OPTIMIZATION OF BRAKE DISC STRUCTURE FOR MASS AND TEMPERATURE REDUCTION USING THE MOPSO ALGORITHM

TỐI ƯU HÓA CẤU TRÚC ĐĨA PHANH NHẰM GIẢM KHỐI LƯỢNG VÀ NHIỆT ĐỘ SỬ DỤNG THUẬT TOÁN MOPSO

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Abstract - This study focuses on optimizing the structure of an automotive brake disc to reduce temperature (Y1) and mass (Y2). The disc thickness (X1), number of hole rows (X2), and number of slots (X3) were considered. A Taguchi OA25(5^3) design was employed to construct a simulation table consisting of 25 modes. The effects of the brake disc design parameters were analyzed using analysis of variance (ANOVA), and the multi-objective particle swarm optimization (MOPSO) algorithm was utilized to determine the optimal values. The ANOVA results indicated that, only X1 had a significant effect on temperature, whereas all three parameters, X1, X2, and X3, significantly influenced the brake disc mass (p<0.001). After convergence, the MOPSO algorithm obtained 20 Pareto-optimal solutions, balancing brake disc temperature and mass. For validation, five optimal designs were reanalyzed, revealing deviations of no more than 1.5%, confirming the reliability of the proposed method.

Key words - Brake Disc; optimization; temperature; multi-objective; particle swarm optimization.

Tóm tắt - Nghiên cứu này tập trung tối ưu hóa cấu trúc đĩa phanh ô tô nhằm giảm nhiệt độ (Y1) và khối lượng (Y2). Độ dày (X1), số hàng lỗ (X2) và số rãnh (X3) của đĩa phanh đã được xem xét. Thiết kế Taguchi OA25(5^3) được sử dụng để xây dựng bảng mô phỏng gồm 25 chế độ. Ảnh hưởng của các thông số thiết kế đĩa của phanh được xem xét bằng phân tích phương sai (ANOVA), đồng thời thuật toán tối ưu hóa bầy đàn đa mục tiêu (MOPSO) được sử dụng để tối ưu hóa xác định giá trị tối ưu. Kết quả ANOVA cho thấy, chỉ có X1 có ảnh hưởng đáng kể đến nhiệt độ trong khi cả ba thông số X1, X2, X3 đều có ảnh hưởng đáng kể đến khối lượng của đĩa phanh (p<0,001). Thuật toán MOPSO sau khi hội tụ đã thu được 20 giải pháp tối ưu Pareto cân bằng giữa nhiệt độ và khối lượng đĩa phanh. Để kiểm chứng, năm thiết kế tối ưu được phân tích lại, cho thấy sai lệch không quá 1,5%, khẳng định độ tin cậy của phương pháp đề xuất.

Từ khóa - Đĩa phanh; tối ưu hóa; nhiệt độ; đa mục tiêu; thuật toán tối ưu hóa bầy đàn.

1. Introduction

The braking system is one of the most crucial components in automotive structures, playing a key role in ensuring operational safety. Among these, the brake disc is a core component, primarily responsible for dissipating the heat generated from the friction between the brake pad and the disc, thereby maintaining stable braking performance. However, one of the major challenges in brake disc design is optimally controlling temperature and mass.

When a vehicle operates at high speeds or performs frequent hard braking in a short period, the brake disc temperature can rapidly increase, exceeding permissible working limits. This can lead to temporary brake fading, surface cracking, or even disc deformation due to uneven thermal expansion [1]. To address this issue, various studies have proposed methods such as improving geometric design, using materials with high thermal conductivity, or integrating heat dissipation structures like ventilation holes and air escape grooves on the disc surface [2, 3]. Additionally, the mass of the brake disc is also a crucial factor. Reducing mass helps improve fuel efficiency [4]. Thigale and Shah [5] used Altair INSPIRE 9.5 software to optimize the brake disc mass, achieving a new disc weight 26.68% lighter than the existing one. Other studies have also examined brake disc mass [6, 7]. This demonstrates that optimizing the design to reduce brake disc mass while maintaining performance is a significant objective in the automotive industry today [8].

In recent years, many optimization methods have been applied to improve the design of mechanical structures in general, including experimental methods, numerical simulations, and optimization algorithms [9, 10]. Numerous previous studies have focused on using finite element analysis (FEA) to simulate heat dissipation and evaluate thermal stress in brake discs [11, 12]. Zheng, Wang and Zhang [13] optimized the natural frequency and mass of brake drums using the response surface method (RSM). Their results showed significant improvements in natural frequency, though changes in brake drum mass were negligible. Several studies employing genetic algorithms, grey wolf optimization, and others related to brake disc optimization have also been published [14, 15].

One of the effective algorithms in multi-objective design optimization is the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm [16]. This is a variant of the Particle Swarm Optimization (PSO) algorithm, enhanced to tackle optimization problems with multiple conflicting objectives. MOPSO simulates the foraging behavior of bird or fish swarms to find optimal design solutions, based on the principle of information sharing among particles in the search space. Compared to some other traditional optimization methods, MOPSO has advantages such as fast convergence speed, the ability to explore diverse Pareto optimal solutions, and the capacity to find balanced designs across different criteria [17]. However, there are few studies published using this algorithm for brake disc structure optimization.

This study optimizes the brake disc structure with three parameters: thickness (X1), number of hole rows (X2), and number of grooves (X3), aiming to reduce the temperature (Y1) and mass (Y2) of the brake disc. First, the brake disc structure is redesigned with 3 varying parameters at five levels using the Taguchi method, and FEA is conducted for each case under the same boundary conditions to determine temperature and mass. Then, an Analysis of Variance (ANOVA) analysis is performed to examine influencing factors, and a mathematical model is constructed to predict the temperature and mass of the brake disc. Finally, the MOPSO algorithm is used to optimize and determine the optimal values, and the results are verified. This study provides a reference solution for brake disc design, helping improve heat dissipation performance, reduce mass, and thus enhance the overall efficiency of the braking system. The research also contributes to the development of modern design methods, applying artificial intelligence and optimization algorithms in engineering.

2. Research methodology

The main steps of the research are illustrated in Figure 1.

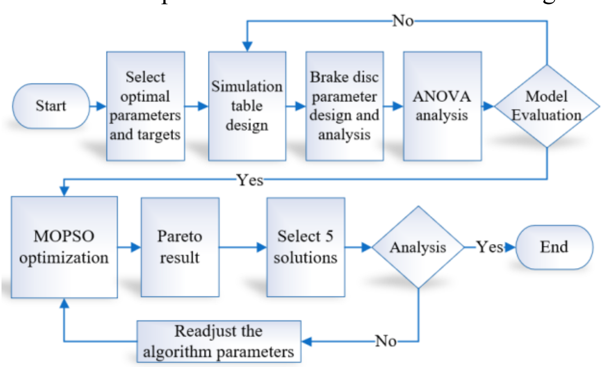

Figure 1. *Flowchart of the study methodology*

2.1. Specifications and modeling of the brake disc

In this study, the brake disc model was designed using SolidWorks software, allowing for the creation of precise and detailed 3D models. The brake disc is made from gray cast iron, a common material in the automotive industry due to its mechanical properties and good heat resistance. The main material specifications are: density of 7200 kg/m³, ultimate tensile strength of 240 MPa, elastic modulus of 1.1×1011 Pa, Poisson's ratio of 0.28 and thermal expansion coefficient of 1.1×10^{-5} °C^{-1}.

The key dimensions of the brake disc, such as diameter and thickness, are clearly shown in Figure 2, referencing an actual brake disc type. The brake disc is designed with a combination of grooves and holes to improve heat dissipation and enhance performance under various conditions. These grooves help increase airflow around the disc surface, thereby reducing temperature during braking. The grooves and holes on the brake disc not only facilitate more efficient heat dissipation but also reduce the overall weight of the brake disc. Therefore, in this study, three parameters of the brake disc are investigated: disc thickness (X1), number of hole rows on the brake surface (X2), and number of grooves (X3).

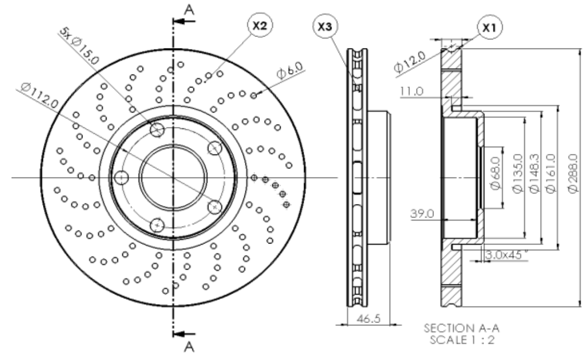

Figure 2. *Main parameters of brake disc*

Based on the analysis requirements and geometric characteristics of the brake disc, a tetrahedral mesh with a size of 1.0 mm is used, with a target quality level for mesh elements set at 5×10^{-2}, resolution level 2, and other parameters set to default.

2.2. Simulation modes table

The study applies the Taguchi orthogonal array design method to determine the number of simulation samples needed. The three parameters of the brake disc (X1, X2, X3) are designed at 5 levels using the Taguchi OA25(5^3) orthogonal array, resulting in a total of 25 simulation analyses for different levels of the brake disc parameters (Table 1). The thickness levels of the brake disc are chosen based on a range that includes values larger and smaller than the actual thickness (25 mm), ensuring effective operation even under conditions where thickness may change due to wear or specific design requirements [18]. Similarly, when selecting the levels for the number of hole rows and grooves on the brake disc, increasing the number of holes and grooves can improve cooling capacity, but excessive numbers may affect the performance and durability of the brake disc [19].

Table 1. *Levels of brake disc parameters*

Level	X1	X2	X3
1	22	18	18
2	23	19	19
3	24	20	20
4	25	21	21
5	26	22	22

The temperature (Y1) and mass (Y2) parameters are selected for analysis and optimization of the brake disc structure due to their crucial roles in influencing durability and production costs. Temperature is a key factor directly affecting the thermal performance of the brake disc, as the heat generated from friction can lead to structural degradation and significantly reduce braking efficiency.

2.3. Boundary conditions and thermal analysis parameters

This study examines the case of a hypothetical vehicle with a total mass m = 1300 kg, traveling at a speed v_t = 90 km/h and braking to a stop at v_0 = 0 km/h. According to regulations, the minimum safe distance when a vehicle is moving at 90 km/h is 70 m. However, the study investigates conditions where the vehicle stops after a distance S = 45 m to increase the severity, aiding in the assessment of the reliability and safety of the braking system.

The braking acceleration *(a)* is determined by:

$$a = \frac{v_t^2 - v_0^2}{2S} = \frac{0 - 25^2}{2 \times 45} = -6.94 m/s^2 \quad (1)$$

The braking time (*t*) is calculated using:

$$t = \frac{-v_t}{a} = \frac{-25}{-6.94} = 3.61 s \quad (2)$$

Assuming the kinetic energy of the vehicle (E_k) is converted into heat, and 70% of the braking power is applied to the front axle of a four-wheeled vehicle. the heat flux (*q*) generated on each side of the disc is calculated as follows:

$$q = \frac{E_k \times 0.5 \times 0.7}{t \times A} \quad (3)$$

Where *A* is the area of the brake disc (m^2), determined from the 3D model of the brake disc as S=0.0896m^2. The kinetic energy (E_k) of the vehicle is given by:

$$E_k = \frac{m \times v^2}{2} = \frac{1300 \times 25^2}{2} = 406250 J \quad (4)$$

Hence, the heat flux is calculated as q=5.07×10^5 W/m^2.

The brake is cooled by convection, using the default convection coefficient for standard air, which is 5 W/m^2k and the initial temperature is 35°C. The total simulation time is 5 seconds, with an initial time step set to 0.01 seconds and a maximum time step set to 0.1 seconds.

2.4. Influence of design parameters on brake disc temperature and mass

To analyze the influence of design parameters on the temperature and mass of the brake disc, this study uses ANOVA to identify significant factors. ANOVA provides *p*-values and *F*-statistics to determine whether there are statistically significant differences between groups. If the p-value is less than the significance level (0.05), it can be concluded that the design factor significantly affects the temperature or mass of the brake disc.

The mathematical model for predicting parameters is constructed using RSM. This method is commonly used to model and analyze the relationships between one or more independent variables (input factors) and one or more dependent variables (output responses). RSM is often applied in optimization processes to find optimal operating conditions in manufacturing and research. The regression model in this study is of the form:

$$Y = \alpha_0 + \sum_{i=1}^{k} \alpha_i X_i + \sum_{i=1}^{k} \alpha_{ii} X_i^2 + \sum_{ij}^{k} \alpha_{ij} X_i X_j + \varepsilon \quad (5)$$

Where *Y* is the objective function corresponding to Y_1 and Y_2; X_i, X_j are the design parameters of the brake disc; α_i is the first-order regression coefficient.; α_{ij} is the interaction regression coefficient describing the simultaneous effect of two factors X_i and X_j; α_{ii} is the second-order regression coefficient describing the effect of factor X_i; α_0 is the intercept of the model and ε is the statistical error related to the mean value.

2.5. MOPSO algorithm

The PSO algorithm was introduced by J. Kennedy and R.C. Eberhart to solve nonlinear functions [20]. This algorithm is based on the social behavior of animals, such as flocks of birds or schools of fish. PSO begins by initializing a population of particles, each representing a potential solution in the search space. Each particle has a position and velocity represented by vectors X_i and V_i. The PSO model is illustrated in Figure 3, where bold lines describe the velocity and position of a particle after each iteration, and dotted lines describe the components of equations (6) and (7), P_i^{best} represents the personal best position of the particle, G_i^{best} represents the global best position of a specific particle in the search space [21]. Additionally, *X* and *Y* denote the horizontal and vertical directions of the search in the solution space, O is the theoretical optimal value to be found.

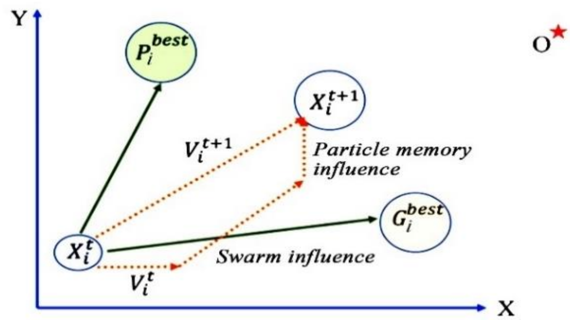

Figure 3. *PSO optimization model overview*

The velocity and position update formulas for particles are shown in equations (6) and (7):

$$\begin{aligned} V_i^{(t+1)} &= \omega \times V_i^{(t)} + C_1 \times r_1 \times (P_i^{best} - X_i^{(t)}) \\ &+ C_2 \times r_2 \times (G_i^{best} - X_i^{(t)}) \end{aligned} \quad (6)$$

$$X_i^{(t+1)} = X_i^{(t)} + V_i^{(t+1)} \quad (7)$$

Where: ω is the inertia weight, which helps adjust the influence of previous velocity; C_1 and C_2 are the personal and social coefficients, respectively; r_1 and r_2 are random numbers in the range [0, 1].

The MOPSO is an extension of PSO, designed to solve optimization problems with multiple objectives. MOPSO not only searches for a single optimal solution but also seeks a set of Pareto optimal solutions, each representing a balance between different objectives. Therefore, particles not only store their personal best position P_i^{best} but also maintain a repository of Pareto solutions. The update of velocity and position in MOPSO is similar to PSO but includes the addition of a leader particle selection algorithm from the Pareto repository. The velocity update equation in MOPSO is introduced in equation (8):

$$\begin{aligned} V_i^{(t+1)} &= \omega \times V_i^{(t)} + C_1 \times r_1 \times (P_i^{best} - X_i^{(t)}) \\ &+ C_2 \times r_2 \times (Leader - X_i^{(t)}) \end{aligned} \quad (8)$$

The *Leader* particle is the position of a particle selected from the Pareto repository, guiding the search process towards better optimal solutions. MOPSO effectively finds Pareto optimal solutions by leveraging swarm intelligence and managing conflicting objectives well. Due to its flexibility and ability to handle complex problems, MOPSO is considered a powerful tool in multi-objective optimization.

3. Research results and discussion

3.1. Thermal and mass analysis results of brake discs

The results of thermal analysis and mass calculation for 25 brake disc models are presented in Table 2.

Figure 4 presents the analysis results for model 14 (X1 = 23mm; X2 = 22; X3 = 18). The total number of nodes and elements are 1,482,850 and 877,215 (Figure 4-a). The simulation analysis results show that the maximum temperature is 158.54°C, primarily concentrated on the outer surface of the brake disc, where the highest heat is generated due to friction during braking (Figure 4-b).

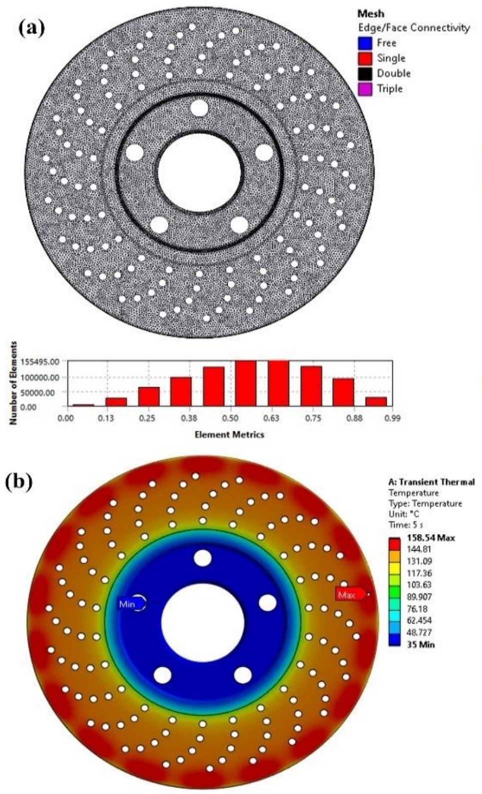

Figure 4. *Detailed simulation results of model number 10: (a) mesh model and (b) maximum temperature*

Table 2. *Brake disc parameters and corresponding thermal analysis results and mass*

No.	X1	X2	X3	Y1	Y2
1	22	18	18	164.02	7.78
2	22	19	19	164.21	7.73
3	22	20	20	164.19	7.68
4	22	21	21	164.36	7.64
5	22	22	22	164.65	7.59
6	23	18	19	158.54	8.06
7	23	19	20	158.61	8.01
8	23	20	21	158.70	7.96
9	23	21	22	158.83	7.92
10	23	22	18	158.54	7.98
11	24	18	20	153.65	8.31
12	24	19	21	153.82	8.29
13	24	20	22	153.89	8.24
14	24	21	18	153.65	8.43
15	24	22	19	153.72	8.26
16	25	18	21	149.72	8.62
17	25	19	22	149.87	8.57
18	25	20	18	149.62	8.64
19	25	21	19	149.70	8.59
20	25	22	20	149.70	8.54
21	26	18	22	146.61	8.89
22	26	19	18	146.35	8.97
23	26	20	19	146.39	8.92
24	26	21	20	146.41	8.87
25	26	22	21	146.46	8.82

3.2. ANOVA analysis

The ANOVA results for the brake disc temperature Y1 indicate a statistically significant model with a p-value < 0.0001, allowing for the assessment of the impact of design parameters (Table 3). The model has an $R^2 = 0.9939$, demonstrating high reliability of the analysis. Among the main factors, parameter X1 has a significant impact with a high F-value of 2049.32 and a significance level of $p < 0.0001$. However, parameters X2 and X3 show insignificant effects with p-values of 0.1656 and 0.6065, respectively. Additionally, interactions between parameters significantly affect Y1. These results indicate a complex and nonlinear relationship between design parameters affecting the brake disc temperature.

Table 3. *ANOVA Results for Y1*

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	1000.29	6	166.72	486.25	< 0.0001
X1	702.63	1	702.63	2049.32	< 0.0001
X2	0.7161	1	0.7161	2.09	0.1656
X3	0.0942	1	0.0942	0.2748	0.6065
X1×X2	2.61	1	2.61	7.61	0.0129
X1×X3	2.11	1	2.11	6.17	0.0231
X2×X3	5.08	1	5.08	14.82	0.0012
Residual	6.17	18	0.3429		
Total	1006.46	24			

Table 4. *ANOVA results for Y2*

Source	Sum of Squares	df	Mean Square	F-value	p-value
Model	4.64	6	0.7735	1096.44	< 0.0001
X1	3.48	1	3.48	4938.68	< 0.0001
X2	0.0176	1	0.0176	24.96	< 0.0001
X3	0.0311	1	0.0311	44.1	< 0.0001
X1×X2	0	1	0	0.0217	0.8845
X1×X3	9.8E-6	1	9.8E-6	0.0139	0.9075
X2×X3	0.0001	1	0.0001	0.1953	0.6638
Residual	0.0127	18	0.0007		
Total	4.65	24			

Table 4 also shows that the statistical model for Y2 is significant with a p-value < 0.0001 và $R^2 = 0.9973$. Among the three parameters surveyed, X1 has the highest impact with an F-value of 4938.68 and a significance level of $p < 0.0001$. Parameters X2 and X3 also show significant effects with F-values of 24.96 and 44.1, respectively. This indicates that all three parameters play an important role in determining the mass of the brake disc. However, interactions between the surveyed parameters show no significant effect as the p-values are greater than 0.05. Figure 5 presents contour plots illustrating the relationship of design parameters to the temperature and mass of the brake disc.

The mathematical models for predicting the parameters Y1 and Y2 are given by the following equations:

$$Y1=384.629 - 3.77891\times X1 - 0.851143 \times X2 - 12.5663\times X3 - 0.252816\times X1\times X2 + 0.227551 \times X1\times X3 + 0.352776\times X2\times X3 \quad (9)$$

$$Y2=0.787388 + 0.323722 \times X1 + 0.0299837\times X2 + 0.0199837\times X3 - 0.000612245 \times X1\times X2 - 0.000489796 \times X1\times X3 - 0.00183673\times X2\times X3 \quad (10)$$

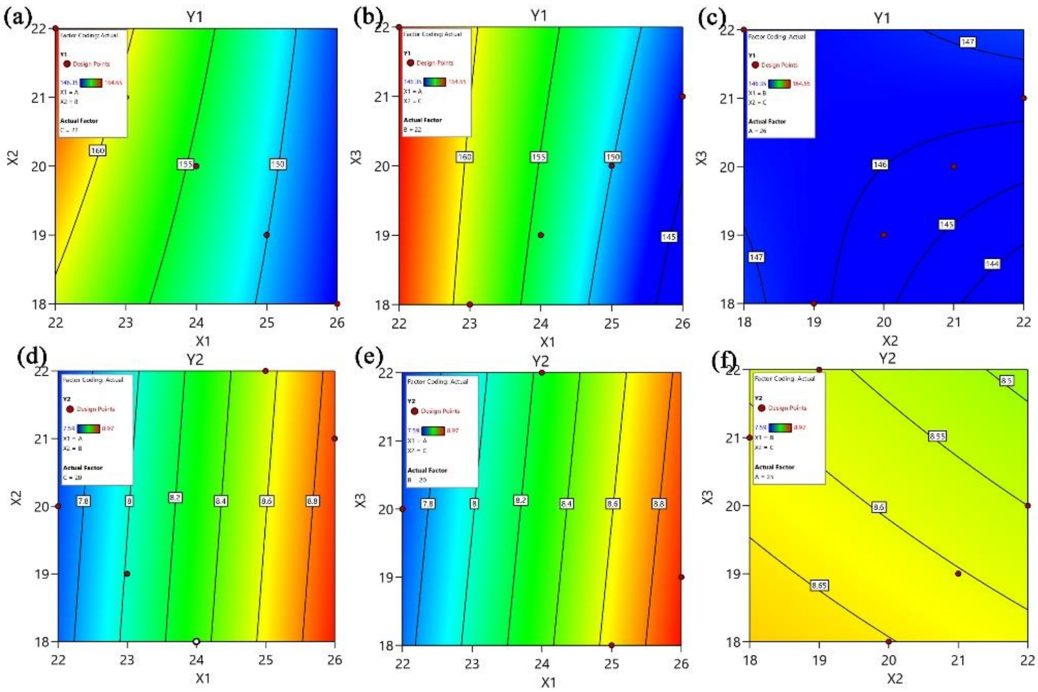

Figure 5. *Contour diagram depicting the influence of brake disc design parameters on Y1 (a, b, c) and Y2 (d, e, f)*

3.3. Optimization of brake disc temperature and mass

The MOPSO algorithm is applied with two objective functions, Y1 and Y2 along with boundary conditions for X1, X2, and X3. The algorithm begins by initializing a population of 100 particles with an archive size of 20. It employs a global search strategy with a maximum of 1000 generations. The inertia weight is initialized at 0.9 and gradually decreases to 0.3 to enhance exploitation in the later stages of the algorithm, when particles need to fine-tune their positions closer to optimal solutions. The personal coefficient is set at 1.5 to encourage individuals to seek better solutions based on their own experience. The social coefficient is also set at 1.5 to promote positive interactions among individuals, allowing the swarm to utilize information from the best individuals without completely relying on them. The maximum velocity is set at 15 to prevent particles from moving too quickly, which could cause them to miss good solutions. Additionally, the mutation rate is 0.3 to introduce diversity in the population, encouraging exploration of unexplored regions in the solution space.

After 1000 generations, the MOPSO algorithm yielded 20 optimal Pareto solutions, each representing a balance between the two objective functions Y1 and Y2 (Figure 6). These solutions demonstrate the algorithm's capability to find optimal solutions in a multi-objective space while satisfying the boundary conditions of the input variables X1, X2, and X3. The corresponding values of the two objective functions for each optimal solution are presented in Table 5.

To validate the Pareto optimal solutions obtained from the MOPSO method, five solutions from the Pareto dataset were used to redesign the brake disc and simulate under the same operating conditions. The simulation results in Table 6 show that the deviation between the optimal values from MOPSO and the simulation results is small, only under 1.5%. These results reinforce the conclusion that the MOPSO method is not only effective in finding optimal solutions but also provides reliable results for the structural optimization of brake discs. This opens up the potential for widespread application of MOPSO in research and engineering design related to complex mechanical systems.

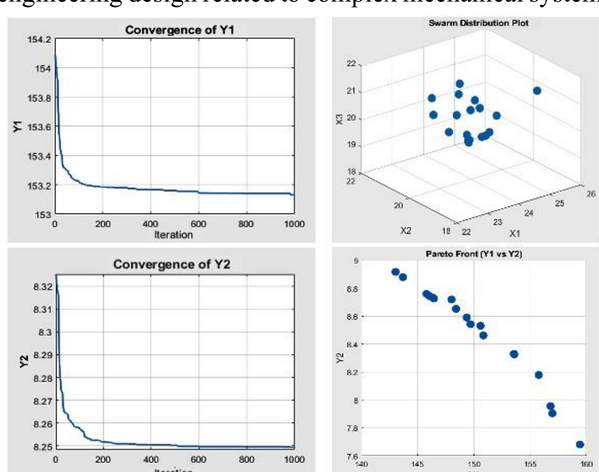

Figure 6. *Convergence, swarm distribution and Pareto plots of optimal solutions*

Table 5. Pareto optimal solutions

No.	X1	X2	X3	Y1	Y2
1	24.82	22	19	149.71	8.54
2	24.58	18	22	150.84	8.46
3	25.39	22	18	146.45	8.73
4	22.00	18	22	159.43	7.68
5	23.58	19	21	155.76	8.18
6	22.00	18	22	159.43	7.68
7	24.96	22	20	150.59	8.53
8	22.95	18	22	156.81	7.96
9	24.27	22	20	153.56	8.33
10	25.39	22	18	146.33	8.73
11	22.74	18	22	156.98	7.90
12	25.43	22	18	146.03	8.75
13	25.87	22	18	143.70	8.88
14	25.12	22	20	149.37	8.59
15	26.00	22	18	143.04	8.92
16	25.25	22	19	148.42	8.65
17	26.00	22	18	143.04	8.92
18	24.19	22	19	153.59	8.33
19	25.42	21	20	148.02	8.72
20	25.48	22	18	145.80	8.76

Table 6. Deviation between MOPSO optimal values and simulation

No.	Y1(Temperature)			Y2 (Mass)		
	MOPSO	Simulation	Deviation (%)	MOPSO	Simulation	Deviation (%)
1	149.71	150.43	0.48	8.54	8.51	0.35
5	155.76	155.73	0.02	8.18	8.16	0.25
10	146.33	148.29	1.32	8.73	8.7	0.34
15	143.04	144.32	0.89	8.92	8.89	0.34
20	145.8	147.99	1.48	8.76	8.74	0.23

4. Conclusion

This study shows that the three design parameters examined thickness, number of hole rows, and number of grooves in the brake disc all significantly affect mass. However, only the thickness of the brake disc is the main factor influencing temperature. The multi-objective particle swarm optimization algorithm identified 20 optimal Pareto solutions by balancing temperature and mass. Validation of five optimal options from the Pareto solutions showed deviations not exceeding 1.5%, confirming the reliability and feasibility of the proposed research method.

However, this study has some limitations. First, it only considers three design parameters and two performance parameters, which may not fully reflect other factors that can affect brake disc performance. Additionally, although the MOPSO method yielded good results, applying this method to other multi-objective optimization problems in braking systems requires further study or combination with other optimization methods. Finally, the simulation model used in the study includes some assumptions that may affect the accuracy of the results, such as not accounting for dynamics, material properties, mesh size, and simulation environment. Therefore, future research should expand the scope of the survey and consider using other optimization algorithms for better comparison and evaluation.

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