

EXPLORING THE POTENTIAL OF SWARM INTELLIGENCE FOR OPTIMAL ENERGY EFFICIENCY IN IoT DOWNLINK SYSTEM

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(Received: May 04, 2024; Revised: June 14, 2024; Accepted: June 17, 2024)

Abstract - This study delves into the energy optimization problem in Internet of Things (IoT) networks. We consider the downlink from multiple antenna Gateway (GW) and single antenna IoT devices. For this challenging nonconvex problem, we initially introduced the well-known zero-forcing beamforming (ZFBF) to eliminate inter-user interference, thereby transforming the energy efficiency maximization problem into a concave-convex fractional problem. Then, instead of applying a combination of ZFBF with power allocation, we propose the Particle Swarm Optimization (PSO) algorithm to allocate power to find the optimized beamforming matrix. Through extensive numerical analysis, we demonstrate the effectiveness of our proposed scheme in terms of energy efficiency and power achieved at the GW. The results underscore the significant benefits of our approach over conventional methods, paving the way for practical and efficient energy optimization in IoT networks.

Key words - IoT; PSO; optimization; energy efficiency; Zero-Forcing; Beamforming

1. Introduction

The Internet of Things (IoT) is a fast-growing technology that connects devices, sensors, and applications to collect and exchange data automatically [1]. It has applications in various industries, including smart homes, cities, industrial automation, and healthcare [2, 3].

Since IoT devices are frequently connected and continuously transmitting data, this leads to high energy consumption demands, making energy efficiency one of the key challenges of IoT. Maintaining the operation of billions of IoT devices requires a stable and energy-efficient power source. Additionally, the battery life of these devices is also a concern, especially for those that are difficult to access for replacement or recharging. Developing low-power technologies and improving the energy efficiency of the devices are essential to minimizing the environmental impact and extending the device's lifespan. Furthermore, optimizing communication protocols and utilizing renewable energy sources can also help enhance the energy efficiency of IoT systems [4, 5].

The concept of energy efficiency addresses the challenge of high energy consumption in IoT systems [5]. Energy efficiency is defined as the ratio between the total data throughput and the total power consumption, which encompasses the power required for various stages of signal processing and transmission [6]. Our main goal is to send the most data (bits) for every unit of energy used (Joule). The problem of maximizing energy efficiency (EEmax) is like finding the perfect balance between how

much data we can send (sum rate) and the total amount of power we use. This balance gives us the best energy efficiency possible [7]. The tradeoff between power and throughput has been studied previously [8], but without accounting for the circuit power consumption. Authors in [8] considered a weighted sum of throughput and power to find the convex hull of the achievable region, by varying the weighting coefficients. The work [9] studied an energy loss optimization scheduling modeling method based on a multi-objective fuzzy algorithm approach. The energy efficiency maximization problem in multiple-input single-output (MISO) downlink channel, subject to the total power and user-specific SINR constraints, is considered in [10].

In recent years, the application of swarm intelligence (SI) techniques for optimization problems has gained significant traction. SI techniques mimic the collective behavior of natural organisms, such as the flocking of birds, the schooling of fish, or the foraging of bees, to search for optimal solutions to complex problems [11]. SI techniques have been widely applied across diverse domains, including Engineering, Finance, Computer Science, and Social Sciences [12].

Swarm intelligence (SI) is a subfield of artificial intelligence (AI) that takes inspiration from the collective behavior of social colonies, such as ant colonies, bird flocks, fish schools, and bee swarms [13]. The primary principle behind SI is that individual agents (or "particles") follow simple rules and interact locally with one another and their environment, leading to complex global behavior or optimization capabilities without centralized control. This decentralized approach allows flexibility, robustness, and scalability, making SI suitable for solving complex optimization and search problems in various domains [14, 15, 16].

A prominent algorithm within the field of swarm intelligence is Particle Swarm Optimization (PSO) [17]. PSO is a heuristic optimization algorithm inspired by the collective behavior of bird species [18]. In PSO, a set of individuals called particles move through a search space to optimize an objective function.

In this paper, we propose a solution using the PSO algorithm to optimize energy efficiency for downlink IoT systems with constraints on service quality and maximum transmit power. We assessed the proposed algorithm's effectiveness by comparing the performance of our IoT downlink system with established benchmarks like the

Zero-Forcing Beamforming (ZFBF) algorithm presented in [10]. Simulations were conducted, and the numerical results confirm that the proposed PSO-based algorithm outperforms existing approaches

The remainder of the paper is organized as follows. In Section II, we introduce the IoT downlink system model. Section III presents the Zero-Forcing (ZF) beamforming design. The proposed PSO-based algorithm is introduced in Section IV. Section V validates the effectiveness of the proposed algorithm through numerical simulations. Finally, conclusions are drawn in Section VI.

Notations: The following notations are used in this paper. Lower-case and upper-case boldface letters are used to denote vectors and matrices. $\mathbf{C}^{N \times M}$ represents the set of all $N \times M$ complex matrices and \mathbf{I}_M denotes an $M \times M$ identity matrix. $|\mathbf{X}|$, \mathbf{X}^H , and $\text{tr}(\mathbf{X})$ denote the determinant, Hermitian transpose, and trace of a matrix \mathbf{X} , respectively. $E\{\cdot\}$ and $\|\cdot\|$ are the expectation and norm operators, respectively. A complex Gaussian random vector variable \mathbf{z} with mean μ and variance σ^2 is represented as $\mathbf{z} \sim \mathcal{CN}(\mu, \sigma^2)$.

2. System model

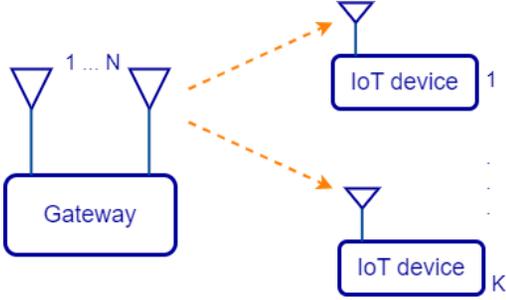


Figure 1. IoT downlink system model.

We consider an IoT system that includes the gateway (GW) equipped N_t transmitting antennas and K single antenna IoT devices in the download channels, as depicted in Figure II. In this system, we consider flat fading channels in the links between the GW and IoT devices to simplify the analysis. The received signal \mathbf{y}_k at the i -th devices can be expressed:

$$\mathbf{y}_k = \mathbf{h}_k \mathbf{w}_k \mathbf{x}_k + \sum_{i=1, i \neq k}^K \mathbf{h}_i \mathbf{w}_i \mathbf{x}_i + \mathbf{n}_k, \quad (1)$$

where \mathbf{h}_k is the channel matrix between the GW and the k th IoT device, \mathbf{x}_k is the transmitted signal for the k -th IoT device, \mathbf{w}_k is the linear precoder, and \mathbf{n}_k denotes the additive white Gaussian (AWGN) with distribution $\mathcal{CN}(0, N_0 \mathbf{I})$.

Let B be the bandwidth, the signal-to-interference-and-noise at the k -th IoT device is given by

$$\gamma_k = \frac{|\mathbf{h}_k \mathbf{w}_k|^2}{\sum_{i=1, i \neq k}^K |\mathbf{h}_i \mathbf{w}_i|^2 + BN_0}, \quad (2)$$

the data rate of the k -th IoT device can be expressed as

$$R_k = B \log(1 + \gamma_k). \quad (3)$$

For the sake of simplicity, we will omit the constant term B in the derivation of the algorithms presented in this paper. The first problem involves minimizing power while satisfying individual quality of service (QoS) constraints.

$$\begin{aligned} SP_{min} &= \min \sum_{k=1}^K |\mathbf{w}_k|^2, \\ s. t & \quad \gamma_k \geq \bar{\gamma}_k, \end{aligned} \quad (4)$$

where γ_k is the threshold associated with the QoS constraint of k -th IoT device.

The second problem aims to maximize spectral efficiency while considering certain power constraints. For instance, the problem of maximizing spectral efficiency with a total transmit power limited by the max transmit power P can be formulated as follows:

$$\begin{aligned} SE_{max} &= \max \sum_{k=1}^K R_k \\ s. t & \quad \sum_{k=1}^K |\mathbf{w}_k|^2 \leq P. \end{aligned} \quad (5)$$

We define the total power consumption at the GW in the downlink channel as

$$P_{tot} = 1/\eta P_{data} + NP_{sync} + P_{sta}, \quad (6)$$

where P_{data} represents the power consumption associated with the transmitted data, η denotes the power amplifier efficiency, P_{sync} refers to the dynamic power consumption related to the power radiation of all circuit blocks in each active RF chain, and P_{sta} represents the static power consumed by the cooling system, power supply, and other components.

The problem of energy efficiency maximization with peruser SINR constraints can be expressed as

$$\begin{aligned} EE_{max} &= \frac{\max \sum_{k=1}^K R_k}{1/\eta \sum_{k=1}^K |\mathbf{w}_k|^2 + P_0} \\ s. t & \quad \sum_{k=1}^K |\mathbf{w}_k|^2 \leq P, \\ & \quad \gamma_k \geq \bar{\gamma}_k, \forall k \in [1, \dots, K]. \end{aligned} \quad (7)$$

The nonconvex nature of the objective function concerning \mathbf{w}_k in equation (7) presents a challenge in determining the optimal design for the EE_{max} problem.

3. Zero-forcing beamforming design

Although the EE_{max} problem is nonconvex, it can still be solved with global convergence and optimality. The EE_{max} problem is a nonlinear fractional program, and the parametric solution method based on Dinkelbach's method [19] has been extensively utilized in the field of wireless communications design to address similar problems.

The difficulty in solving equation (7) arises from the nonconvex nature of the objective function, which is a result of inter-user interference. An effective approach to address this challenge is to employ the zero-forcing method [20]. It has been demonstrated that the zero-forcing beamforming (ZFBF) method is highly effective for MIMO systems, thanks to the significant degrees of freedom it provides. In ZFBF, the inter-user interference is completely eliminated, i.e. $\mathbf{h}_k \mathbf{w}_i = 0, \forall k \neq i$. If we define $\tilde{\mathbf{H}} = [\mathbf{h}_1^T, \dots, \mathbf{h}_{k-1}^T, \mathbf{h}_{k+1}^T, \dots, \mathbf{h}_K^T]^T$, the precoder matrix can be chosen to eliminate inter-user interference by $\mathbf{w}_k = \mathbf{M}_k \tilde{\mathbf{w}}_k$ [21], where \mathbf{M}_k is an orthonormal basis of the null space of $\tilde{\mathbf{H}}$. The EE_{max} problem with ZFBF reduces to the following problem

$$\begin{aligned} EE_{max} &= \max \frac{\sum_{k=1}^K \log(1 + |\tilde{\mathbf{h}}_k \tilde{\mathbf{w}}_k|^2)}{1/\eta \sum_{k=1}^K |\tilde{\mathbf{w}}_k|^2 + P_0}, \\ s. t & \quad \sum_{k=1}^K |\tilde{\mathbf{w}}_k|^2 \leq P, \end{aligned}$$

$$|\tilde{\mathbf{h}}_k \tilde{\mathbf{w}}_k|^2 \geq \bar{\gamma}_k, \forall k \in [1, \dots, K], \quad (8)$$

where $\tilde{\mathbf{h}}_k = \mathbf{h}_k \mathbf{M}_k$.

The optimal solution \mathbf{w}^{*k} can be found by solve the following optimization problem

$$\begin{aligned} EE_{max} &= \max \frac{\sum_{k=1}^K \log(1+p_k \frac{\tilde{\mathbf{h}}_k'}{|\tilde{\mathbf{h}}_k|})}{1/\eta \sum_{k=1}^K |\tilde{\mathbf{w}}_k|^2 + P_0}, \\ s. t \quad &\sum_{k=1}^K p_k \leq P, \\ &p_k |\tilde{\mathbf{h}}_k'| \geq \bar{\gamma}_k, \forall k \in [1, \dots, K], \end{aligned} \quad (9)$$

where p_k is the power allocation for k -th IoT device.

The optimal solution of (9) can be found by the bisection method [22].

4. Beamforming design based on the particle swarm optimization algorithm

The advantage of ZFBF is its simplicity, however, the use of the bisection method can lead to slow convergence and instability for non-smooth objective functions. In this section, we proposed energy efficiency maximization using the PSO algorithm.

At each iteration, the position ($\mathbf{x}_{i,k}$) and velocity ($\mathbf{v}_{i,k}$) of each particle k of population i are updated using the following equations:

$$\mathbf{v}_{i,k}^{(t+1)} = \omega \mathbf{v}_{i,k}^{(t)} + c_1 r_1 (\mathbf{p}_{i,k} - \mathbf{x}_{i,k}) + c_2 r_2 (\mathbf{p}_g - \mathbf{x}_{i,k}), \quad (10)$$

$$\mathbf{x}_{i,k}^{(t+1)} = \mathbf{x}_{i,k}^{(t)} + \mathbf{v}_{i,k}^{(t)}, \quad (11)$$

where $\mathbf{v}_{i,k}^{(t)}$ denotes the velocity of population i particle k at iteration t , $\mathbf{x}_{i,k}^{(t)}$ is the position of population i particle k at iteration t , $\mathbf{p}_{i,k}$ is the personal best position of population i particle k , \mathbf{p}_g denotes the global best position among all particles, ω is the inertia weight controlling the impact of the previous velocity, c_1 and c_2 are acceleration constants controlling the impact of personal and global best positions, respectively, r_1 and r_2 are random numbers sampled from a uniform distribution.

The movement of particles is guided by their own experience ($\mathbf{p}_{i,k}$) and the shared knowledge of the swarm (\mathbf{p}_g), allowing them to efficiently explore the search space and converge towards optimal solutions. PSO is widely used in optimization problems across various domains due to its simplicity and effectiveness in finding solutions to complex optimization tasks.

To solve (7), we let $\mathbf{W} = \mathbf{U}\mathbf{P}^{1/2}$, where $\mathbf{U} = \mathbf{H}^H (\mathbf{I}_K + \mathbf{P}/K\mathbf{H}\mathbf{H}^H)^{-1}$, $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_K]$, $\mathbf{H} = [\mathbf{h}_1^T, \dots, \mathbf{h}_K^T]^T$, $\mathbf{P} = \text{diag}(p_1, \dots, p_K)$ [18, 19]. The problem (7) can be restated as follows:

$$\begin{aligned} EE_{max} &= \max \frac{\sum_{k=1}^K \log(1+p_k)}{1/\eta \sum_{k=1}^K p_k |\mathbf{u}_k|^2 + P_0}, \\ s. t \quad &\sum_{k=1}^K p_k |\mathbf{u}_k|^2 \leq P, \\ &p_k \geq \bar{\gamma}_k, \forall k \in [1, \dots, K], \end{aligned} \quad (12)$$

Let $\mathbf{x}_k = \mathbf{w}_k$ be the beamforming vector to be found using the PSO algorithm, which is computed based on the

cost function of (12). We use the projection method [22] to solve the optimization problem with constraint (12) by defining the feasible region \mathcal{F} as follows:

$$\mathcal{F} = \left\{ \mathbf{x}_k = \mathbf{w}_k \mid \sum_{k=1}^K p_k \leq \frac{P}{|\mathbf{u}_k|^2} \right\} \quad (13)$$

If \mathbf{x}_k not in the feasible region \mathcal{F} , we project \mathbf{x}_k into the feasible region as

$$\hat{\mathbf{x}}_k = \sqrt{\frac{P}{\sum_{k=1}^K \mathbf{x}_k |\mathbf{u}_k|^2}} \mathbf{x}_k \quad (14)$$

We define the objective function as

$$obj_func = \frac{\sum_{k=1}^K \log(1+p_k)}{1/\eta \sum_{k=1}^K p_k |\tilde{\mathbf{w}}_k|^2 + P_0}. \quad (15)$$

Algorithm 1 Applying PSO algorithm to maximize EE

Input:

- $N, K, P, P_0, \mathbf{h}_k, \eta, \max_iter, pop_size, \bar{\gamma}_k$.
- Random : ω, c_1, c_2

Output:

- Power allocation vector $\mathbf{p}_{out} = [p_1, \dots, p_K]$

Initialization:

- $t = 0, f_{p_g} = -\infty, f_k = -\infty$,
- Calculation \mathbf{U} .

For $i = 1 : pop_size$ do

For $k = 1 : K$ do

Random population $\mathbf{x}_{i,k}$ and velocity $\mathbf{v}_{i,k}$ with $\mathbf{x}_{i,k}, \mathbf{v}_{i,k} \geq \frac{\bar{\gamma}_k}{\mathbf{u}_k}$.

Calculate $fitness_k = obj_func$ of $\mathbf{x}_{i,k}$ by (15)

If ($fitness_k > f_k$) then

$\mathbf{p}_{i,k} = \mathbf{x}_{i,k}, f_k = fitness_k$

EndIf

If ($fitness_k > f_{p_g}$) then

$\mathbf{p}_g = \mathbf{x}_{i,k}, f_{p_g} = fitness_k$

EndIf

EndFor

EndFor

While ($t < \max_iter$)

For $i = 1 : pop_size$ do

For $k = 1 : K$ do

Determine the velocity $\mathbf{v}_{i,k}$ of k -th particle by (10)

Determine the new position $\mathbf{x}_{i,k}$ by (11)

Bound $\mathbf{x}_{i,k}$ by lower bound $\frac{\bar{\gamma}_k}{\mathbf{u}_k}$ and (14)

Calculate $fitness_k = obj_func$ of $\mathbf{x}_{i,k}$ by (15)

If ($fitness_k > f_k$) then

$\mathbf{p}_{i,k} = \mathbf{x}_{i,k}, f_k = fitness_k$

EndIf

If ($fitness_k > f_{p_g}$) then

$\mathbf{p}_g = \mathbf{x}_{i,k}, f_{p_g} = fitness_k$

EndIf

EndFor

EndFor

EndWhile

Update $\mathbf{p} = \mathbf{p}_g$

5. Numerical results

In this section, numerical simulation results are provided to evaluate the performance of the proposed PSO algorithm for the design beamforming matrix in the downlink IoT system to maximize energy efficiency. We consider quasi-static frequency flat Rayleigh fading channels and adopt a macrocell configuration, where the path loss in decibels (dB) is modeled as $128.1 + 37.6 \log(d)$ with the distance d measured in kilometers [23]. The simulation parameters of the system are described in Table 1. In the simulations, the distance of each IoT device to the GW is randomly chosen from the range of 0.1 to 1 kilometers. The transmit power budget of the gateway (GW) over the total bandwidth is $B = 10\text{MHz}$. The noise power is $BN_0 = 104\text{dBm}$, the circuit power consumption is $P_{sta} = 33\text{dBm}$, and the power amplifier efficiency is $\eta = 0.35$. The shadow fading is modeled as a log-normal distribution with a standard deviation of 8dB . Without loss of generality, we set the SINR threshold of all IoT devices $\bar{\gamma}_k = 0\text{dB}$.

Table 1. System simulation parameters

Parameters	Value
d	$0.1 \div 1$ (km)
B	10 (MHz)
η	0.35
$\bar{\gamma}_k$	0 (dB)
P_{sta}	33 (dBm)
BN_0	104 (dBm)
N	16
K	{2, 4, ..., 16}
P	{17, 18, ..., 23}

Figure 2 presents the energy efficiency as a function of the maximum transmit power. We can see that the proposed approach may significantly improve the energy efficiency compared to the ZFBF method, especially when the transmit power is low. When the transmission power increases, the performances of the proposed and the ZFBF methods tend to converge. The reason is that with high transmission power, the SNR is high, allowing the ZFBF scheme to achieve optimal solutions as it can effectively eliminate interference.

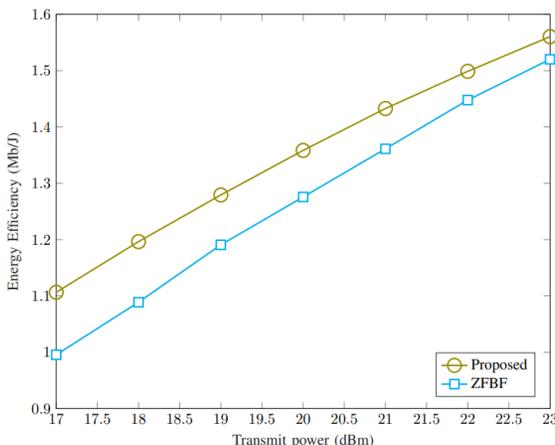


Figure 2. The average of the energy efficiency (Mb/J) as a function of the maximum transmit power (dBm), where $N=16$, $K=8$

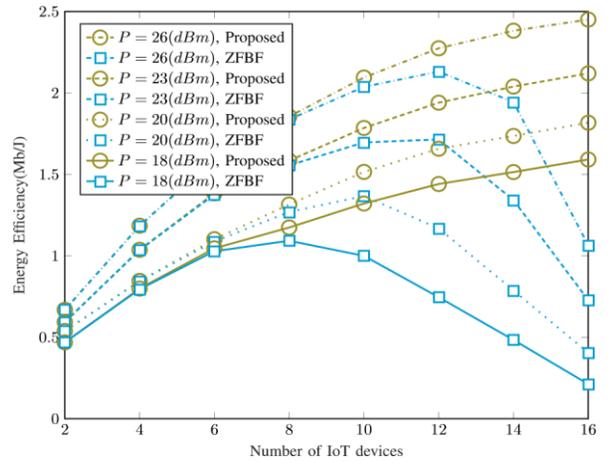


Figure 3. The average of the energy efficiency (Mb/J) as a function of the number of IoT devices, where $N=16$

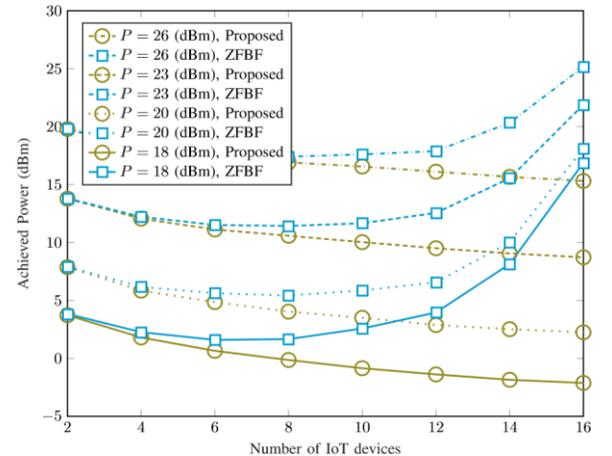


Figure 4. The average of the energy efficiency (Mb/J) as a function of the number of IoT devices, where $N=16$

In Figure 3, representing energy efficiency by the number of IoT devices. In this figure, we see that the PSO algorithm has better performance than the ZFBF method in the high number of IoT devices. As the number of IoT devices increases, finding the optimal solution in the ZFBF method becomes more difficult, because the constraints become more "strict". Meanwhile, the beamforming matrix of the proposed method is based on the channel direction but was rotated to balance the transmit power and is orthogonal to the inter-user channels [19].

In Figure 4, we compare the system's transmit power according to the number of IoT devices when changing the transmit power constraint. The figure shows that the proposed optimization algorithm has a lower output power than ZFBF when the constraints are tighter. The proposed method is more "flexible" than ZFBF, especially effective when increasing the difficulty of the constraints. That is, the power consumption of the proposed method will be smaller than that of the ZFBF method.

6. Conclusion

This paper addresses the challenge of maximizing energy efficiency in an IoT downlink system. A PSO-based algorithm is proposed for designing beamforming matrices to achieve this goal. Simulations confirmed that the PSO-based algorithm significantly outperforms Zero-Forcing

Beamforming (ZFBBF). This finding suggests PSO is a promising approach for tackling optimization problems in future wireless communication systems.

Acknowledgement: This research is funded by the Ministry of Education and Training under Project B2024.DNA.19.

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