2D TRAIN-TRACK-BRIDGE INTERACTION ANALYSIS CONSIDERING THE EFFECT OF BALLASTED TRACKS

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(Received: September 08, 2024; Revised: October 12, 2024; Accepted: October 15, 2024)

DOI: 10.31130/ud-jst.2024.509E

Abstract - This paper investigates the effect of ballasted tracks on the dynamic interaction between train, track, and bridge systems. A 2D interaction model is developed, incorporating detailed representations of the bridge, train, and track, with and without ballasted tracks. The Cat Linh - Ha Dong urban railway line on a simply supported bridge in Hanoi is used as a case study to perform parametric analysis, evaluating how variations in track parameters influence the dynamic responses of the bridge, train, and track. The numerical results reveal a significant dynamic vertical coupling between the bridge decks, highlighting the critical importance of including ballasted tracks in the modeling process of these structures.

Key words - Train-track-bridge interaction; ballasted track; FEM; vertical dynamic response; urban railway line

1. Introduction

Ensuring stable urban traffic flow is a pressing issue in major cities of Vietnam, especially in the capital, Hanoi. Therefore, the construction of urban railways is essential and a modern solution to address this problem. Currently, several urban railway lines are being constructed and have undergone trial operations, showing promising results in reducing urban traffic congestion. However, during the operation and maintenance phases, there are still unresolved issues that need to be studied to ensure safety in operation. Among these, the dynamic interaction between trains, tracks, and bridges has been a topic of concern in recent years. The choice of using ballasted or non-ballasted tracks is also a factor that needs to be considered, as it impacts the dynamic effects on railway bridges under the load of passing trains [1].

In Vietnam, several researchers have studied the structural dynamics of railway bridge systems. For instance, Truong and Le [2] researched and analyzed the impact of uneven rail profiles on the dynamic characteristics of high-speed railways. Nguyen [3] presented studies and analyses on the dynamic behavior of non-ballasted railway structures using the finite element model. Pham et al. [4] proposed a time-history analysis method to analyze the dynamics of high-speed railway structures for the North-South High-Speed Railway project.

Globally, many countries have had the advantage of developing urban railway networks early, and numerous studies on the interaction between trains, tracks, and bridges have been conducted by researchers. For example, Zhai et al. [1] systematically presented a modern evaluation model of train–track–bridge dynamic interaction. Su et al. [5] utilized field measurements and simulation analysis to study the dynamic interaction between high-speed trains and reinforced concrete viaducts. A review of the literature indicates that research on the dynamic interaction between trains, tracks, and bridges remains limited, with few studies focusing specifically on the effects of ballasted tracks on the dynamic responses of these systems.

It is crucial to examine the differences and performance between ballasted and non-ballasted railway bridges. This evaluation is significant during the initial design phase of future projects, as it affects the weight of bridge structures, with non-ballasted bridges being lighter than their ballasted counterparts. Furthermore, considering the removal of ballast from bridges may facilitate the design of slimmer and more cost-efficient bridge structures that still meet dynamic requirements.

In light of these considerations, this paper aims to assess the impact of different track systems on the dynamic interaction within the train–track–bridge system. The objective is to analyze and evaluate the relationship between the vertical response of the bridge encompassing acceleration and displacement of the bridge deck and the vertical response of the vehicle, which represents the interaction force between the vehicle's wheels and the rails. This analysis is based on a 2D finite element model and classical theories of railway vehicle dynamics. The parametric study focuses on the Cat Linh – Ha Dong urban railway line with a simply supported bridge span.

2. Numerical modeling

2.1. 2D Train-Track-Bridge Interaction Model

The theoretical model for analyzing the dynamic interaction between the train, track, and bridge in this study employs a combined 2D finite element approach based on the documentation of D. Cantero et al [6]. This model is used to investigate the bridge's response to a passing train for both ballasted and non-ballasted track systems. It encompasses the train, track, and bridge systems.

In this model, the train is represented as a series of vehicles and is modeled using a mass-spring system. The track is depicted as a beam supported by a spring system, while the bridge is represented as a 2D Euler-Bernoulli beam with a simply supported span. All subsystems are integrated into a unified model.

The equations of motion for both the train and the infrastructure are formulated using mass, stiffness, damping, and force matrices. The nonlinear Hertzian contact theory is applied within the 2D train-track-bridge interaction model to account for wheel-rail contact loss and to describe the interaction forces between the wheels and track irregularities.

The interaction between the train, track, and bridge is a combination of subsystems defined through the equation of motion:

$$\mathrm{M}_x\ddot{\mathrm{u}}_x + \mathrm{C}_x\dot{\mathrm{u}}_x + \mathrm{K}_x\mathrm{u}_x = \mathrm{F}_x \tag{1}$$

where M, C, and K are the mass, viscous damping, and stiffness matrices distributed along the beam, respectively; F is the vector of modal and external forces; ü, u̇, and u are the vectors of acceleration, velocity, and displacement, respectively. Meanwhile, x denotes the subsystem that the matrix refers to and can be replaced by V, T, or B to represent the vehicle, track, and bridge subsystems, respectively. The combined equation of motion for this interaction simulation [7] is written in block matrix form as follows:

$$\begin{pmatrix} M_V & 0 & 0 \\ 0 & M_T & 0 \\ 0 & 0 & M_B \end{pmatrix}\begin{Bmatrix} \ddot{u}_V \\ \ddot{u}_T \\ \ddot{u}_B \end{Bmatrix} + \begin{pmatrix} C_V & C_{V,T} & 0 \\ C_{T,V} & C_T & C_{T,B} \\ 0 & C_{B,T} & C_B \end{pmatrix}\begin{Bmatrix} \dot{u}_V \\ \dot{u}_T \\ \dot{u}_B \end{Bmatrix} + \begin{pmatrix} K_V & K_{V,T} & 0 \\ K_{T,V} & K_T & K_{T,B} \\ 0 & K_{B,T} & K_B \end{pmatrix}\begin{Bmatrix} u_V \\ u_T \\ u_B \end{Bmatrix} = \begin{Bmatrix} F_V \\ F_T \\ F_B \end{Bmatrix} \tag{2}$$

where matrices of the vehicles-track interaction are represented by the subscript 'V, T' or 'T, V'. Besides, the matrices of the track-bridge interaction induced by the continuous springs and dampers between the track and bridges are donated by the subscripts 'T, B' or 'B, T'.

The implementation of the train–track–bridge interaction model and the integration of subsystems through the equations of motion are executed using MATLAB. This software allows for the precise and efficient computation of individual matrices through advanced algorithms. Furthermore, the interaction forces between the wheels and the rails are characterized using the nonlinear Hertzian theory [8], as detailed in the following equation:

$$F = \begin{cases} k_C(u_w - u_r - r_w), (u_w - u_r - r_w) > 0 \\ 0, (u_w - u_r - r_w) \leq 0 \end{cases} \tag{3}$$

where F is the interaction force between the wheel and the rail, k_C, u_w, u_r, and r_w are the Hertzian contact stiffness constant, vertical displacement of the wheel, vertical displacement of the rail, and track surface irregularity, respectively.

Random track irregularities are characterized using power spectral density (PSD) functions. While different agencies utilize various PSD functions, this study employs the PSD function provided by the Federal Railroad Administration (FRA), defined by the following formula:

$$S(\omega) = \frac{A_v\omega_2^2(\omega^2 + \omega_1^2)}{\omega^4(\omega^2 + \omega_2^2)} \tag{4}$$

This spectral density depends on the circular spatial frequency $\Omega = 2\pi/\lambda$ where λ is the wavelength of the irregularity. By Equation (4), ω_1 and ω_2 are described wave lengths in the range, A_v is used to define track class.

The calculation procedure, details of the constants and detailed examples of PSD implementation can be found in the works by Du Kim and Warnitchai [9] and Ferrara [10].

2.1.1. Train model

The train is represented by a finite element model, defined by the combination of individual subsystems including the car body with 2 degrees of freedom, bogies, and wheels, each with 4 degrees of freedom. The model of the train running on randomly uneven tracks is shown in Figure 1.

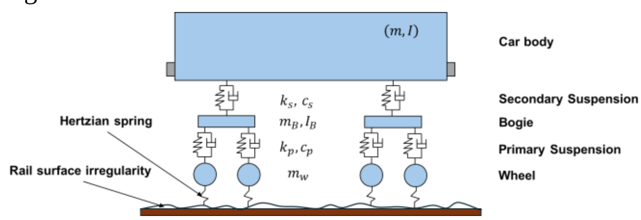


Figure 1. *Model of train*

Table 1. *Train model characteristics based on the Cat Linh-Ha Dong train*

Parameters	Symbol	Value	Unit
Main body mass	m_v	22400	kG
Main body moment of inertia	I_v	23200	$kG.m^2$
Main body length	L_v	12.6	m
Additional distance	L_E	3.46	m
Number of bogies	N_b	2	
Bogies masses	m_B	3520	kG
Bogies moments of inertia	I_B	1430	$kG.m^2$
Bogies length	L_B	2.2	m
Number of wheels	N_w	4	
Wheel masses	m_w	1539	kG
Primary suspension stiffness	k_{pi}	1.7×10^6	N/m
Primary suspension viscous damping	c_{pi}	10^4	Ns/m
Secondary suspension stiffness	k_{si}	0.45×10^6	N/m
Secondary suspension viscous damping	c_{si}	6×10^4	Ns/m

The geometric and mechanical input parameters of the train model depend on the specific type of train being modeled, with the details of the train model parameters provided in Table 1. The main body of the train is modeled as rigid beams with mass m_v and moment of inertia I_v. The primary and secondary suspension systems are represented by parallel linear spring systems k_p, k_s and viscous dampers c_p, c_s. The wheels are specifically modeled as masses on the rails, connected to the bogies through the primary suspension system.

2.1.2. Track on bridge system model

In this study, both the track and the bridge are modeled using a 2D finite element approach. The bridge is treated as having a simply supported span, while the track is represented as a beam supported by a multilayered structure. This multilayered structure includes masses that simulate sleepers and ballast, which are connected to the beam through spring and damper systems that represent rail pads, the ballast layer, and the sub-ballast. Both the track and the bridge are modeled as Euler-Bernoulli beams.

The ballasted track model comprises rails, rail pads, sleepers, ballast, and sub-ballast, all placed directly on the bridge. In contrast, the non-ballasted track model generally includes rails, rail pads, sleepers, and sleeper pads, with these components placed directly on the bridge. Detailed configurations for both ballasted and non-ballasted track models are illustrated in Figures 2 and 3. Technical specifications for the track system are outlined in Table 2.

Table 2. *Track properties*

Parameters		Symbol	Value	Unit
Rail	Young's modulus of rail material	E_R	2.059×10^{11}	N/m^2
	Second moment of area	I_R	6.434×10^{-5}	m^4
	Mass per unit length of rail	μ_R	121.28	kG/m
	Damping	c_R	0.1	%
Pads	Vertical stiffness of pad	k_P	6.5×10^7	N/m
	Vertical viscous damping of pad	c_P	7.5×10^4	Ns/m
Sleepers	Distance between sleepers	L_S	0.6	m
	Mass of each sleeper	m_S	251	kG

Table 3. *List of beam properties*

Parameters	Symbol	Value	Unit
Bridge span	L	30	m
Modulus of elasticity	E	3.45×10^{10}	N/m^2
Section's second moment of area	I	1.22126	m^4
Damping ratio	η	1	%
Mass per unit length of the bridge	μ	11634	kG/m

In that, the track model is defined by four parameters including Young's modulus of rail material (E_R), the second moment of area (I_R), mass per unit length of rail (μ_R) and damping (c_R). The sleepers are represented as masses (m_S) at an equal distance (L_S) connected to the rail by the spring and dashpot system represents the pad with stiffness (k_P) and viscous damping (c_P). For the ballasted track, the ballast layer is characterized as concentrated mass (m_{BA}) interacting with the sleepers through a spring and dashpot system with stiffness (k_{BA}) và viscous damping (c_{BA}).

The bridge model is implemented using the finite element method (FEM) [11] based on five parameters, specifically bridge span length (L), modulus of elasticity (E), section's second moment of area (I), damping ratio (η) and Mass per unit length of the bridge (μ). Finally, input parameters for the bridge are listed in Table 3.

a. Model of ballasted track

Figure 2 provides a detailed depiction of the track model structure resting on ballast. The ballast blocks are represented as masses. The technical specifications are detailed in Table 4.

Table 4. *List of ballast on bridge properties*

Parameters		Symbol	Value	Unit
Ballast	Mobilized ballast mass	m_{BA}	531.4	kG
	Ballast vertical stiffness	k_{BA}	137.75×10^6	N/m
	Ballast vertical viscous damping	c_{BA}	5.88×10^4	Ns/m
Sub-Ballast	Sub-Ballast vertical stiffness	k_{SB}	77.5×10^6	N/m
	Sub-Ballast vertical viscous damping	c_{SB}	3.115×10^4	Ns/m

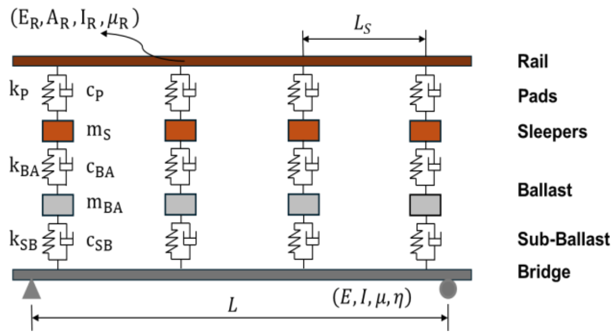


Figure 2. *Model of ballasted track*

To determine mobilized ballast mass and ballast stiffness, the procedure proposed by Zhai et al [12] has been applied, in which the mobilized ballast mass is determined through the expression:

$$m_{BA} = \rho_b \left[\begin{array}{c} l_b h_b (l_e + h_b tg\alpha) + l_e (h_b^2 - h_0^2) tg\alpha \\ + \frac{4}{3} (h_b^3 - h_0^3) tg^2\alpha \end{array} \right] \quad (5)$$

And ballast stiffness is calculated as follows:

$$k_{BA} = \frac{k_{BA_1} k_{BA_2}}{k_{BA_1} + k_{BA_2}} \quad [6]$$

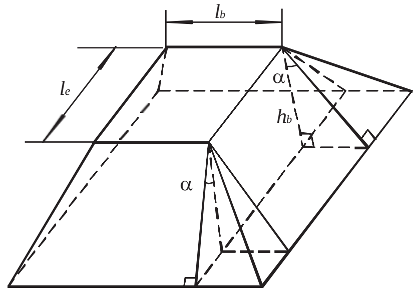
with

$$k_{BA_1} = \frac{2(l_e - l_b) tg\alpha}{\ln \left[\frac{(l_e l_s)}{(l_b (l_e + l_s - l_b))} \right]} E_b \quad [7]$$

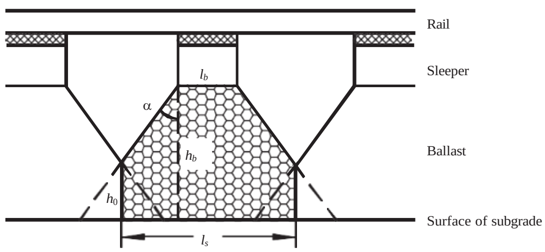
$$k_{BA_2} = \frac{l_s (l_s - l_b + 2 l_e + 2 h_b tg\alpha) tg\alpha}{l_b - l_s + 2 h_b tg\alpha} E_b \quad [8]$$

Where ρ_b is the density of ballast, h_b is the depth of ballast, l_e is the effective supporting length of half sleeper, l_b is the width of the sleeper underside, α is the ballast stress distribution angle, $h_0 = h_b - (l_s - l_b)/(2tg\alpha)$ is the height of the overlapping regions, and E_b is the elastic modulus of the ballast.

The definition of the model parameters for the ballast layer beneath the sleepers used in formulas (5), (6), (7), and (8) is illustrated in Figure 3.



(a) Model of the ballast under one rail support point



(b) The modified model of ballast

Figure 3. *Proposed model for estimating mobilized ballast mass and ballast stiffness (Zhai et al [12])*

b. Model of ballastless track

In this model, the sleeper structure is installed directly on the concrete base, effectively substituting the traditional sub-ballast layer. This arrangement is further supported by an under-sleeper pad (PU), which plays a crucial role in cushioning and distributing the loads from the sleepers to the concrete base. The configuration and properties of the under-sleeper pad are illustrated in Figure 4, which provides a detailed view of how the pad is integrated into the system.

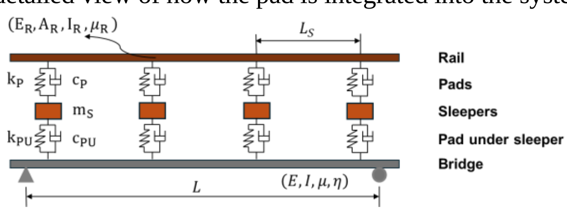


Figure 4. *Model of ballastless track*

The PU is designed to enhance the overall performance of the track by providing additional damping and reducing vibrations that could otherwise be transmitted to the concrete base. Its technical specifications, including thickness and viscous damping are comprehensively detailed in Table 5.

Table 5. *List of pads under sleeper properties*

Parameters		Symbol	Value	Unit
Pad under sleeper	Pad under sleeper vertical stiffness	k_{PU}	1.2×10^8	N/m
	Pad under sleeper viscous damping	c_{PU}	6×10^5	Ns/m

2.2. Influencing the effect of ballasted tracks

The interaction between ballasted tracks and bridges has a significant impact on the dynamic behavior of the railway system, particularly under dynamic loading conditions caused by passing trains. The importance of this relationship is highlighted by several studies. For instance, Tahiri et al. [13] demonstrated that the nonlinear characteristics of ballasted tracks can influence the bridge's dynamic response, potentially leading to reduced critical speeds and increased resonance amplitudes, which may pose safety concerns during operation. Similarly, Stollwitzer et al. [14] provided experimental findings indicating that the dynamic properties of ballast often diverge from standard specifications, affecting the stability and overall performance of railway bridges. Furthermore, the shear forces exerted by ballast are crucial in the dynamic response of railway bridges, especially for shorter spans where large vertical accelerations can increase maintenance costs and safety risks, as discussed by J. Chordà-Monsonís et al. [15]. Heavy-load tracks with ballast significantly affect bridge dynamics, with factors such as rail irregularities, axle loads, and bridge parameters playing key roles in system performance. This study aims to explore the effects of both ballasted and non-ballasted tracks on the dynamic responses of bridges, including acceleration at midspan, vertical displacement, and wheel-rail interaction forces, to elucidate the differences in performance and implications of various track structures.

2.3. Solution method

The Newmark Beta method is used in this study to solve the coupled equations of motion for the train-track-bridge interaction simulation at each time step. Given that the nonlinear contact between the vehicle and the track depends on position and time, the Newmark Beta method is employed to enhance the efficiency of the solving process.

3. Validation of train-bridge dynamic interaction responses with experimental results

The vertical displacement of the bridge at midspan, obtained from the analysis of the train-track-bridge interaction model, is compared with the experimental results obtained by Xia et al [16] under the Gouhe bridge on the Qin-Shen HSR line when the Pioneer EMU passes at a speed of 270 km/h, which is considered in this case study.

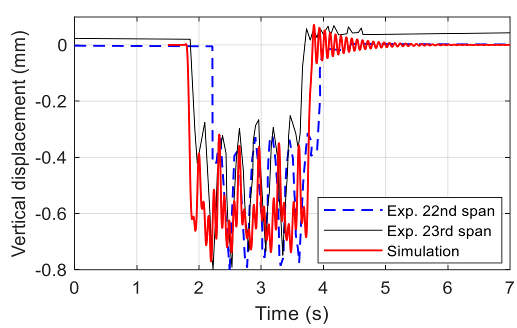


Figure 5. *Vertical displacement at the midspan for the Pioneer EMU moving at v = 270 km/h*

The bridge model has a simply supported 24 m-span double-track PC box-girder structure. In addition, the Pioneer EMU train has 6 cars, of which the 1st, 3rd, 4th, and 6th are motor cars and the 2nd and 5th are trailer cars, and the total length of each car is 25.5 m. The calculated maximum displacement of the bridge at the midspan based on the model is shown in Figure 5 and the value of the experimental result is approximately 0.79 and 0.82 mm, respectively. From the results, it can be seen that the calculated displacement time history curve is very close to the experimental results presented in the study by Xia et al [16], demonstrating the excellent suitability of the proposed analytical and computational model for simulating the dynamic response of the train-track-bridge system.

4. Description of case study

The Cat Linh - Ha Dong urban railway line is part of the Hanoi urban rail network, with a total mainline length of 13.05 km and the entire line designed as an elevated track. The main span structure of the Cat Linh - Ha Dong railway bridge consists of simply supported spans ranging from 18.5 m to 32 m in length, with a typical cross-section being a box-shaped profile, as shown in Figure 6. The structure is designed according to double-track railway standards, with a maximum speed of 80 km/h and an operational speed of 35 km/h. In this study, the bridge model is represented in the train-track-bridge interaction simulation with a span length of 30 m and a simple span structure. Additionally, other model parameters such as Young's modulus, moment of inertia, and damping coefficient are detailed in Table 3 above.

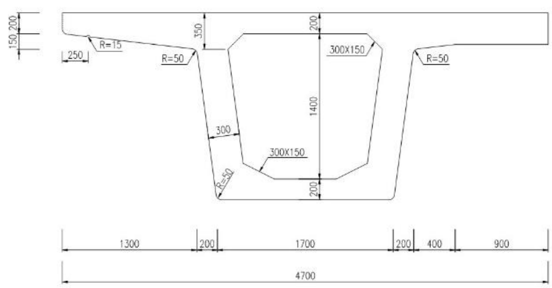


Figure 6. *Cross section of beam*

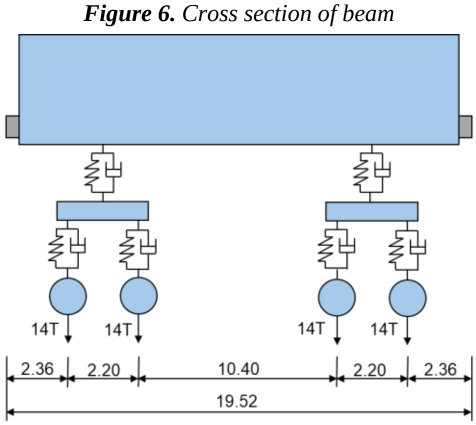


Figure 7. *Design load of the Cat Linh – Ha Dong train*

The train used on the Cat Linh - Ha Dong urban railway line consists of 4 cars, each 19.52 meters long, with a distance of 2.2 meters between the two bogies on each side, and an axle load of 140 kN per axle. The specific parameters of the train currently in operation are shown in Figure 7 and Table 1.

5. Parametric analysis

5.1. Simulation results

This study performs a simulation analysis of the train-track-bridge interaction for both ballasted and non-ballasted tracks, as modeled in Section 2.1.2, at various speeds of 35, 50, 65, and 80 km/h. The results are presented and discussed in this section. The dynamic responses based on the train-track-bridge interaction simulation at 35 km/h for both ballasted and non-ballasted track cases include the acceleration and displacement of the bridge deck, as well as the vertical response of the vehicle, which is the interaction force between the vehicle's wheels and the rails. These relationships are illustrated in Figures 8, 9, 10, and 11.

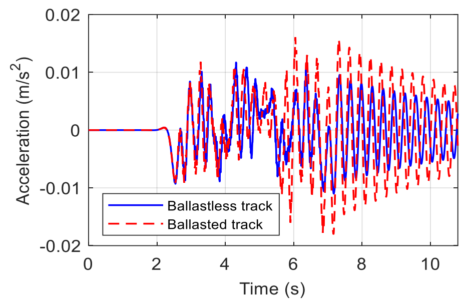


Figure 8. *Midpoint acceleration of bridge for ballasted track and ballastless track at 35 km/h*

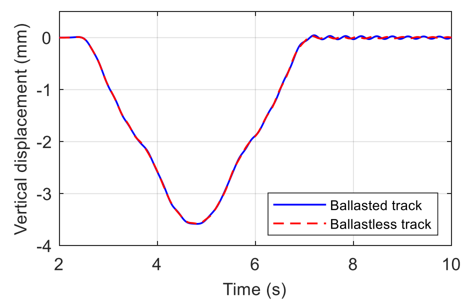


Figure 9. *Midpoint displacements of bridge for ballasted track and ballastless track at 35 km/h*

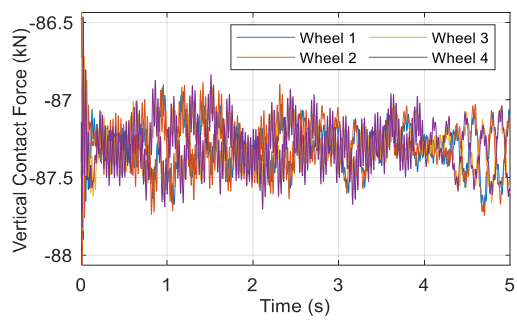


Figure 10. *Maximum value of vertical contact force of bridge for ballasted track at 35 km/h*

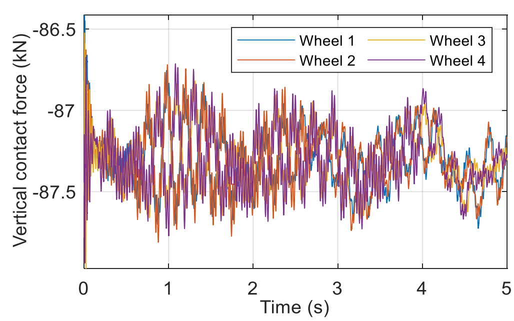


Figure 11. *Maximum value of vertical contact force of bridge for ballastless track at 35 km/h*

5.2. Parametric analysis

Figure 12 presents a comparison of the maximum acceleration values across the bridge span over time as the train passes by at different speeds. It can be observed that at speeds below 50 km/h, there is no significant difference in the bridge's acceleration. However, the bridge's response shows a strong correlation with the train's speed, with acceleration values increasing as the speed rises. The graph indicates that the acceleration of the bridge with ballast is higher compared to the bridge without ballast.

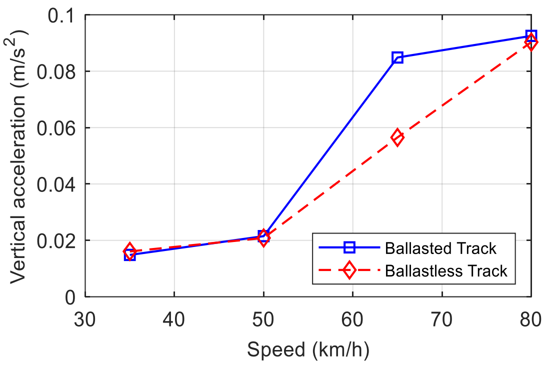


***Figure 12.** Maximum value of midpoint acceleration of bridge at different train speeds*

Furthermore, the relationship between the train speed and the displacement between bridge spans when using two different types of track is evaluated in Figure 13. The results clearly show that the displacement between bridge spans increases as the train speed rises for both types of tracks. This indicates that the train speed directly affects the vibrations and deformation of the bridge. Figure 13 shows that the displacement between bridge spans for ballasted tracks increases more rapidly with speed compared to non-ballasted tracks. This issue may be due to the ballast layer absorbing some of the vibrational energy, but as the speed increases rapidly, this absorption effect diminishes. Meanwhile, non-ballasted tracks have a stiffer structure compared to ballasted tracks, which helps reduce the vibrations and deformation of the bridge, resulting in a slower and less pronounced increase in bridge displacement compared to ballasted tracks.

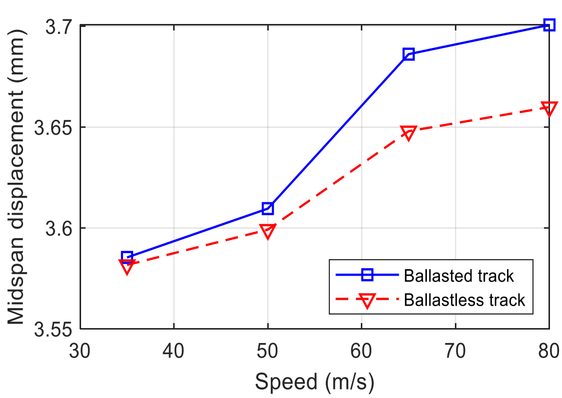


***Figure 13.** Midpoint displacement of bridge*

Some differences in the interaction forces between the wheels and the rails are illustrated in Figure 14. The chart in Figure 14 shows that as the speed increases, significant loss of contact between the wheels and the rails occurs.

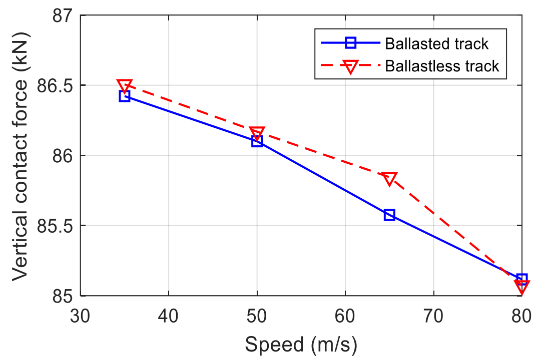


***Figure 14.** Maximum value of vertical contact force of train*

6. Conclusions

This study employs a comprehensive 2D finite element model combined with classical railway vehicle dynamics theories to explore the responses of both ballasted and non-ballasted track systems. The interaction simulation is then conducted for the Cat Linh - Ha Dong railway bridge to explore how the ballasted tracks affect the dynamic response of the whole system. Key findings can be summarized as the following:

The simulation results indicate that the acceleration experienced by the bridge is significantly higher when using ballasted tracks compared to non-ballasted tracks. This difference is attributed to the increased dynamic load transmitted through the ballast layer, which affects the bridge's stability and performance.

It was observed that vertical displacement increases more rapidly with ballasted tracks as train speeds increase. This implies that the bridge experiences greater deformation and potential structural stress under higher speeds with ballasted tracks.

The interaction between the wheels and the rails shows a notable loss of contact as speed rises, particularly with ballasted tracks. This loss of contact can lead to reduced efficiency in load transfer and potential safety issues, emphasizing the need for careful consideration of track type in design and maintenance.

The findings suggest that non-ballasted tracks offer several advantages over ballasted tracks in urban railway settings. Specifically, non-ballasted tracks result in reduced acceleration experienced by the bridge, slower rates of vertical displacement increase, and minimized wheel-rail contact loss. These benefits contribute to enhanced overall performance and potentially lower maintenance costs.

The results of this study highlight the significant impact of track type on the dynamic response of railway bridges. The increased acceleration and vertical displacement associated with ballasted tracks, along with the loss of wheel-rail contact, underscore the advantages of using non-ballasted tracks in urban rail systems. Non-ballasted tracks not only mitigate the adverse effects observed with ballasted systems but also contribute to improved operational efficiency and safety. The insights gained from this simulation provide valuable information for the design and optimization of future railway infrastructure, particularly in urban environments where dynamic performance and structural integrity are crucial.

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