithm must be designed to ensure efficient structure. Active isolation is an effective method to protect infrastructure and structures from damage caused by strong earthquakes. However, design and implementation require advanced knowledge and techniques.

There are many active control algorithms for active isolator to reduce seismic response reduction. PID control can effectively control simple damping systems and may need to be fine-tuned. LQR is an optimal control method based on a linear system model. It optimizes an objective function based on the linear dynamics of the system to achieve the best performance. H-infinite control method attempts to optimize the performance of the damping system [4]. Fuzzy logic control uses fuzzy thinking to control the system [5]. It is suitable for non-linear systems and can handle complex control rules. The robust control method aims to resist changes in the system or environmental disturbances without losing stability [6-8].

In seismic protection of structures using active control, uncertain parameters are inevitable. Several methods have recently been proposed to overcome these problems, using adaptive algorithms [9, 10]. There has been research into a new adaptive control method using the Lyapunov Barrier Function (LBF) to address the performance requirements for earthquake isolation systems, such as limiting structural responses [11, 12]. This method allows online updating of uncertain system parameters, ensuring that these parameters converge to their actual values, thereby enhancing control performance for completely isolated systems. Although BLF control is sufficient to trade-off between conflicting requirements, it often fails to keep the immediate system performance (e.g., overshoot, convergence rate) within defined limits. It can lead to a more complex parameter-tuning process to improve implementation comfort by adhering to significant output constraints. Recently, Prescribed Performance Function Control (PPFC) is an advanced control strategy designed to achieve precise and predefined performance objectives in complex dynamic systems [13-14]. Unlike traditional control methods that focus on stabilizing a system around a set point, PPFC goes further by specifying desired performance criteria or functions the system must follow. It offers a flexible framework where engineers can define a prescribed performance function, often as a function of time, and the control system will actively work to ensure that the system's behavior adheres to this desired performance trajectory. This approach is precious in applications where precise tracking of performance specifications, such as position, velocity, or other dynamic parameters, is critical, including robotics, aerospace, and manufacturing processes. PPFC's ability to enforce prescribed performance criteria makes it a powerful tool for achieving high-precision control and ensuring the desired behavior of complex systems.

In this paper, an adaptive control strategy for earthquake-resistant systems with completely unknown isolator parameters is proposed. The novelty of prescribed performance-based adaptive control lies in effectively combining these two aspects. Incorporating specified performance requirements into the adaptive control framework ensures that the control system not only adapts to changes in the system dynamics but also achieves the desired performance specifications. This integration offers several advantages. Firstly, it provides a systematic approach to designing control systems with specific performance requirements. Secondly, it allows for flexibility in handling uncertainties and disturbances, thus enhancing the robustness of the control system. Lastly, the serial–parallel estimation model was adopted for parameter vector and error model. An adaptive algorithm is applied to compensate online for unknown dynamics, reducing the modeling accuracy requirement. To further enhance the control performance, an adaptive control strategy was designed using a Prescribed Performance Function (PPF) and the corresponding error transformation. This allows retaining both temporal performance measures (e.g., maximum overshoot, convergence rate) and predefined steady state constraints on the system's vertical variation, according to a priority.

2. Control design

2.1. System dynamics

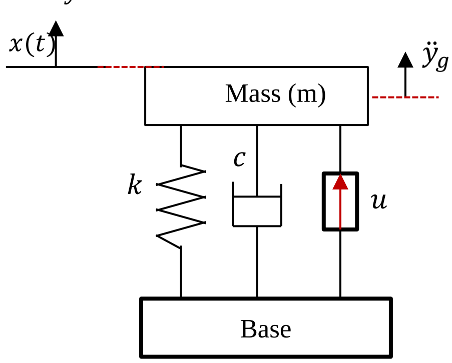

Figure 1. *SDOF active isolation system*

Consider a single degree of freedom (SDOF) active isolation system of a structure consisting of a spring, a damper, and an actuator installed in parallel, as shown in Figure 1. The motion equation of the structure can be written as

$$m\ddot{x} + c\dot{x} + kx = u + m\ddot{y}_g, \tag{1}$$

where m, c and k are the mass, damping coefficient, and stiffness of the system, respectively; $\ddot{y}_g$ is the acceleration excitation from an earthquake; u is the active control force.

2.2. Prescribed performance bounds (PPB) based controller design

The motion Eq. (1) is rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \rho u - Y\vartheta + \ddot{y}_g, \end{cases} \tag{2}$$

where $x_1 = x$; $x_2 = \dot{x}\ \rho = 1/m$, $Y = \left[{}^{k}/_{m}\ {}^{c}/_{m}\right]$, $\vartheta = [x_1\ x_2]$.

The Prescribed Performance Bound (PPB) based controller design is used in this study. The PPB model calculation involves creating a mathematical representation of the dynamics and performance of the system being controlled. The model captures the system behavior and ensures stability with control inputs. Through the model, the controller design process aims to optimize control actions to meet and maintain desired performance criteria, thereby confirming that system behavior complies with predetermined limits, which are essential in applications with strict requirements for performance, safety, and control system efficiency.

We choose the function $\varphi(t)$ and $e = x_1$ as

$$\varphi(t) = (\varphi_0 - \varphi_\infty)e^{\alpha t} + \varphi_\infty, \tag{3}$$

where $\varphi_0 > \varphi_\infty$ and $\alpha > 0$ are the design parameters.

The motion x_1 can be retained by the following prescribed performance bound

$$-\underline{\delta}\varphi(t) < x_1(t) < \bar{\delta}\varphi(t)\ \forall t > 0, \tag{4}$$

where $\underline{\delta}, \bar{\delta}$ are positive constants chosen by designers.

We define a smooth and strictly increasing function $S(z)$ of the transformed error $z \in R$,

$$-\underline{\delta} < S(z) < \bar{\delta}, \forall z, \in L_\infty \tag{5a}$$

$$\operatorname{Lim}_{z_1 \to +\infty} (S(z)) = \bar{\delta};\ \lim_{z_1 \to -\infty} (S(z)) = -\underline{\delta} \tag{5b}$$

From the prosperities $S(z_1)$, the PPF condition can be rewritten as

$$x_1(t) = \varphi(t)\, S(z_1);\ z_1 = S^{-1}\left[\frac{x_1}{\varphi}\right]. \tag{6}$$

To facilitate the control design to stabilize z_1 in Eq. (6), we choose the function $S(z_1)$ as

$$S(z_1) = \frac{\bar{\delta}e^{z_1} - \underline{\delta}e^{-z_1}}{e^{z_1} + e^{-z_1}}. \tag{7}$$

Then the transformed error z_1 is derived as

$$z_1 = S^{1}\left[\frac{x_1(t)}{\varphi(t)}\right] = \frac{1}{2}\ln\frac{\mu(t) + \underline{\delta}}{\bar{\delta} - \mu(t)}, \tag{8}$$

where $\mu(t) = x_1(t)/\varphi(t)$, consequently, the derivative of the transformed error dynamic as

$$\dot{z}_1 = \frac{\partial S^{-1}}{\partial \mu}\dot{\mu} = \frac{1}{2}\left[\frac{1}{\mu + \underline{\delta}} - \frac{1}{\mu - \bar{\delta}}\right]\left(\frac{\dot{x}_1}{\varphi} - \frac{x_1\dot{\varphi}}{\varphi^2}\right)$$

$$= \tau\left(x_2 - \frac{x_1\dot{\varphi}}{\varphi}\right), \tag{9}$$

where

$$\tau = \frac{1}{2\varphi}\left[\frac{1}{\mu+\underline{\delta}} - \frac{1}{\mu-\bar{\delta}}\right]. \quad (10)$$

Note that, τ can be calculated based on x_1, φ and fulfils $0 \le \tau \le \tau_m$ for constants $\tau_m > 0$ as long as x_1 is bounded,

Furthermore, we can obtain from (4) and (10) that

$$\ddot{z}_1 = \dot{\tau}\left(x_1 - \frac{x_1\dot{\varphi}}{\varphi}\right) + \tau\left(\dot{x}_2 - \frac{x_2\dot{\varphi}}{\varphi} - \frac{x_1\ddot{\varphi}}{\varphi} + \frac{x_1\dot{\varphi}^2}{\varphi^2}\right)$$
$$= \dot{\tau}\left(x_1 - \frac{x_1\dot{\varphi}}{\varphi}\right) - \tau\left(\dot{x}_2 + \frac{x_2\dot{\varphi}}{\varphi} + \frac{x_1\ddot{\varphi}}{\varphi} - \frac{x_1\dot{\varphi}^2}{\varphi^2}\right) + \tau\left(\rho F_{MRE} - Y\psi - \ddot{y}_g\right). \quad (11)$$

The PPF condition of x_1 can be guaranteed as long as z_1 can be controlled to be bounded by means of proposed control u. The following sliding surface equation is defined in terms of z_1 as,

$$S = \Lambda z_1 + \dot{z}_1, \quad (12)$$

where $\Lambda > 0$ is a positive constant.

Directly differentiating S(t) in Eq. (12) and considering Eqs. (9) and (11), it is yield that

$$\dot{S} = \Lambda\dot{z}_1 + \ddot{z}_1$$
$$= \Lambda\tau\left(x_2 - \frac{x_1\dot{\varphi}}{\varphi}\right) + \dot{\tau}\left(x_2 - \frac{x_1\dot{\varphi}}{\varphi}\right) - \tau\left(\frac{x_2\dot{\varphi}}{\varphi} + \frac{x_1\ddot{\varphi}}{\varphi} - \frac{x_1\dot{\varphi}^2}{\varphi^2}\right) + \tau\left(\rho F_{MRE} - Y\psi - \ddot{y}_g\right)$$
$$= P + \tau\rho u - \tau Y\psi - \tau\ddot{y}_g, \quad (13)$$

where

$$P = \Lambda\tau\left(x_2 - \frac{x_1\dot{\varphi}}{\varphi}\right) + \dot{\tau}\left(x_2 - \frac{x_1\dot{\varphi}}{\varphi}\right) - \tau\left(\frac{x_2\dot{\varphi}}{\varphi} + \frac{x_1\ddot{\varphi}}{\varphi} - \frac{x_1\dot{\varphi}^2}{\varphi^2}\right).$$

In the process of updating the estimated parameter vectors $\hat{Y}$, updating the error model provides better error control effect. Regarding this, the following serial–parallel estimation model is adopted in this paper,

$$\dot{\hat{S}} = P + \tau\hat{\rho}u - \tau\hat{Y}\psi - \tau\ddot{y}_g + \beta\tilde{S}, \quad (14)$$

where $\hat{S}$ is the state of the serial-parallel estimation model, $\beta > 0$ is a gain constant, and $\tilde{S} = S - \hat{S}$ is the prediction error $\tilde{S}$,

$$\dot{\tilde{S}} = \dot{S} - \dot{\hat{S}} = \tau\tilde{\rho}u - \tau\tilde{Y}\psi - \beta\tilde{S}. \quad (15)$$

Then, the proposed controller is designed as follows,

$$u = \frac{1}{\tau\hat{\rho}}\left[-k_1 S - P + \tau\hat{Y}\psi + \tau\ddot{y}_g\right]. \quad (16)$$

The update algorithms are proposed as

$$\dot{\hat{Y}}^T = S\tau\psi - \tilde{S}\tau\psi \quad (17a)$$

$$\dot{\hat{\rho}} = S\tau F + \tilde{S}\tau u, \quad (17b)$$

Applying the Schwartz inequality,

$$-\sigma_1\tilde{Y}\hat{Y}^T \le -\frac{\sigma_1\|\tilde{Y}\|^2}{2} + \frac{\sigma_1\|Y\|^2}{2} \quad (18a)$$

$$-\sigma_2\tilde{\rho}\hat{\rho} \le -\frac{\sigma_2\|\tilde{\rho}\|^2}{2} + \frac{\sigma_2\|\rho\|^2}{2}. \quad (18b)$$

Theorem: Consider the dynamic system (2), taking the controller in (16) and adaptive law (17) into account, if the initial condition $-\underline{\delta}\varphi(0) < x_1(0) < \bar{\delta}\varphi(0)$ is satisfied. All parameters are bounded and errors are within allowable limits $S \to \Omega$.

Proof: Consider the Lyapunov function candidate as,

$$V = \tfrac{1}{2}S^2 + \tfrac{1}{2}\tilde{Y}^T\tilde{Y} + \tfrac{1}{2}\tilde{\rho}^2 + \tfrac{1}{2}\tilde{S}^2 \quad (19)$$

The time derivative of V_1 can be written,

$$\dot{V} = S\dot{S} + \tilde{Y}^T\dot{\tilde{Y}} + \tilde{\rho}\dot{\tilde{\rho}} + \tilde{S}\dot{\tilde{S}}$$
$$= S\left(P + \tau(\hat{\rho} - \tilde{\rho})F - \tau\ddot{y}_g\right) + \tilde{Y}^T\dot{\tilde{Y}} + \tilde{\rho}\dot{\tilde{\rho}} + \tilde{S}\dot{\tilde{S}}$$
$$= S\left(P + \tau\hat{\rho}\left(\frac{1}{\tau\hat{\rho}}(-k_1 S_2 - P + \tau\hat{Y}\psi + \tau\ddot{y}_g)\right) + \tau\tilde{\rho}F - \tau Y\psi - \tau\ddot{y}_g\right) - \tilde{Y}^T\dot{\hat{Y}} - \tilde{\rho}\dot{\hat{\rho}} + \tilde{S}\dot{\tilde{S}}$$
$$= -k_1 S^2 + S\left(\tau\hat{Y}\psi - \tau Y\psi\right) - \tilde{Y}^T\dot{\hat{Y}} + S_1\tau F\tilde{\rho} - \tilde{\rho}\dot{\hat{\rho}} + \tilde{S}(\tau\tilde{\rho}u - \tau\tilde{Y}\psi - \beta\tilde{S})$$
$$= -k_1 S^2 - \beta\tilde{S}^2 + \tilde{Y}S\tau\psi - \tilde{S}\tau\tilde{Y}\psi - \tilde{Y}\dot{\hat{Y}}^T + S\tau F\tilde{\rho} + \tilde{S}\tau\tilde{\rho}u - \tilde{\rho}\dot{\hat{\rho}}$$
$$= -k_1 S^2 - \beta\tilde{S}^2 + \tilde{Y}\left(S\tau\psi - \tilde{S}\tau\psi - \dot{\hat{Y}}^T\right) + \tilde{\rho}\left(S\tau F + \tilde{S}\tau u - \dot{\hat{\rho}}\right). \quad (20)$$

Eq. (19) can be rewritten,

$$\dot{V} = -k_1 S^2 - \beta\tilde{S}^2 - \frac{\sigma_1\|\tilde{Y}\|^2}{2} - \frac{\sigma_2\|\tilde{\rho}\|^2}{2} + \frac{\sigma_1\|Y\|^2}{2} + \frac{\sigma_2\|\rho\|^2}{2} \quad (21)$$

Further, the following inequality hold

$$\dot{V} \le -C_1 V + C_2, \quad (22)$$

where $C_1 = \min\{k_1, \beta, \sigma_1/2, \sigma_2/2\}$ and $C_2 = \sigma_1\|Y\|^2/2 + \sigma_2\|\rho\|^2/2$.

By integrating of Eq. (18) over $[0, t]$, it yields as

$$V(t) \le (V(0) - C_1/C_2)e^{-Ct} + C_2/C_1$$
$$\le V(0) + C_2/C_1. \quad (23)$$

According to (18), V_1 is exponential convergence, i.e., S is exponential convergence. The sliding mode surface S will converge to the following compact set $S \to \Omega$, $\Omega = \sqrt{2V(0)e^{-A_1 t} + 2\,C_2/C_1}$.

3. Simulation

The effectiveness of the proposed control algorithm was evaluated through simulations conducted on a single-degree-of-freedom integral isolation system. System parameters of the SDOF include $m = 30$, $k = 20000\ N/m$, $c = 200\ Ns/m$, which correspond to a one-story steel frame building with a ratio of 1:10 in height. The excitation acceleration was derived from the El Centro earthquake, as shown in Figure 2. The simulation results were compared with those obtained using a traditional sliding control algorithm and passive isolation system. Figure 3 shows a significant reduction in mass response clearly when the proposed control algorithm was applied.

This outcome demonstrates the superiority of the proposed approach over the traditional methods, highlighting the effectiveness of the active control technique in minimizing vibrations and enhancing system stability. Table 1 shows the root mean square (RMS) and maximum values of the mass response, which decrease significantly when using the controller. The proposed control method is improved compared to sliding mode control. Compared with the passive isolation method, the proposed controller achieves a reduction ratio of 40% for the RMS value and 38% for the maximum displacement value.

Table 1. *Displacement values of response to earthquake excitation*

	RMS values [mm]	Maximum values [mm]
Passive-off	2.2 (1)	9 (1)
Sliding mode control	1.4 (0.63)	7 (0.77)
Proposed control	0.9 (0.4)	3.5 (0.38)

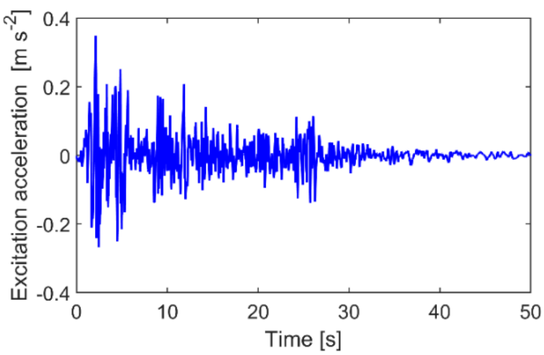

Figure 2. *The excitation acceleration ($\ddot{y}_g$)of El Centro earthquake is used for simulation*

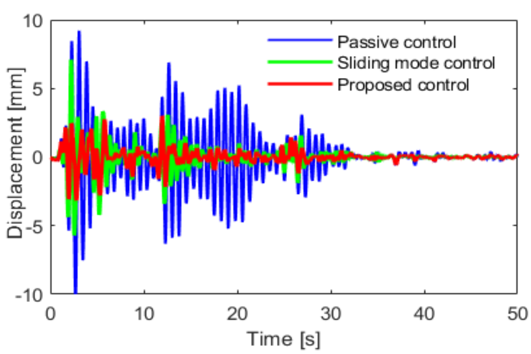

Figure 3. *The displacement responses elicited when different algorithms are used under earthquake excitation*

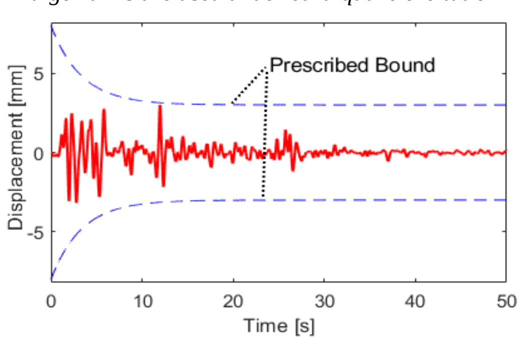

Figure 4. *The displacement response of SDOF using proposed controller*

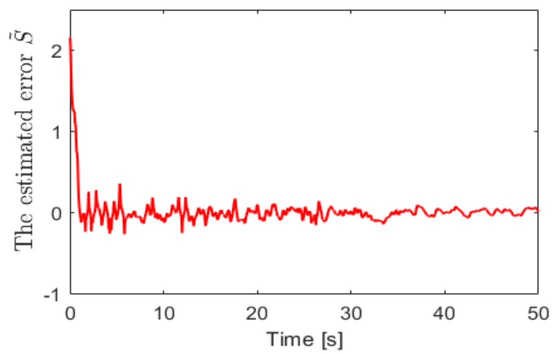

Figure 5. *The estimated error of $\tilde{S}$ using proposed controller*

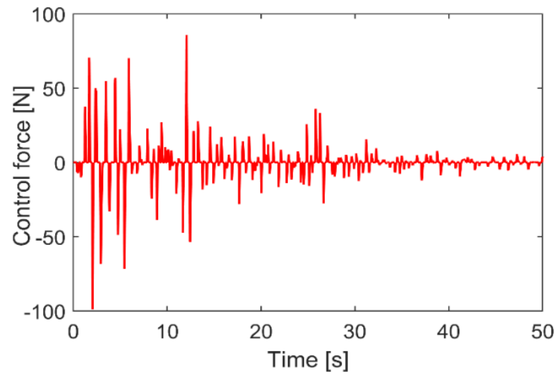

Figure 6. *The force response of the proposed controller*

In addition, further analysis reveals that the displacement achieved by implementing the proposed controller in this research that always remains within the predefined boundary, as illustrated in Figure 4. This indicates that the proposed controller effectively regulates and limits the displacement within the desired range. Additionally, Figure 5 portrays the outcomes obtained from monitoring the error function, indicating how accurately the system responds to discrepancies between the desired output and the actual output. Adopting the serial-parallel estimation model for the parameter vector and error model has yielded significant benefits in system analysis and performance evaluation. The model could accurately estimate the system's state, which is crucial for achieving effective control. Moreover, the control force, as depicted in Figure 6, represents the applied force that the controller exerts to modulate the system's behavior.

4. Conclusions

In conclusion, this study presents the application of PPB control for the isolation control of a SDOF system. The proposed control scheme is evaluated through simulation studies comparing it to the sliding control and passive control methods. The simulation results demonstrate that the proposed controller is effective in isolating the system under earthquake excitation. The proposed control method is improved compared to sliding mode control. Compared with the passive isolation method, the proposed controller achieves a reduction ratio of 40% for the RMS value and 38% for the maximum displacement value. Adopting the serial-parallel estimation model for the parameter vector and error model in this study has proven effective in estimation. Using the serial-parallel estimation model has enhanced the understanding and characterization of the system, leading to improved performance and better control strategies. The proposed control ensures asymptotic tracking stability and maintains errors within acceptable limits. In comparison to traditional control methods, the use of PPB based controller offers significant advantages in terms of sustaining the system's performance. With its ability to provide effective vibration isolation, the proposed design holds promise for enhancing the overall sustainability of such systems.

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