STUDY ON THE EFFECT OF BOND-SLIP MODELS AT THE STEEL-CONCRETE INTERFACE ON THE FLEXURAL CAPACITY OF REINFORCED CONCRETE BEAMS

NGHIÊN CỨU ẢNH HƯỞNG CỦA CÁC MÔ HÌNH LỰC BÁM DÍNH GIỮA BÊ TÔNG VÀ CỐT THÉP ĐẾN KHẢ NĂNG CHỊU UỐN CỦA DẦM BÊ TÔNG CỐT THÉP

Dinh Hoang Phan[1,2], Chinh Van Nguyen[1], Quang Hieu Bui[1]*

[1]The University of Danang - University of Science and Technology, Vietnam
[2]Tin An Construction Consulting Joint Stock Company, Vietnam

*Corresponding author: bqhieu@dut.udn.vn

(Received: April 04, 2025; Revised: May 28, 2025; Accepted: June 11, 2025)

DOI: 10.31130/ud-jst.2025.23(7).186

Abstract - This article investigates the effect of four bond-slip models at the steel-concrete interface including Eligehausen, CEB-FIP, Morris, Harajli on the ultimate bending capacity of reinforced concrete (RC) beam. A three-dimensional finite element model in Abaqus used nonlinear connector element CONN3D2 was constructed to model the bond-slip behavior between longitudinal reinforcements and concrete. The investigated results not only show a high agreement between the numerical and experimental results but also indicate the effect of bond-slip models on the flexural strength of RC beams. Within the four investigated models, Morris-model provides the highest bond strength leading to the highest flexural capacity of RC being highest among four investigated cases, consequently the ultimate bending capacity of beam used this model was also highest.

Key words - Bond-Slip modes; Non-linear Spring element; Reinforced concrete beams; Abaqus; Bending capacity.

Tóm tắt - Bài báo khảo sát sự ảnh hưởng của bốn mô hình lực bám dính giữa bê tông và cốt thép bao gồm mô hình Eligehausen, CEB-FIP, Morris, Harajli đến khả năng chịu uốn cực đại của dầm bê tông cốt thép. Mô hình phần tử hữu hạn ba chiều trong Abaqus được xây dựng với phần tử lò xo phi tuyến CONN3D2 được sử dụng để mô phỏng sự bám dính giữa cốt thép dọc và bê tông. Kết quả khảo sát cho thấy, khả năng chịu uốn cực đại của dầm bê tông cốt thép không những tương đồng với kết quả thí nghiệm, mà còn phản ánh được sự ảnh hưởng của các mô hình lực bám dính. Trong 4 môn hình được nghiên cứu, mô hình Morris có lực dính lớn nhất trong số các mô hình được khảo sát, từ đó khả năng chịu lực uốn của dầm bê tông cốt thép là lớn nhất.

Từ khóa - Mô hình lực bám dính; Phần tử lò xo phi tuyến; Dầm bê tông cốt thép; Abaqus; Khả năng chịu uốn.

1. Introduction

Reinforced concrete (RC) is a composite material consisting of concrete and steel reinforcement, in which the concrete primarily resists compressive stress while the steel reinforcement excels at resisting tensile stress. The performance of RC elements is highly dependent on the bond at the concrete and steel interface. Several studies have been conducted to investigate the factors affecting the bond strength between concrete and steel, including the compressive strength of concrete, the adhesive force of cement mortar, the surface roughness of the reinforcement, and the corrosion of steel bars in concrete. N. T. Hieu [1] clearly demonstrated the influence of concrete compressive strength on bond strength, showing that the bond strength is directly proportional to the compressive strength of concrete. N. N. Tan et al. [2] conducted the experiments to indicate that: (a) when the degree of reinforcement corrosion is in the range of 0–2%, the bond stress increases compared to non-corroded reinforcement; (b) when the average degree of corrosion is about 6.5%, the bond stress can decrease by up to 30%; (c) and when the corrosion level exceeds 8.4%, the bond stress significantly decreases by 37.5–62%.

Experimental studies to determine the relationship between bond strength and slip between concrete and steel reinforcement have attracted many researchers. Recently, a comprehensive review of bond-slip models between concrete and reinforcement was conducted by Y. Zheng et al. [3]. In this study, the authors summarized previous research on bond-slip models, evaluated the advantages and disadvantages of these models through numerical simulation, identified the limitations of existing studies, and provided reliable guidance for future research. In general, two common methods are used to determine the bond-slip relationship: the pull-out test and the beam bending test. For example, the pull-out test was used in the experiments of R. Eligehausen et al. [4], CEB-FIP [5], and G. Morris et al. [6], while M. Harajli et al. [7] used the beam bending test.

On the other hand, numerical simulations, especially finite element analysis, have also been widely used to assess the impact of bond strength between concrete and steel reinforcement on the load-carrying capacity of RC members. N. V. Chinh et al. [8] used Abaqus software to evaluate the reduction in flexural capacity of RC beams subjected to four-point bending due to corrosion effects. H. Tang et al. [9] presented five finite element models using Abaqus to predict the load-bearing capacity of beams, including two un-strengthened beams and three strengthened but corroded beams that had been previously designed and tested by the authors. The accuracy of those FEM models was concluded to closely match the experimental data. In these above studies, the authors considered the effect of reinforcement corrosion on the bond-slip relationship between steel and concrete, but only the bond model from the CEB-FIP Model Code [5] was adopted for investigation.

This paper presents an overview of bond-slip models between concrete and reinforcement, with a detailed study of four models: Eligehausen, CEB-FIP, Morris, and Harajli. Furthermore, a numerical investigation of the influence of these models on the four-point flexural capacity of RC beams is conducted using finite element analysis in Abaqus. In particular, the nonlinear connector element CONN3D2 is presented in detail to simulate the nonlinear characteristics of the bond-slip relationship between longitudinal reinforcement and concrete. The results show that the ultimate flexural capacity of RC beams from the models agrees well with experimental tests and depends on the bond-slip models. Morris model shows the highest bond strength, resulting in the highest ultimate load-carrying capacity of RC beams.

2. Overview of bond-slip models between concrete and reinforcement

Several models describing the relationship between bond strength and slip at concrete and steel interface have been proposed based on experimental results. Among these, the pull-out test is used to evaluate the bond strength by pulling the steel bar out of the concrete, while the beam test assesses the bond strength under actual flexural and shear conditions commonly found in RC structures. Both methods have their own advantages and disadvantages, and researchers have developed various bond-slip models based on these methods to more accurately describe the degradation process of the bond between steel and concrete. The pull-out test has been adopted by researchers such as R. Eligehausen et al. [4], G. Morris et al. [6], X. Liang [10], Y. Xu [11], and G. M. Verderame et al. [12], whereas the beam test was conducted by M. Harajli et al. [7]. Recently, Y. Zheng et al. [3] performed a comprehensive review of models describing the bond-slip relationship between concrete and reinforcement.

Among these models, four bond-slip models- Eligehausen [4], CEB-FIP [5], Harajli [7], and Morris [6]- are presented in detail in the following subsections. Eligehausen et al. [4] began studying the bond-slip model between concrete and reinforcement in the 1980s. Their objective was to develop a simulation model for the bond behavior of ribbed steel bars anchored in concrete under pull-out loading conditions. In the 1990s, based on the results of Eligehausen et al. [4], the bond-slip model was included in the CEB-FIP Model Code [5], with formulas and parameters provided in detail for practical application. However, the model by G. Morris et al. [6] showed higher bond strength values than those of the Eligehausen [4] and CEB-FIP [5] models, due to differences in experimental data. Three above bond-slip models are based on pull-out test results whereas Harajli et al. [7] proposed a bond-slip model using beam bending tests. Their results clearly indicate that the bond stress between concrete and reinforcement from Harajli models is significantly lower than that of Eligehausen et al. [4], CEB-FIP [5], and Morris [6] models.

2.1. Eligehausen model [4]

Eligehausen model presents the relationship between bond stress and slip at the concrete and steel interface in Equation (1):

$$\tau = \tau_{max} \cdot \left(\frac{s}{s_1}\right)^{\alpha} \quad \text{when} \quad 0 \le s \le s_1 \quad (1a)$$

$$\tau = \tau_{max} \quad \text{when} \quad s_1 < s \le s_2 \quad (1b)$$

$$\tau = \tau_{max} - (\tau_{max} - \tau_f).\left(\frac{s - s_2}{s_3 - s_2}\right) \quad \text{when} \quad s_2 < s \le s_3 \quad (1c)$$

$$\tau = \tau_f \quad \text{when} \quad s_3 < s \quad (1d)$$

The parameters for determining the relationship between bond stress and slip are shown in Table 1.

Table 1. *Parameters of Eligehausen model*

s_1 (mm)	s_2 (mm)	s_3 (mm)	α	τ_{max} (MPa)	τ_f (MPa)
1	3	10	0.4	Eq. (2)	Eq. (3)

The maximum bond stress in this model is given in Equation (2):

$$\tau_{max} = \frac{\tau_{max(f_c'=30MPa)}}{\sqrt{\frac{30}{f_c'}}} \quad (2)$$

Where:

$\tau_{max(f_c'=30MPa)} = 13.5(MPa)$ is the maximum bond stress corresponding to concrete with a compressive strength of f_c' = 30 MPa.

The residual bond stress in this model is given in Equation (3):

$$\tau_f = \frac{\tau_{f(f_c'=30MPa)}}{\sqrt{\frac{30}{f_c'}}} \quad (3)$$

$\tau_{f(f_c'=30MPa)} = 5(MPa)$ is the residual bond stress corresponding to concrete with a compressive strength of f_c' = 30 MPa

2.2. CEB-FIP model [5]

CEB-FIP model shows the relationship between bond stress and slip at the concrete and steel interface in Equation (4):

$$\tau = \tau_{max} \cdot \left(\frac{s}{s_1}\right)^{\alpha} \quad \text{when} \quad 0 \le s \le s_1 \quad (4a)$$

$$\tau = \tau_{max} \quad \text{when} \quad s_1 < s \le s_2 \quad (4b)$$

$$\tau = \tau_{max} - (\tau_{max} - \tau_f).\left(\frac{s - s_2}{s_3 - s_2}\right) \quad \text{when } s_2 < s \le s_3 \text{ (4c)}$$

$$\tau = \tau_f \quad \text{when } s_3 < s \text{ (4d)}$$

The parameters for determining the relationship between bond stress and slip are shown in Table 2.

Table 2. *Parameters of CEB-FIP model*

s_1 (mm)	s_2 (mm)	s_3 (mm)	α	τ_{max} (MPa)	τ_f (MPa)
1	3	10	0.4	Eq. (5)	Eq. (6)

The maximum bond stress in this model is given in Equation (5):

$$\tau_{max} = 2.5\sqrt{f_c^{'}} \quad (5)$$

The residual bond stress in this model is given in Equation (6):

$$\tau_f = 0.4\tau_{max} \quad (6)$$

2.3. Morris model [6]

Morris model presents the relationship between bond stress and slip at the concrete and steel interface in Equation (7):

$$\tau = \tau_{max} \cdot \left(\frac{s}{s_1}\right)^{\alpha} \quad \text{when } 0 \le s \le s_1 \text{ (7a)}$$

$$\tau = \tau_{max} \quad \text{when } s_1 \le s \le s_2 \text{ (7b)}$$

$$\tau = \tau_{max} - (\tau_{max} - \tau_f).\left(\frac{s - s_2}{s_3 - s_2}\right) \quad \text{when } s_2 \le s \le s_3 \text{ (7c)}$$

$$\tau = \tau_f \quad \text{when } s_3 < s \text{ (7d)}$$

Table 3. *Parameters of Morris model*

s_1 (mm)	s_2 (mm)	s_3 (mm)	α	τ_{max} (MPa)	τ_f (MPa)
1	3	10	0.4	Eq. (8)	Eq. (9)

The maximum bond stress in this model is given in Equation (8):

$$\tau_{max} = 3.1\sqrt{f_c^{'}} \quad (8)$$

The residual bond stress in this model is given in Equation (9):

$$\tau_f = 0.4\tau_{max} \quad (9)$$

2.4. Harajli model [7]

Harajli model presents the relationship between bond stress and slip at the concrete and steel interface in Equation (10):

$$\tau = \tau_{cr} \cdot \left(\frac{s}{s_{1h}}\right)^{0,3} \quad \text{when } 0 \le s \le s_{1h} \text{ (10a)}$$

$$\tau = \tau_{max} + (\tau_{max} - \tau_{cr})\frac{(s - s_{2h})}{(s_{2h} - s_{1h})} \quad \text{when } s_{1h} < s \le s_{2h} \text{ (10b)}$$

$$\tau = \beta\tau_{max}\left(\frac{s}{s_{2h}}\right)^{-0.5} \quad \text{when } s_{2h} < s \le s_3 \text{ (10c)}$$

The parameters for determining the relationship between bond stress and slip:

Table 4. *Parameters of Harajli model*

s_{1h} (mm)	s_{2h} (mm)	s_3 (mm)	τ_{max} (MPa)	τ_{cr} (MPa)	V_f %	β
0.025	0.5	10	4.35	3.04	0	0.7

The maximum bond stress in this model is given in Equation (11):

$$\tau_{max} = 0.75\sqrt{f_c^{'}}\left(\frac{c}{d}\right)^{\frac{2}{3}} \quad (11)$$

Where:

c: Width of the concrete cover.

d: Diameter of the reinforcing bar.

The bond stress at the onset of cracking in this model is given in Equation (12):

$$\tau_{cr} = 0.7\tau_{max} \quad (12)$$

The slip value s_{1h} at position τ_{cr} is given in Equation (13). The formula for s_{1h} is derived based on the bond stress-slip curve:

$$s_{1h} = 1.5c_0 e^{\left(\frac{1}{0.3}\right)\ln\left(\frac{\tau_{cr}}{2.57\sqrt{f_c^{'}}}\right)} \quad (13)$$

The slip value s_{2h} at position τ_{max} is given in Equation (14):

$$s_{2h} = 0.15c_0 e^{1.8\left[\left(\frac{\tau_{max}}{2.57\sqrt{f_c^{'}}}\right)^2 - 1\right]} \quad (14)$$

3. Finite element model of RC beam under four-point bending

Abaqus software [13] was used to numerically simulate the influence of bond-slip models between concrete and reinforcement on the flexural capacity of beams under the four-point bending test. Figures 2, 3, and 4 show the details of the three-dimensional finite element model used to simulate the RC beam. In this model, concrete is modeled using eight-node C3D8R elements with the Concrete Damaged Plasticity Material Model (CDPM), while longitudinal reinforcement and stirrups are simulated using T3D2 truss elements with a bilinear plasticity model. The details of this simulation in Abaqus CAE are implemented in Models/ Materials/ Concrete Damaged Plasticity and Models/ Materials/ Plastic, respectively. The supports and loading plates are simulated with rigid R3D4 elements and interact with the RC beam through a sliding friction contact model with a coefficient of friction of 0.1. Mesh sensitivity analysis is performed to select the mesh sizes for each element type as follows: concrete is 20 mm, longitudinal reinforcement and stirrups are 50 mm.

To investigate the bond-slip models between concrete and reinforcement in Section 2, nonlinear node-to-node connector elements (CONN3D2) are used. The virtual reinforcement bars were placed at the same location as the longitudinal reinforcement. It should be noted that the stiffnesses of these dummy bars is selected not to affect the load-bearing capacity of the beam and to avoid numerical errors. The EMBED constraint in Abaqus is used to link these virtual bars to the concrete. Next, the nodes of the elements simulating the longitudinal reinforcement are connected to the nodes of the virtual reinforcement bars using CONN3D2 elements. Note that the axial stiffness of the CONN3D2 elements is assigned according to the bond-slip models investigated in Section 2, while the stiffness in the other directions is set to be much higher than that of the adjacent elements [8, 9]. The details of this simulation process in Abaqus CAE are performed in the following steps:

- Step 1: Define the nonlinear properties of the CONN3D2 elements in Models/Connector Sections.
- Step 2: Define the CONN3D2 elements using the Wire feature in Models/Assembly.
- Step 3: Assign the properties from Step 1 to the elements in Step 2 in Models/Assembly/Connector Assignments.

The loading method used in this paper is displacement control. The displacement of the loading plates gradually increased from zero to the target displacement. In addition, a nonlinear static analysis algorithm is used. Specifically, Figure 2 shows the concrete elements, Figure 3 shows the reinforcement elements, and Figure 4 shows the CONN3D2 elements.

3.1. Simulation of RC beam

The four-point bending test beams conducted by N. V. Chinh et al. [8] is selected as a numerical example in this article. The geometric parameters of the RC beam are described in the figures below.

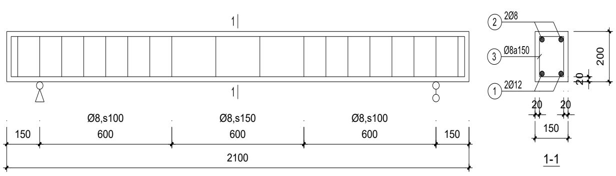

Figure 1. *Details of RC beams [8]*

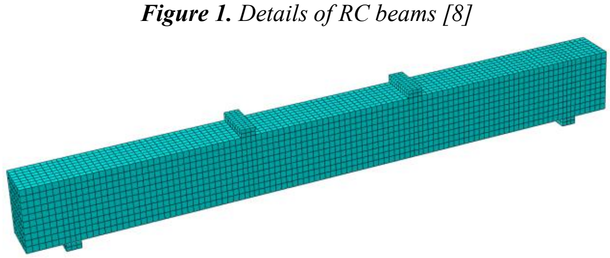

Figure 2. *Details of beam elements in Abaqus*

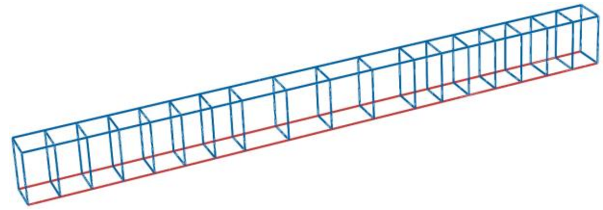

Figure 3. *Details of longitudinal and stirrup elements in Abaqus*

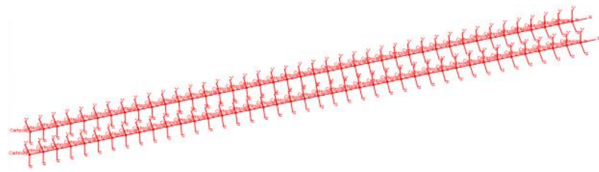

Figure 4. *Details of CONN3D2 elements in Abaqus*

3.2. Material parameters

3.2.1. Concrete

The parameters of concrete used in the model are shown below:

Table 5. *Parameters of concrete*

f_c' (MPa)	f_t (MPa)	E (MPa)	$\frac{f_{b0}}{f_{c0}}$	K	Dilation (°)
17	1.15	32500	1.16	0.6667	35

3.2.2. Reinforcement

The parameters of reinforcement used in the model are shown below:

Table 6. *Parameters of reinforcement*

Reinforcement	E_s (MPa)	f_y (MPa)	f_u (MPa)	ε_u
$\phi 8$	210000	205.40	383.50	0.3034
$\phi 12$	210000	383.60	545.80	0.1782

4. Survey results and discussion

4.1. Bond-slip models

Based on the properties of concrete and reinforcement described in Section 3, four bond-slip models were calculated and are shown in Figure 5. Figure 6 presents the bond-slip relationship in the slip range from 0 to 0.03 which is not clearly shown in Figure 5.

Figure 5 shows that the maximum bond stress in the Morris model [6] provides the highest bond strength (maximum bond stress), with $\tau_{\max} = 12.8$ MPa. The difference in maximum stress between the Eligehausen [4] and CEB-FIP [5] models is not significant as these maximum bond stresses were 10.16 MPa and 10.3 MPa, respectively. Notably, for the beam test, the bond-slip model of Harajli [7] is completely different from the other three models of pull-out tests. The maximum bond stress of Harajli model is 4.35 MPa which is about 34% of the $\tau_{\max}$ from Morris model [6].

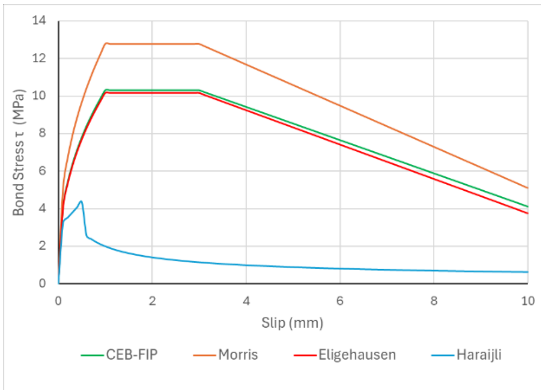

Figure 5. *Bond stress-slip curves of concrete and reinforcement*

Based on the results in Figures 5 and 6, for pull-out tests, the values from the Morris model [6] are the highest, followed by the CEB-FIP [5] model, and finally Eligehausen [4]. For the Harajli [7] model, the values are the lowest, as this model is based on beam bending tests, which are fundamentally different from the other three models, leading to a different curve shape. When the bond stress between concrete and reinforcement reaches its limit in the Harajli [7] model, the bar will slip out of the concrete.

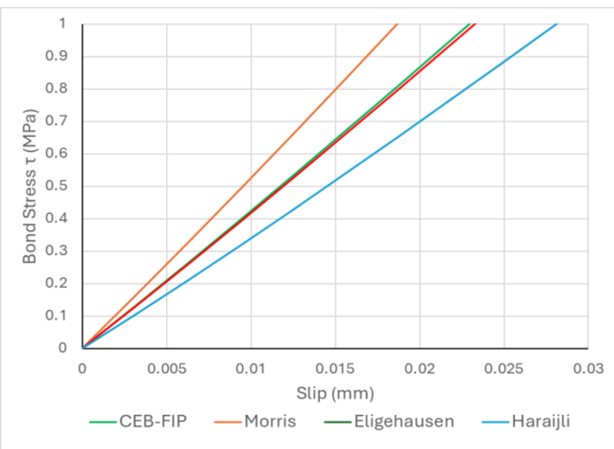

Figure 6. *Bond stress-slip curves of concrete and reinforcement when slip value varies from 0 to 0.03*

These models will be used to define the nonlinear properties of the CONN3D2 elements along the longitudinal direction of the main reinforcement to simulate the bond between concrete and steel. The nonlinearity of the CONN3D2 element is expressed in the relationship between force and slip, so the bond stress must be converted to force as in Equation (15):

$$F_i = A_i \times \tau \qquad (15)$$

Where:

τ : Bond stress value from the models.

A_i : Nominal area of the CONN3D2 element, determined by Equation (16):

$$A_i = 2\pi r l \qquad (16)$$

Where:

r: Radius of the reinforcement bar.

l: Distance between spring nodes, which is l = 50 mm in this paper.

The values calculated using the above formulas will be assigned to the CONN3D2 elements.

4.2. Load–displacement relationship of RC beams with different bond-slip models

The relationship between mid-span displacement and applied load from the finite element model is shown in Figure 7, compared with the experimental results of N. V. Chinh et al. [8]. It can be seen that the load - displacement relationships obtained from four finite element models corresponding to four investigated bond-slip models are quite similar and do not differ much from the experimental results. This can be explained by the fact that under normal working conditions, the bond strength between concrete and reinforcement is very high, resulting in almost no slip between concrete and steel. The maximum slips between concrete and reinforcement of four finite element models corresponding to the four bond-slip models are shown in Table 7.

Table 7. *Maximum slip distance*

Model	Morris	CEB-FIP	Eligehausen	Harajli
Maximum slip (mm)	0.0685	0.0836	0.0832	0.1049

To clearly see the influence of the bond-slip models on the flexural capacity of RC beams, Figure 9 magnifies the load–displacement curves at mid-span. It can be seen that the differences between the models are quite small. However, even though this difference is small, it still indicates that the bond stress affects the flexural capacity of RC beams. The results from the simulation models and the experimental model by N. V. Chinh et al. [8] are shown in Figures 7 and 8, indicating that the shape of the load–displacement curve from the experiment and the simulation are similar. According to the four simulation models implemented in Abaqus, the ultimate load values for the Morris [6] model are the highest at 67.017 kN, followed by CEB-FIP [5] at 66.914 kN, Harajli [7] at 66.871 kN, and finally Eligehausen [4] at 66.866 kN (see Table 8).

Table 8. *Ultimate load P_{max} (KN)*

Model	Experimental result [8]	Morris	CEB-FIP	Harajli	Eligehausen
P_{max} (KN)	67.51	67.017	66.914	66.871	66.866

Although the maximum bond stress (τ_{max}) of the Harajli [7] model is only 43% of the maximum bond stress of the Eligehausen [4] model, the difference in the maximum flexural load of the RC beam corresponding to two bond-slip models is not significant, only about 0.75%. Similarly, the flexural load obtained from Morris [6] and CEB-FIP [5] models are also like Harajli and Eligehausen models. For example, compared with the Eligehausen [4] model, the ultimate flexible load is about 0.23% and 0.07% for the Morris [6] and CEB-FIP [5] models, respectively. This is attributed to the insignificant influence of bond slip models on the ultimate flexural capacity of RC beams under normal working conditions.

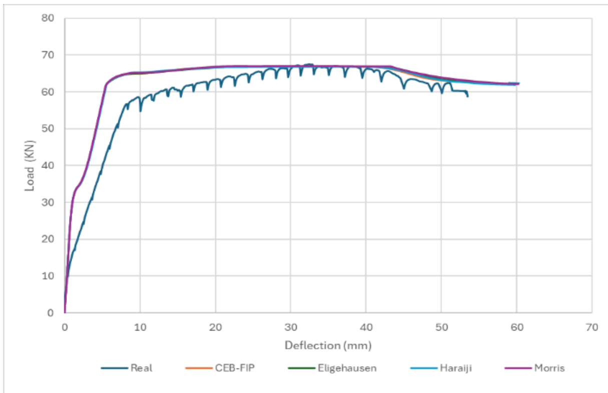

Figure 7. *Load deflection curves of RC beams with investigated bond-slip models*

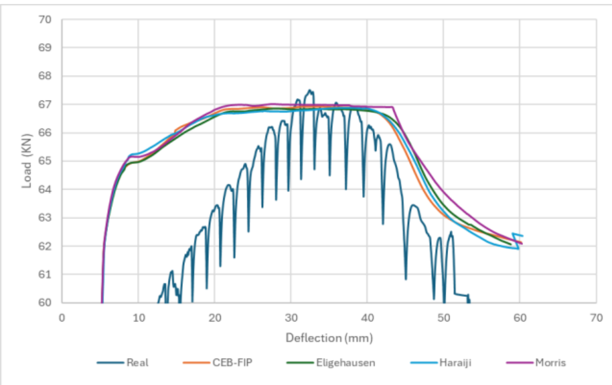

Figure 8. *Load deflection curves of RC beams with investigated bond-slip models when load varies from 60kN to 70kN*

5. Conclusion

This paper has provided an overview of four bond-slip models between reinforcement and concrete, including the Eligehausen, CEB-FIP, Morris, and Harajli models. In addition, a three-dimensional finite element model was developed using these bond-slip models to evaluate their influence on the flexural capacity of RC beams. The conclusions of this paper are listed as follows:

- The highest and lowest bond strength (maximum bond stress) are obtained from Morris model (based on pull-out test) and Harajli model (based on beam test), respectively.

- Under normal working conditions, the bond strength between concrete and reinforcement is sufficiently high so as not to affect the flexural capacity of RC beams. The load – displacement relationship of RC beams, as investigated using the finite element model corresponding to the four bond-slip models, is almost identical to the experimental results.

- The flexural capacity of RC beams with the Morris bond-slip model is only slightly higher than that of the Harajli bond-slip model.

Data statement

The Python script for defining the properties of the CONN3D2 element in this paper is available from the corresponding author upon reasonable request.

REFERENCES

[1] N. T. Hieu, "Effect of concrete compressive strength on steel-concrete bond", *Journal of Structural Engineering and Construction Technology,* vol. 18, pp. 66-71, 2015.

[2] N. N. Tan, T. A. Dung, N. C. The, T. B. Tuan, and L. T. Anh, "An experimental study on identify the influence of reinforcement corrosion on steel-concrete bond stress", *Journal of Science and Technology in Civil Engineering-HUCE,* vol. 12, no. 6, pp. 29-38, 2018.

[3] Y. Zheng, C. Fan, J. Ma, and S. Wang, "Review of research on Bond–Slip of reinforced concrete structures", *Construction and Building Materials,* vol. 385, pp. 131437, 2023, doi: https://doi.org/10.1016/j.conbuildmat.2023.131437.

[4] R. Eligehausen, V. Ciampi, V. V. Bertero, and E. P. Popov, *Analytical model for concrete anchorages of reinforcing bars under generalized excitations*. College of Engineering, University of California Berkeley, CA, USA, Science Report No. UCB/EERC-82/83, November 1982.

[5] *CEB-FIP model code 1990: Design code*, Thomas Telford Publishing, 1993.

[6] G. Morris, D. Bull, and B. Bradley, "Monotonic and cyclic bond behaviour of deformed bars in reinforced concrete structures", in Proc. of. NZSEE, pp. 761-770, 2015.

[7] M. Harajli, B. Hamad, and K. Karam, "Bond-slip response of reinforcing bars embedded in plain and fiber concrete", *Journal of materials in civil engineering,* vol. 14, no. 6, pp. 503-511, 2002.

[8] V. C. Nguyen., Q. H. Bui, and P. Lambert, "Experimental and numerical evaluation of the structural performance of corroded reinforced concrete beams under different corrosion schemes", *Structures,* vol. 45, pp. 2318-2331, 2022/11/01/ 2022, doi: https://doi.org/10.1016/j.istruc.2022.10.043.

[9] H. Tang, J. Peng, L. Xiao, X. Liu, and J. Zhang, "Numerical Simulation of Corroded Reinforced Concrete Beam Strengthened by a Steel Plate with Different Strengthening Schemes", *Advances in Civil Engineering,* vol. 2020, no. 1, pp. 4236943, 2020, doi: https://doi.org/10.1155/2020/4236943.

[10] X. Liang, "*An investigation of bond-slip behavior of reinforcing steel subjected to inelastic strains*", Master Thesis, Dept. Civil Eng., Iowa State University, Ames, Iowa, 2013.

[11] Y. Xu, "A simplified model of reinforced concrete stick-slip instantonal relationship", *Engineering Mechanics A,* vol. 2, pp. 34-38, 1997.

[12] G. M. Verderame, G. De Carlo, P. Ricci, and G. Fabbrocino, "Cyclic bond behaviour of plain bars. Part II: Analytical investigation", *Construction and building Materials,* vol. 23, no. 12, pp. 3512-3522, 2009.

[13] D. Systèmes, "*Abaqus analysis user's guide (6.14)*", Dassault Systèmes: Vélizy-Villacoublay, France, 2014.