EXISTENCE OF SOLUTIONS FOR THE SYSTEM OF VECTOR QUASIEQUILIBRIUM PROBLEMS

SỰ TỒN TẠI NGHIỆM CHO HỆ BÀI TOÁN TỰA CÂN BẰNG VÉC TƠ

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Abstract - In this paper, we study the systems of generalized multivalued strong vector quasiequilibrium problem (in short (SQVEP)) real locally convex Hausdorff topological vector spaces. This problem includes as special cases the generalized strong vector quasi-equilibrium problems, quasi-equilibrium problems, equilibrium problems and variational inequality problems. Then, we establish an existence theorem for its solutions by using fixed-point theorem. Moreover, we also discuss the closedness of the solution set for (SQVEP). The results presented in the paper improve and extend the main results of Long et al [Math. Comput. Model,47 445--451 (2008)] and Plubtieng - Sitthithakerngkietet [Fixed Point Theory Appl. doi:10.1155/2011/475121 (2011)].

Key words - system vector quasiequilibrium problems; multivalued; fixed-point theorem; existence; closedness.

Tóm tắt - Trong bài báo này, chúng tôi nghiên cứu hệ bài toán tựa cân bằng véc tơ đa trị mạnh tổng quát (viết tắt, (SQVEP)) trong các không gian véc tơ tô pô Hausdorff thực lồi địa phương. Bài toán này chứa rất nhiều bài toán đặc biệt như bài toán tựa cân bằng véc tơ mạnh tổng quát, bài toán tựa cân bằng, bài toán cân bằng và bài toán bất đẳng thức biến phân. Sau đó, chúng tôi thiết lập một định lý cho sự tồn tại nghiệm của nó bởi sử dụng định lý điểm bất động. Ngoài ra, chúng tôi cũng thảo luận tính đóng của tập nghiệm cho (SQVEP). Kết quả hiện tại trong bài báo là cải thiện và mở rộng các kết quả chính của Long cùng các tác giả [Math. Comput. Model,47 445--451 (2008)] và Plubtieng - Sitthithakerngkietet Sitthithakerngkietet [Fixed Point Theory Appl. doi:10.1155/2011/475121 (2011)].

Từ khóa - hệ bài toán tựa cân bằng véctơ; đa trị; định lý điểm bất động; sự tồn tại; tính đóng.

1. Introduction and Preliminaries

Let X, Y, Z be real locally convex Hausdorff topological vector spaces $A\subseteq X$ and $B\subseteq Y$ be two nonempty compact convex subsets and $C\subset Z$ be a solid pointed closed convex cone. Let $K_i, P_i: A\times A\to 2^A$, $T_i: A\times A\to 2^B$ and $F_i: A\times B\times A\to 2^Z$, $i=1,2$ be multifunctions. We consider the following system generalized strong vector quasiequilibrium problems (in short, (SQVEP):

(SQVEP): Find $(\bar{x},\bar{u})\in A\times A$ and $\bar{z}\in T_1(\bar{x},\bar{u})$, $\bar{v}\in T_2(\bar{x},\bar{u})$ such that $\bar{x}\in K_1(\bar{x},\bar{u}), \bar{u}\in K_2(\bar{x},\bar{u})$ satisfying

$$F_1(\bar{x},\bar{z},y)\subset C, \forall y\in P_1(\bar{x},\bar{u}),$$
$$F_2(\bar{u},\bar{v},y)\subset C, \forall y\in P_2(\bar{x},\bar{u}).$$

We denote that $\Sigma(F)$ is the solution set of (SQVEP).

Next, we recall some basic definitions and their some properties.

Definition 1.1 *(See [1, 3])* Let X, Z be two topological vector spaces, A be a nonempty subset of X and $F: A\to 2^Z$ be a multifunction.

(i) F is said to be *lower semicontinuous (lsc) at* x_0 if $F(x_0)\cap U\neq\emptyset$ for some open set $U\subseteq Z$ implies the existence of a neighborhood N of x_0 such that, for all $x\in N, F(x)\cap U\neq\emptyset$. said to be lower semicontinuous in A if it is lower semicontinuous at each $x_0\in A$.

(ii) F is said to be *upper semicontinuous (usc) at* x_0 if for each open set $U\supseteq F(x_0)$, there is a neighborhood N of x_0 such that $U\supseteq F(N)$. F is said to be upper semicontinuous in A if it is upper semicontinuous at each $x_0\in A$.

(iii) F is said to be *continuous at* x_0 if it is both lsc and usc at x_0. F is said to be continuous *at* $x_0\in A$ if it is continuous at each $x_0\in A$.

(iv) F is said to be *closed* if $\text{Graph}(F)=\{(x,y): x\in A, y\in F(x)\}$ is a closed subset in $A\times Y$. F is said to be closed in A if it is closed at each $x_0\in A$.

Definition 1.2 *(See [1, 2])* Let X, Z be two topological vector spaces, A be a nonempty subset of X and $F: A\to 2^Z$ be a multifunction and $C\subset Z$ is a nonempty closed convex cone.

(i) F is called *upper* C *-continuous at* $x_0\in A$, if for any neighborhood U of the origin in Z, there is a neighbourhood V of x_0 such that

$$F(x)\subset F(x_0)+U+C, \forall x\in V.$$

(ii) F is called *lower* C *-continuous at* $x_0\in A$, if for any neighborhood U of the origin in Z, there is a neighborhood V of x_0 such that

$$F(x_0)\subset F(x)+U-C, \forall x\in V.$$

Definition 1.3 *(See [3])* Let X and Z be two topological vector spaces and A is a nonempty convex subset of X. A set-valued mapping $F: A\to 2^Z$ is said to *be properly* C *-quasiconvex* if for any $x,y\in A$ and $t\in[0,1]$, we have

either $F(x)\subset F(tx+(1-t)y)+C$

or $F(y)\subset F(tx+(1-t)y)+C$.

Lemma 1.1 (See [3]) Let X, Z be two topological vector spaces, A be a nonempty convex subset of X and $F: A\to 2^Z$ be a multifunction.

(i) If F is upper semicontinuous at $x_0 \in A$ with closed values, then F is closed at $x_0 \in A$;

(ii) If F is closed at $x_0 \in A$ and Z is compact, then F is upper semicontinuous at $x_0 \in A$.

(iii) If F has compact values, then F is usc at x_0 if and only if, for each net $\{x_\alpha\} \subseteq A$ which converges to x_0 and for each net $\{y_\alpha\} \subseteq F(x_\alpha)$, there are $y \in F(x_0)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y.$

Lemma 1.1 (See [1,5]) Let A be a nonempty compact subset of a locally convex Hausdorff vector topological space Z. If $M: A \to 2^A$ is upper semicontinuous and for any $x \in A$, $M(x)$ is nonempty, convex and closed, then there exists an $x^* \in A$ such that $x^* \in M(x^*)$.

2. Main Results

Definition 2.1. Let X, Z be two real locally convex Hausdorff topological vector spaces and A be a nonempty compact convex subset of X, and $C \subset Z$ is a nonempty closed convex cone. Suppose $F: A \to 2^Z$ be a multifunction. F is said to be *strongly C-quasiconvex in A* if $\forall x_1, x_2 \in A, \forall \lambda \in [0,1]$, $F(x_1) \subset C$ and $F(x_2) \subset C$. Then, it follows that

$$F(\lambda x_1 + (1-\lambda)x_2) \subset C.$$

Remark 2.1 In the Definition 2.1, if we let $X = A = Z = \mathbb{R},\ C = R_-$, and let $F: \mathbb{R} \to \mathbb{R}$ be a single-valued mapping. Then, we have • $\forall x_1, x_2 \in A, \forall t \in [0,1]$, if $F(x_1) \le 0$, $F(x_2) \le 0$, then $F((1-t)x_1 + tx_2)) \le 0$. This means that F is modified 0-level quasiconvex, since the classical quasiconvexity says that $\forall x_1, x_2 \in A, \forall \lambda \in [0,1]$,

$$F((1-\lambda)x_1 + \lambda x_2)) \le \max\{F(x_1), F(x_2)\}.$$

Now, we let X and Z be two topological vector spaces, A be a nonempty convex subset of X $F: A \to 2^Z, \theta \in Z$ and $C \subset Z$ be a solid pointed closed convex cone, we will use the following level-sets types:

$$L_{\ge\theta}F := \{x \in A : F(x) \subset \theta + C\};$$
$$L_{\ngeq\theta}F := \{x \in A : F(x) \not\subset \theta + C\}.$$

Theorem 2.1 *For each $\{i = 1,2\}$, assume for the problem (SQVEP) that*

(i) K_i is upper semicontinuous, $K_i(x,u)$ is nonempty closed convex, for all $(x,u) \in A \times A$ and P_i is lower semicontinuous and nonempty;

(ii) T_i is upper semicontinuous and compact-valued and $T_i(x,u)$ is nonempty convex, for all $(x,u) \in A \times A$;

(iii) for all $(x,z) \in A \times B$, $F_i(x, z, P_i(x,u)) \subset C$;

(iv) for all $(z,y) \in B \times A$, $F_i(.,z,y)$ is strongly C-quasiconvex;

(v) for all $z \in B$, $L_{\ge 0}F_i(.,z,.)$ is closed.

Then $\Sigma(F) \neq \emptyset$. Moreover $\Sigma(F)$ is closed.

Proof. For all $(x,z,u,v) \in A \times B \times A \times B$, define be set-valued mappings: $\Psi, \Pi : A \times B \times A \to 2^A$ by

$\Psi(x,z,u) = \{a \in K_1(x,u) : F(a,z,y) \subset C, \forall y \in P_1(x,u)\}$, and

$\Pi(x,v,u) = \{b \in K_2(x,u) : F(b,v,y) \subset C, \forall y \in P_2(x,u)\}.$

Step 1. Show that $\Psi(x,z,u)$ and $\Pi(x,v,u)$ are nonempty.

Indeed, for all $(x,z,u) \in A \times B \times A$ and $(x,v,u) \in A \times B \times A$, for each $\{i = 1,2\}$, $K_i(x,u), P_i(x,u)$ are nonempty. Thus, by assumption (iii), we have $\Psi(x,z,u)$ and $\Pi(x,v,u)$ are nonempty.

Step 2. Show that $\Psi(x,z,u)$ and $\Pi(x,v,u)$ are convex subsets of A.

Let $a_1, a_2 \in \Psi(x,z,u)$ and $\alpha \in [0,1]$ and put $a = \alpha a_1 + (1-\alpha)a_2$. Since $a_1, a_2 \in K_1(x,u)$ and $K_1(x,u)$ is a convex set, we have $a \in K_1(x,u)$. Thus, for $a_1, a_2 \in \Psi(x,z,u)$, we have

$$F_1(a_1, z, y) \subset C, \forall y \in P_1(x,u),$$
$$F_1(a_2, z, y) \subset C, \forall y \in P_1(x,u).$$

By (iv), F_1(., z, y) is strongly C-quasiconvex. $F_1(\alpha a_1 + (1-\alpha)a_2, z, y) \subset C, \forall \alpha \in [0,1],$

i.e., $a \in \Psi(x,z,u)$. Therefore, $\Psi(x,z,u)$ is a convex subset of A. Similarly, we have $\Pi(x,v,u)$ is a convex subset of A.

Step 3. $\Psi(x,z,u)$ and $\Pi(x,v,u)$ are closed subsets of A.

Let $\{a_n\} \subseteq \Psi(x,z,u)$ with $a_n \to a_0$. Then, $a_n \in K_1(x,u)$. Since $K_1(x,u)$ is a closed subset of A, it follow that $a_0 \in K_1(x,u)$. By the lower semicontinuity of P_1, we have $\forall y_0 \in P_1(x,u)$ and any net $\{(x_n, u_n)\} \to (x,u)$, there exists a net $\{y_n\}$ such that $y_n \in P_1(x_n, u_n)$ and $y_n \to y_0$. As $a_n \in \Psi(x,z,u)$, we have

$$F_1(a_n, z, y_n) \subset C. \quad (2.1)$$

By assumption (v), (2.1) yields that

$$F_1(a_0, z, y_0) \subset C,$$

i.e., $a_0 \in \Psi(x,z,u)$. Therefore, $\Psi(x,z,u)$ is closed. Similarly, we also have $\Pi(x,v,u)$ is closed.

Step 4. Now, we need to show that $\Psi(x,z,u)$ and $\Pi(x,v,u)$ are upper semicontinuous.

First, we show that $\Psi(x,z,u)$ is upper semicontinuous. Indeed, since A is a compact set and $\Psi(x,z,u) \subset A$. Hence $\Psi(x,z,u)$ is compact. Now we need to show that Ψ is a closed mapping. Indeed, Let a net $\{(x_n, z_n, u_n) : n \in I\} \subset A \times B \times A$ such that $(x_n, z_n, u_n) \to (x,z,u) \in A \times B \times A$, and let $a_n \in \Psi(x_n, z_n, u_n)$ such that $a_n \to a_0$.

Now we need to show that $a_0 \in \Psi(x,z,u)$. Since $a_n \in K_1(x_n,u_n)$ and K_1 is upper semicontinuous, we have $a_0 \in K_1(x,u)$. Suppose to the contrary $a_0 \notin \Psi(x,z,u)$. Then, $\exists y_0 \in P_1(x,u)$ such that

$$F_1(a_0,z,y_0) \not\subset C. \quad (2.2)$$

By the lower semicontinuity of P_1, there is a net $y_n \in P_1(x_n,u_n)$ such that $y_n \to y_0$. Since $a_n \in \Psi(x_n,z_n,u_n)$, we have

$$F_1(a_n,z_n,y_n) \subset C. \quad (2.3)$$

By the condition (v) we have

$$F_1(a_0,z,y_0) \subset C. \quad (2.4)$$

This is the contradiction between (2.2) and (2.4). Thus, $a_0 \in \Psi(x,z,u)$. Hence, Ψ is upper semicontinuous. Similarly, we also have $\Pi(x,v,u)$ is upper semicontinuous.

Step 5 Now we need to the solutions set $\Sigma(F) \neq \emptyset$.

Define the set-valued mappings $\Theta, \Xi : A\times B\times A: \to 2^{A\times B}$ by

$$\Theta(x,z,u) = (\Psi(x,z,u), T_1(x,u)), \forall (x,z,u) \in A\times B\times A$$

and

$$\Xi(x,v,u) = (\Pi(x,v,u), T_2(x,u)), \forall (x,v,u) \in A\times B\times A.$$

Then Θ, Ξ are upper semicontinuous and $\forall (x,z,u) \in A\times B\times A$, $\forall (x,v,u) \in A\times B\times A$,

$\Theta(x,z,u)$ and $\Theta(x,v,u)$ be two nonempty closed convex subsets of $A\times B\times A$.

Define the set-valued mapping $H:(A\times B)\times(A\times B) \to 2^{(A\times B)\times(A\times B)}$ by

$H((x,z),(u,v)) = (\Theta(x,z,u), \Xi(x,v,u))$, for all $((x,z),(u,v)) \in (A\times B)\times(A\times B)$.

Then H is also upper semicontinuous and $\forall ((x,z),(u,v)) \in (A\times B)\times(A\times B)$, $H((x,z),(u,v))$ is a nonempty closed convex subset of $(A\times B)\times(A\times B)$.

By Lemma 1.1 there exists a point $((\hat{x},\hat{z})(\hat{u},\hat{v})) \in (A\times B)\times(A\times B)$ such that $((\hat{x},\hat{z})(\hat{u},\hat{v})) \in H((\hat{x},\hat{z})(\hat{u},\hat{v}))$, that is

$$(\hat{x},\hat{u}) \in \Theta(\hat{x},\hat{z},\hat{u}),\ (\hat{u},\hat{v}) \in \Xi(\hat{x},\hat{u},\hat{v})$$

which implies that $\hat{x} \in \Psi(\hat{x},\hat{z},\hat{u}), \hat{u} \in T_1(\hat{x},\hat{u})$ and $\hat{u} \in \Pi(\hat{x},\hat{u},\hat{v}), \hat{v} \in T_2(\hat{x},\hat{u})$. Hence, there exist $(\hat{x},\hat{u}) \in A\times A, \hat{z} \in T_1(\hat{x},\hat{u}), \hat{v} \in T_2(\hat{x},\hat{u})$ such that $\hat{x} \in K_1(\hat{x},\hat{u}), \hat{u} \in K_2(\hat{x},\hat{u})$ satisfying

$$F_1(\hat{x},\hat{z},y) \subset C, \forall y \in P_1(\hat{x},\hat{u}), \text{ and}$$

$$F_2(\hat{u},\hat{v},y) \subset C, \forall y \in P_2(\hat{x},\hat{u}),$$

i.e., (SQVEP) has a solution.

Step 6. Now we prove that $\Sigma(F)$ is closed.

Indeed, let a net $\{(x_n,u_n), n\in I\} \in \Sigma(F)$: $(x_n,u_n) \to (x_0,u_0)$. As $(x_n,u_n) \in \Sigma(F)$, there exist $z_n \in T_1(x_n,u_n), v_n \in T_2(x_n,u_n)$,

$x_n \in K_1(x_n,u_n), u_n \in K_2(x_n,u_n)$ such that

$$F_1(x_n,z_n,y) \subset C, \forall y \in P_1(x_n,u_n). \text{ and}$$

$$F_2(u_n,v_n,y) \subset C, \forall y \in P_2(x_n,u_n).$$

Since K_1, K_2 are upper semicontinuous and closed-valued. Thus, K_1, K_2 are closed. Hence, $x_0 \in K_1(x_0,u_0), u_0 \in K_2(x_0,u_0)$. Since T_1, T_2 are upper semicontinuous and $T_1(x_0,u_0), T_2(x_0,u_0)$ are compact. There exist $z \in T_1(x_0,u_0)$ and $v \in T_2(x_0,u_0)$ such that $z_n \to z, v_n \to v$ (taking subnets if necessary), we have $z \in T(x_0)$ such that $z_n \to z$. By the condition (v), we have

$$F_1(x_0,z,y) \subset C, \forall y \in P_1(x_0,u_0), \text{ and}$$

$$F_2(u_0,v,y) \subset C, \forall y \in P_2(x_0,u_0).$$

This means that $(x_0,u_0) \in \Sigma(F)$. Thus $\Sigma(F)$ is a closed set. □

Remark 2.2

(a) If $K_1(\bar{x},\bar{u}) = P_1(\bar{x},\bar{u}) = S_1(\bar{x})$, $K_2(\bar{x},\bar{u}) = P_2(\bar{x},\bar{u}) = S_2(\bar{x})$, $T_1(\bar{x},\bar{u}) = T_1(\bar{x})$, $T_2(\bar{x},\bar{u}) = T_2(\bar{x})$, then (SQVEP) become to (SGSVQEPs) in [4].

(b) If $K_1(\bar{x},\bar{u}) = P_1(\bar{x},\bar{u}) = K_2(\bar{x},\bar{u}) = P_2(\bar{x},\bar{u}) = S(\bar{x})$, $T_1(\bar{x},\bar{u}) = T_2(\bar{x},\bar{u}) = T(\bar{x})$, then (SQVEP) become to (SGSVQEP) in [2].

Remark 2.3 In this special case as Remark 2.2, Theorem 2.1 reduces to Theorem 3.1 in [4] and Theorem 3.1 in [2]. However, our Theorem 2.1 is stronger than Theorem 3.1 in [4] and Theorem 3.1 in [2]. Moreover, we omit the assumption F is lower (-C)-continuous in Theorem 3.1 in [4] and Theorem 3.1 in [2].

The following example shows that in this the special case, all the assumptions of Theorem 2.1 may be satisfied, but Theorem 3.1 in [4] and Theorem 3.1 in [2] are not fulfilled.

Example 2.1 Let $X = Y = Z = R$, $A = B = [-1,1]$, $C = [0,+\infty)$ and let $K_1(x) = K_2(x) = [0,1]$, $F:[-1,1] \to 2^R$

and $$T_1(x,u) = T_2(x,u) = \begin{cases} [0,\frac{3}{2}] & \text{if } x_0 = u_0 = \frac{1}{3}, \\ [0,\frac{1}{2}] & \textit{otherwise.} \end{cases}$$

and $F_1(x,z,y) = F_2(u,v,y) =$

$$F(x) = \begin{cases} [\frac{1}{3},\frac{1}{2}] & \text{if } x_0 = \frac{1}{3}, \\ [\frac{1}{2},1] & \textit{otherwise.} \end{cases}$$

We show that all assumptions of Theorem 2.1 are satisfied. So by this Theorem the considered problem has solutions. However, F is not lower $(-C)$ -continuous at $x_0=\frac{1}{3}$. Indeed, we let a neighborhood $U=[-\frac{1}{8},\frac{1}{8}]$ of the origin in Z, then for any neighborhood $V=[\frac{1}{3}-\varepsilon,\frac{1}{3}+\varepsilon]$ of $x_0=\frac{1}{3}$, where $\varepsilon>0,$ choose $\frac{1}{3}\neq x^*\in V$. We have,

$$F(x_0)=F(\frac{1}{3})=[\frac{1}{3},\frac{1}{2}]\not\subset F(x^*)+U+C$$
$$=[\frac{1}{2},1]+[-\frac{1}{8},\frac{1}{8}]+\mathbb{R}_+$$
$$=[\frac{3}{8},\frac{9}{8}]+\mathbb{R}_+,$$

Also, Theorem 3.1 in [4] and Theorem 3.1 in [2] does not work.

The following example shows that the assumption (v) of Theorem 2.1 is strictly weaker than the assumption upper C-continuous in [2,4]

Example 2.2 Let $X=Y=Z=R$,
$A=B=[-1,1],C=[0,+\infty)$ and let
$K_1(x)=K_2(x)=[0,1],T_1(x,u)=T_2(x,u)=\{1\}$,
$F:[-1,1]\to 2^R$ and
$F_1(x,z,y)=F_2(u,v,y)=$

$$F(x)=\begin{cases}[1,\frac{3}{2}] & \text{if } x_0=\frac{1}{3},\\ [\frac{1}{3},\frac{1}{2}] & \textit{otherwise.}\end{cases}$$

We show that all assumptions of Theorem 2.1 are satisfied. So (SQVEP) has solution. However, F is not upper C -continuous at $x_0=\frac{1}{3}$. Indeed, we let a neighborhood $U=[-\frac{1}{3},\frac{1}{3}]$ of the origin in Z, then for any neighborhood $V=[\frac{1}{3}-\varepsilon,\frac{1}{3}+\varepsilon]$ of $x_0=\frac{1}{3}$, where $\varepsilon>0,$ choose $\frac{1}{3}\neq x^*\in V$. We see that,

$$F(x^*)=[\frac{1}{3},\frac{1}{2}]\not\subset F(x_0)+U+C$$
$$=[1,\frac{3}{2}]+[-\frac{1}{3},\frac{1}{3}]+\mathbb{R}_+$$
$$=[\frac{2}{3},\frac{11}{6}]+\mathbb{R}_+,$$

Also, Theorem 3.1 in [4] and Theorem 3.1 in [2] does not work.

The following example shows that our strongly C -quasiconvexity is strictly weaker than the C-quasiconvexity in [2,4].

Example 2.3 Let $X=Y=Z=R$,
$A=B=[0,1],C=[0,+\infty)$ and let
$K_1(x)=K_2(x)=[0,1],T_1(x,u)=T_2(x,u)=[1,2],$
$F:[0,1]\to 2^R$
and $F_1(x,z,y)=F_2(u,v,y)=$

$$F(x)=\begin{cases}[\frac{3}{2},2] & \text{if } x_0=\frac{1}{2},\\ [\frac{1}{3},\frac{1}{2}] & \textit{otherwise.}\end{cases}$$

We show that all assumptions of Theorem 2.1 are satisfied. However, F is not properly C -quasiconvex. Indeed, we let $\lambda=\frac{1}{2}$ and $x_1=0,x_2=1$. Then,

$$F(x_1)=F(0)=[\frac{1}{3},\frac{1}{2}]\not\subset F(x_1\lambda+(1-\lambda)x_2)+C$$
$$=F(\frac{1}{2})+\mathbb{R}_+=[\frac{3}{2},2]+\mathbb{R}_+,$$
$$F(x_2)=F(1)=[\frac{1}{3},\frac{1}{2}]\not\subset F(x_1\lambda+(1-\lambda)x_2)+C$$
$$=F(\frac{1}{2})+\mathbb{R}_+=[\frac{3}{2},2]+\mathbb{R}_+$$

Thus, it gives case where Theorem 2.1 can be applied but Theorem 3.1 in [4] and Theorem 3.1 in [2] does not work.

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REFERENCES

[1] Aubin J. P., Ekeland I, *Applied Nonlinear Analysis*, John Wiley and Sons, New York, 1984.

[2] Long X. J., Huang N. J., Teo K.L., Existence and stability of solutions for generalized strong vector quasi-equilibrium problems, *Math Computer Model.* 47, 445-451 (2008).

[3] Luc D. T, *Theory of Vector Optimization: Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag Berlin Heidelberg, 1989.

[4] Plubtieng S., Sitthithakerngkiet K, On the existence result for system of generalized strong vector quasi-equilibrium problems, *Fixed Point Theory Appl.* Article ID 475121, doi:10.1155/2011/475121 (2011).

[5] Yu J, Essential weak efficient solution in multiobjective optimization problems, *Nonlinear Anal. TMA.*, 70 (2009), 1528-1535.

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